LUND INSTITUTE
OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

Nonlinear Control and Servo Systems (FRTN05)

Exam - March 9, 2011 at 14.00–19.00

Points and grades

All answers must include a clear motivation. The total number of points is 25. The maximum number of points is specified for each subproblem. Most subproblems can be solved independently of each other. *Preliminary* grades:

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3: $12 - 16.5$ points

4: $17 - 21.5$ points

5: $22 - 25$ points

Accepted aid

Either one of the textbooks by Glad/Ljung, Slotine/Li or Khalil is allowed. Copies of lecture notes and laboratory PMs will be available at the exam. Standard mathematical tables and “Formelsamling i reglerteknik”/”Collection of Formulae” are also allowed. Pocket calculator.

Results

The exam results will be posted on March 22, by the latest, on the noticeboard at the Department of Automatic Control and on the course homepage `http://www.control.lth.se/course/FRTN05`. The corrected exams will be displayed March 22, 12.30-13.00 in the conference room next to lab C on the first floor of the M-building.

Note!
In many cases the sub-problems can be solved independently of each other.

Good Luck!

1. Figure 1 shows four different phase portraits (A-D).

a. Each phase portrait corresponds to one of the systems (i)-(iv) below. Which phase portrait belongs to what system?

(i) $\dot{x}_1 = x_2$
$\dot{x}_2 = -x_1 + x_2(1 - 3x_1^2 - 2x_2^2)$

(ii) $\dot{x}_1 = x_2$
$\dot{x}_2 = -8x_1 - 2x_2 - x_1^4$

(iii) $\dot{x}_1 = -x_2$
$\dot{x}_2 = x_1$

(iv) $\dot{x}_1 = -\sin x_2$
$\dot{x}_2 = x_1$

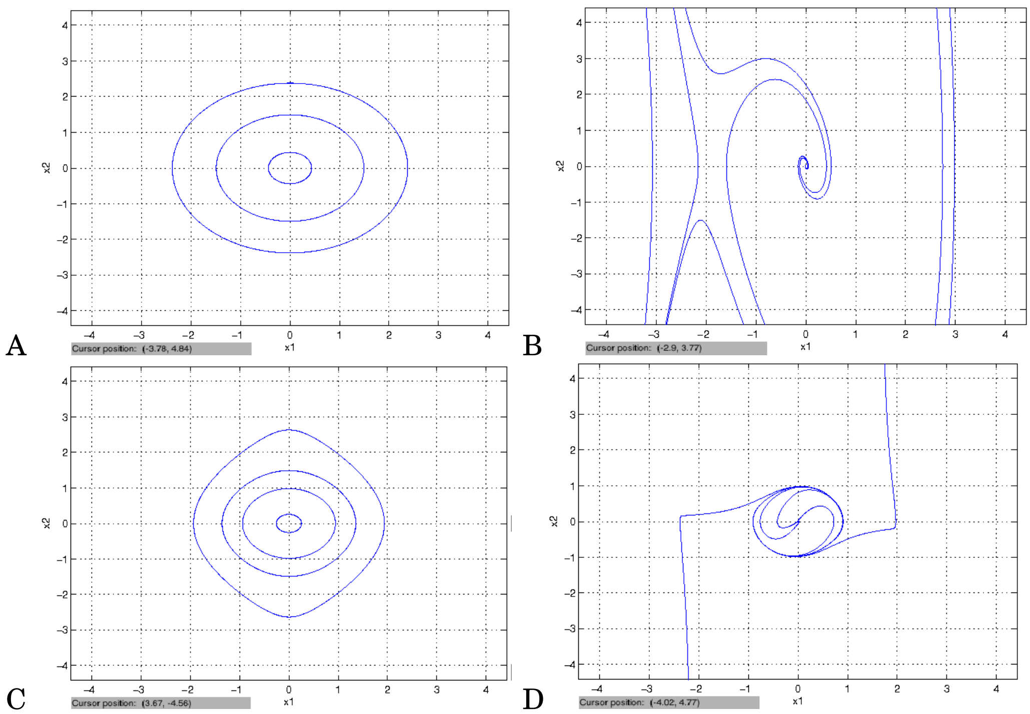

Figure 1 The phase portraits in problem 1

(2 p)

b. Mark the direction of the solutions in the four phase portraits in figure 1.
(2 p)

2. Consider the nonlinearity $f(\cdot)$ shown in figure 2.

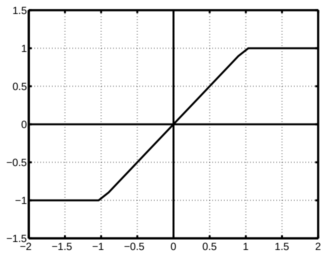

Figure 2 The nonlinearity in problem 2.

a. Where do such nonlinearities appear in practice? (0.5 p)

b. Determine the minimal sector $[k_1, k_2]$ such that $k_1 \leq \frac{f(y)}{y} \leq k_2$ for $y \neq 0$. (1 p)

c. The Nyquist curve of an open loop stable process $P(s)$ is shown in figure 3. Consider the feedback connection of this system and the static nonlinearity in figure 2, as shown in figure 4. Can the closed loop system be proved BIBO stable using the circle criterion? Is the closed loop locally stable when $r = 0$? (1.5 p)

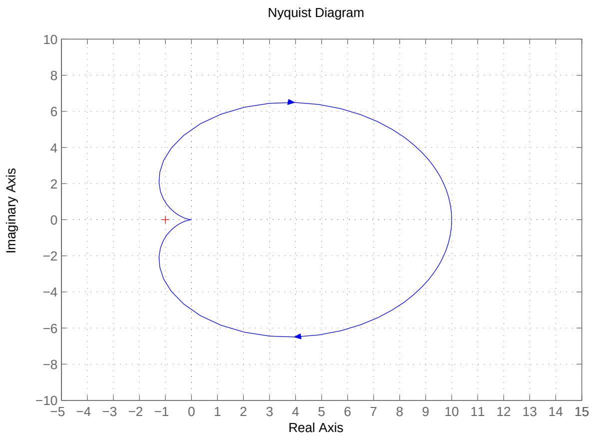

Figure 3 The Nyquist-curve in problem 2.

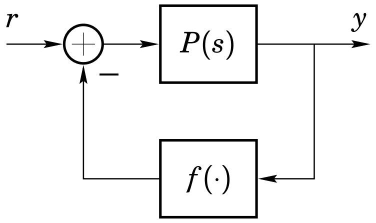

Figure 4 Feedback system in problem 2.

3. Consider a linear system under relay feedback as shown in figure 5, where the relay amplitude is 1 and $G(s) = \frac{50}{s(s+1)(s+10)}$.

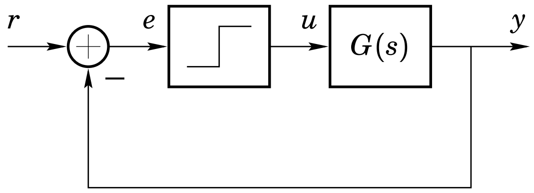

Figure 5 The system in problem 3.

a. Use the describing function method to predict whether the system exhibits a limit cycle. If a limit cycle is predicted, what is the amplitude and frequency of this limit cycle? (3 p)

b. The Ziegler-Nichols frequency response method uses the gain and period time of a limit cycle to compute PID controller parameters. The method is described in the last sheet of the "Collection of Formulae". How can the setup in **a.** be used to estimate K_0 and T_0 from experiments?
(No calculations needed.) (1 p)

4. An inverted pendulum can be described by $\ddot{\theta} = \omega_0^2 \sin\theta - \frac{\omega_0^2}{g} u \cos\theta$,
where ω_0 is the natural frequency of the pendulum, θ the deviation angle from the top position ($\theta = 0$) and u the control signal.

a. Introduce states and write the system on state-space form. (1 p)

b. The total energy of the pendulum is given by

$$E(\theta, \dot{\theta}) = \cos\theta - 1 + \frac{1}{2\omega_0^2}\dot{\theta}^2$$

Design a controller to swing up the pendulum to the top position. (3 p)

5. A sliding mode controller is to be designed to stabilize the origin for the following nonlinear system.

$$\begin{aligned}\dot{x}_1 &= -x_1^2 + x_2 + u \\ \dot{x}_2 &= x_1 + x_2\end{aligned}$$

Choosing a suitable sliding set σ is a crucial part of the control design. In this exercise, three different sets, σ_1, σ_2 and σ_3 are suggested. Calculate the equivalent control, u_{eq}, and the sliding mode dynamics for each of the suggested switching sets and decide which sliding set is the best to use. Is the equivalent control well-defined for all sliding sets? For the specific sliding set you choose, write the control law for the complete sliding mode controller.

$$\begin{aligned}\sigma_1(x) &= x_1 - x_2 \\ \sigma_2(x) &= x_1 + 4x_2 \\ \sigma_3(x) &= x_1^2 - x_2\end{aligned}$$

(4 p)

6. Consider the problem

$$\min \frac{1}{2} \int_0^T u(t)^2 dt$$

when

$$\begin{cases} m\ddot{x}(t) + kx(t) = u(t) \\ x(0) = 0, \quad \dot{x}(0) = 0 \\ x(T) = a, \quad \dot{x}(T) = 0 \end{cases}$$

$m = 1$, $k = 2$ and $a = 3$. Verify that the optimal input must have the form

$$\hat{u}(t) = A\cos(\omega t) + B\sin(\omega t)$$

Determine ω. (3 p)

7. Find a stabilizing control law for the system below. Note that there are two independent inputs, u_1 and u_2.

$$\begin{aligned} \dot{x}_1 &= x_1^2(x_1 + 2x_2) \\ \dot{x}_2 &= x_2^2 + u_1 \\ \dot{x}_3 &= -x_3 + 2x_4 \\ \dot{x}_4 &= x_4 + u_2 \end{aligned}$$

(3 p)