Nonlinear Control and Servo Systems

Lecture 7

- Compensation for saturations (anti-windup)
- Friction models
- Friction compensation

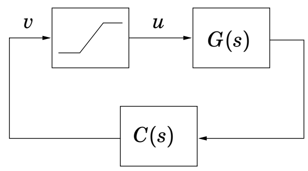
Today's Goal

- To be able to design and analyze antiwindup schemes for
  - PID,
  - state-space systems,
  - and Kalman filters (observers).
  - (polynomial designs (RST-controllers)),
- To understand common models of friction.
- To design and analyze friction compensation schemes.

Material

- Lecture slides
- Handout from CCS book, pp. 310-313, 331-336
- Handout from PhD-thesis by Henrik Olsson, 1996

Windup – The Problem



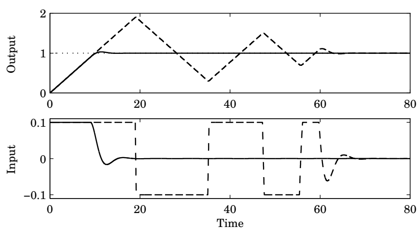
The feedback path is broken when u saturates

The controller $C(s)$ is a dynamic system

Problems when controller is unstable (or stable but not AS)

Example: I-part in PID-controller

Example-Windup in PID Controller

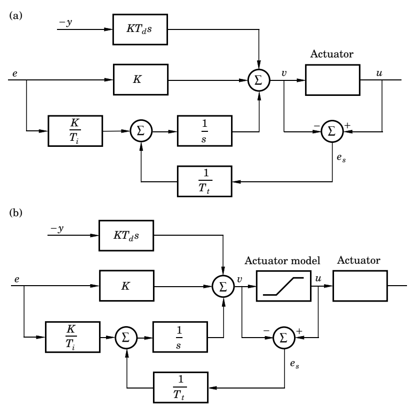


Dashed line: ordinary PID-controller

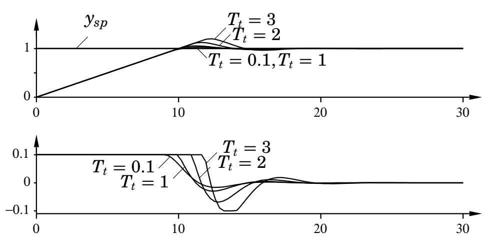
Solid line: PID-controller with anti-windup

Anti-windup for PID-Controller ("Tracking")

Anti-windup (a) with actuator output available and (b) without



Choice of Tracking Time T_t

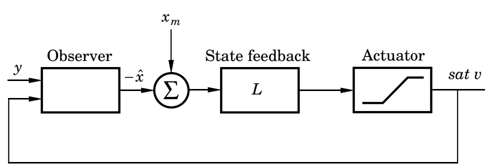


With very small T_t (large gain $1/T_t$), spurious errors can saturate the output, which leads to accidental reset of the integrator. Too large T_t gives too slow reaction (little effect).

The tracking time T_t is the design parameter of the anti-windup.

Common choices: $T_t = T_i$ or $T_t = \sqrt{T_i T_d}$.

State feedback with Observer



$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + B\text{sat } v + K(y - C\hat{x}) \\ v &= L(x_m - \hat{x})\end{aligned}$$

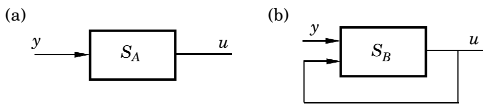
$\hat{x}$ is estimate of process state, x_m desired (model) state.
Need model of saturation if sat v is not measurable

Antiwindup – General State-Space Controller

State-space controller:

$$\begin{aligned}\dot{x}_c(t) &= Fx_c(t) + Gy(t)\\ u(t) &= Cx_c(t) + Dy(t)\end{aligned}$$

Windup possible if F is unstable and u saturates.

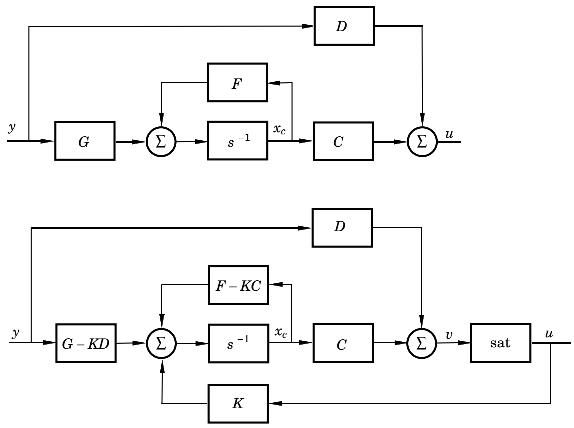


Idea: Rewrite representation of control law from (a) to (b). such that:

(a) and (b) have same input-output relation

(b) behaves better when feedback loop is broken, if S_B stable

State-space controller without and with anti-windup:



Antiwindup – Polynomial Controller

Let $A_{aw}(s)$ be the desired characteristic polynomial of the anti-windup observer. Adding $A_{aw}(s)$ to (both sides of)

$$Ru = Tr - Sy$$

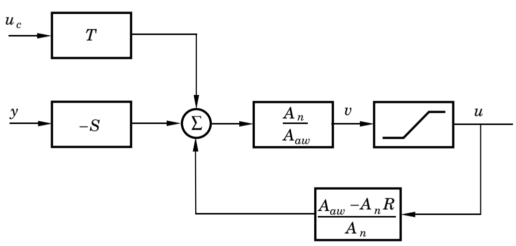
yields

$$A_{aw}u = Tr - Sy + (A_{aw} - R)u$$

This suggests the controller with anti-windup

$$\begin{aligned}A_{aw}v &= Tr - Sy + (A_{aw} - R)u\\ u &= \text{sat } v\end{aligned}$$

A Generalization



The freedom in A_n can be used to shape the response to errors due to saturation

Antiwindup – General State-Space Controller

Mimic the observer-based controller:

$$\begin{aligned}\dot{x}_c &= Fx_c + Gy + K\underbrace{(u - Cx_c - Dy)}_{=0}\\ &= (F - KC)x_c + (G - KD)y + Ku\\ &= F_0x_c + G_0y + Ku\end{aligned}$$

Design so that $F_0 = F - KC$ has desired stable eigenvalues

Then use controller

$$\begin{aligned}\dot{x}_c &= F_0x_c + G_0y + Ku\\ u &= \text{sat }(Cx_c + Dy)\end{aligned}$$

Polynomial Controller Design

Process $A(p)y(p) = B(p)u(p)$ and polynomial controller (RST-controller)

$$R(p)u = T(p)u_c - S(p)y, \quad p = d/dt$$

gives the closed-loop system

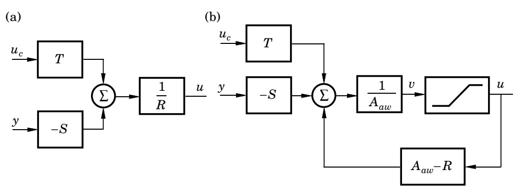
$$y = \frac{BT}{AR + BS}r = \frac{B_m}{A_m}r$$

where B_m/A_m is the desired response model. Choose R and S such that

$$AR + BS = A_mA_o$$

where A_o corresponds to observer dynamics.

Antiwindup – Input Output Form

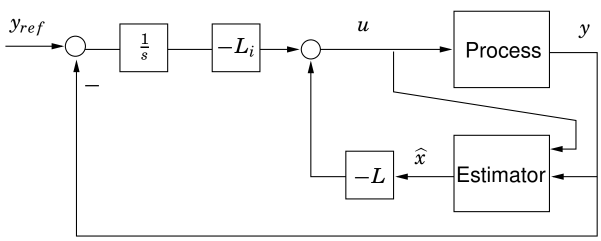


Common choices

- $A_{aw} = A_o$ (same as observer)
- $A_{aw} = R$ (no antiwindup)

5 Minute Exercise

How would you do antiwindup for the following state-feedback controller with observer and integral action ?



Saturation

A more systematic treatment?

Interested students: Look at articles by Teel, Packard, Dahleh

Optimal control theory (later)

Multi-loop Anti-windup (Cascaded systems)

Difficult problem, several suggested solutions

Turn off integrator in outer loop when inner loop saturates

Friction

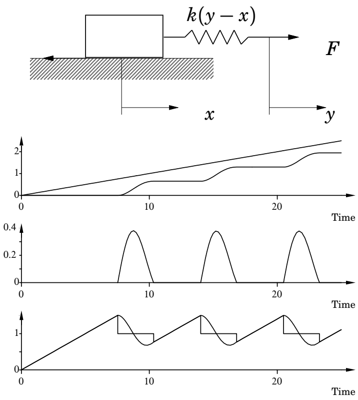
Almost is present almost everywhere

- Often bad
  - Friction in valves and mechanical constructions
- Somtimes good
  - Friction in brakes
- Sometimes too small
  - Earthquakes

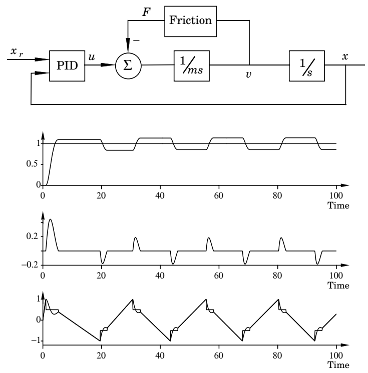
Problems

- How to model friction
- How to compensate for friction

Stick-slip Motion



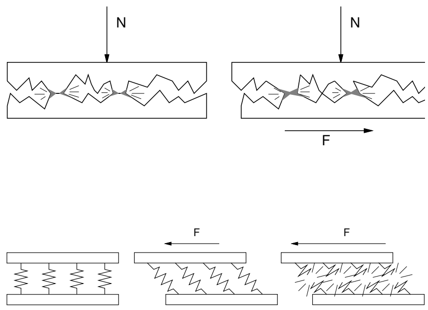
Position Control of Servo with Friction – Hunting



3 Minute Exercise

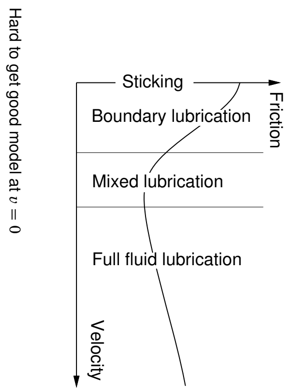
What are the signals in the previous plots? What model of friction has been used in the simulation?

Friction



Lubrication Regimes

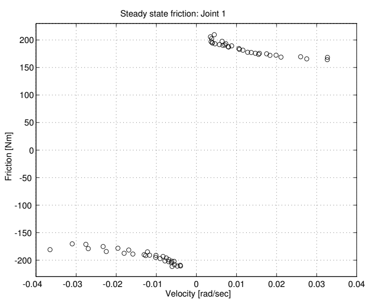
Hard to get good model at $v = 0$



Stribeck Effect

For low velocity: friction increases with decreasing velocity

Stribeck (1902)



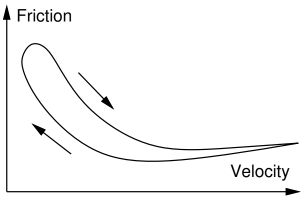
Frictional Lag

Dynamics are important also outside sticking regime

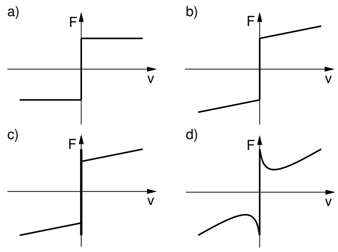
Hess and Soom (1990)

Experiment with unidirectional motion $v(t) = v_0 + a\sin(\omega t)$

Hysteresis effect!



Classical Friction Models



c) $$F(t) = \begin{cases} F_c \text{ sign } v(t) + F_v v(t) & v(t) \neq 0 \\ \max(\min(F_e(t), F_s), -F_s) & v(t) = 0 \end{cases}$$

$F_e(t) =$ external applied force , F_c, F_v, F_s constants

Advanced Friction Models

See handout from Olsson's PhD-thesis

- Karnopp model
- Armstrong's seven parameter model
- Dahl model
- Bristle model
- Reset integrator model
- Bliman and Sorine
- Wit-Olsson-Åström

Demands on a model

To be useful for control the model should be

- sufficiently accurate,
- suitable for simulation,
- simple, few parameters to determine.
- physical interpretations, insight

Pick the simplest model that does the job! If no stiction occurs the $v = 0$-models are not needed.

Friction Compensation

- Lubrication
- Integral action (beware!)
- Dither
- Non-model based control
- Model based friction compensation
- Adaptive friction compensation

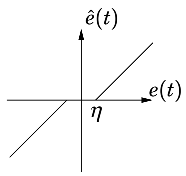
Integral Action

- The integral action compensates for any external disturbance
- Good if friction force changes slowly ($v \approx$ constant).
- To get fast action when friction changes one must use much integral action (small T_i)
- Gives phase lag, may cause stability problems etc

Deadzone - Modified Integral Action

Modify integral part to $I = \frac{K}{T_i}\int^t \hat{e}(t)d\tau$

where input to integrator $$\hat{e} = \begin{cases} e(t) - \eta & e(t) > \eta \\ 0 & |e(t)| < \eta \\ e(t) + \eta & e(t) < -\eta \end{cases}$$

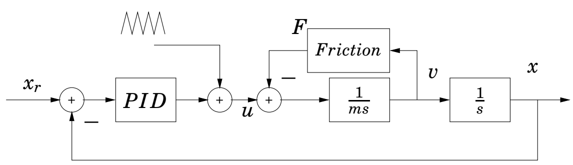


Advantage: Avoid that small static error introduces limit cycle

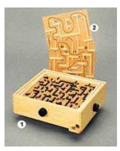
Disadvantage: Must accept small error (will not go to zero)

Mechanical Vibrator–Dither

Avoids sticking at $v = 0$ where there usually is high friction by adding high-frequency mechanical vibration (dither)

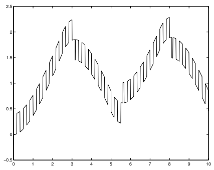


Cf., mechanical maze puzzle (labyrintspel)



The Knocker

Combines Coulomb compensation and square wave dither



Tore Hägglund, Innovation Cup winner + patent 1997

Result: $e = a - \widehat{a} \to 0$ as $t \to \infty$,

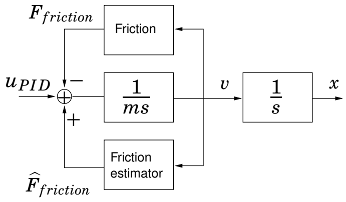
since

$$\begin{aligned} \frac{de}{dt} &= -\frac{d\hat{a}}{dt} = -\frac{dz}{dt} + km\frac{d}{dt}|v| \\ &= -ku_{PID}\text{sgn}(v) + km\dot{v}\text{sgn}(v) \\ &= -k\text{sgn}(v)(u_{PID} - m\dot{v}) \\ &= -k\text{sgn}(v)(F - \hat{F}) \\ &= -k(a - \hat{a}) \\ &= -ke \end{aligned}$$

Remark: Careful with $\frac{d}{dt}|v|$ at $v = 0$.

Adaptive Friction Compensation

Coulomb Friction $F = a\text{sgn}(v)$



Assumption: v measurable.
Friction estimator:

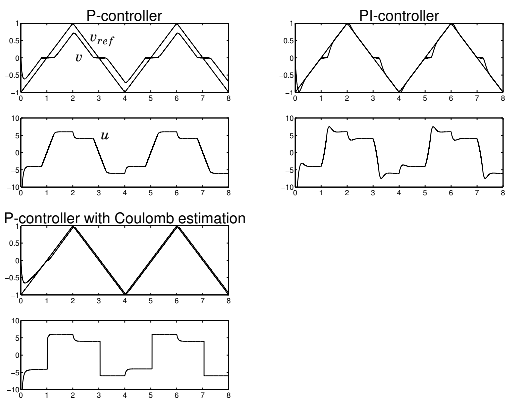
$$\begin{aligned} \dot{z} &= ku_{PID}\ \text{sgn}\ v \\ \widehat{a} &= z - km|v| \\ \widehat{F}_{friction} &= \widehat{a}\ \text{sgn}\ v \end{aligned}$$

Example–Friction Compensation

Velocity control with

a) P-controller
b) PI-controller
c) P + Coulomb estimation

Results



Next Lecture

- Backlash
- Quantization