Nonlinear Control (FRTN05)

Computer Exercise 5

Last updated: May 8, 2008

The following exercises should be solved with help of computer programs and optimization tools (Optimica[1]). Remember that computers can help to give insight, but you still need to understand the underlying theory.

Before you start to solve the problem, you must do the following steps to initiate the software:

1. Download the file `init_CE5.sh` from the web–page.
2. Run the script by typing "`sh init_CE5.sh`" on the command line.
3. change the directory by typing "`cd FRTN05Lab`"

You can now start to solve the problems.

1. 2D Double Integrator Consider the following model of a two dimensional double integrator:

$$\begin{aligned} \ddot{x}(t) &= u_x(t) \\ \ddot{y}(t) &= u_y(t) \end{aligned} \tag{1}$$

We would like to find trajectories that transfers the state of the system from $(-1.5, 0)$ to $(1.5, 0)$ in 4.5s with the control constraint $u_x^2 + u_y^2 \leq 1$. This constraint ensures that the resulting force has a magnitude equal to or less than 1. Interpret the solution from the optimization problem without any path constraints and compare with your intuition. As a second step we would like to see how the solution changes if we impose the condition $y \geq \cos x - 0.2$ (avoiding a certain area in the XY-plane). Finally, the control energy needed to transfer the state should be minimized. This gives us the following optimal control formulation

$$\min_u \int_0^{4.5} u_x(t)^2 + u_y(t)^2 dt$$

subject to

$$\begin{aligned} \ddot{x}(t) &= u_x(t) \\ \ddot{y}(t) &= u_y(t) \\ x(0) &= -1.5, \quad x(4.5) = 1.5 \\ y(0) &= 0, \quad y(4.5) = 0 \\ \dot{x}(0) &= 0, \quad \dot{x}(4.5) = 0 \\ \dot{y}(0) &= 0, \quad \dot{y}(4.5) = 0 \\ 1 &\geq u_x(t)^2 + u_y(t)^2 \\ \\ y(t) &\geq \cos x(t) - 0.2 \qquad \text{(Try both with and without)} \end{aligned} \tag{2}$$

The dynamics of the double integrator system is given by the following Modelica model:

[1] For an overview of Optimica see hand-out on course home page.

```
model DoubleIntegrator2d
  input Real ux;
  input Real uy;
  Real x(start=-1.5);
  Real vx(start=0);
  Real y(start=0);
  Real vy(start=0);
equation
  der(x)=vx;
  der(vx)=ux;
  der(y)=vy;
  der(vy)=uy;
end DoubleIntegrator2d;
```

and the Optimica description of the optimization problem is given by:

```
class optDI2d
optimization
  grid(finalTime=fixedFinalTime(finalTime=4.5),nbrElements=100);
  minimize(lagrangeIntegrand=ux^2+uy^2);
subject to
  terminal x=1.5;
  terminal vx=0;
  terminal y=0;
  terminal vy=0;
  ux^2+uy^2<=1;
   // y>=cos(x)-0.2;  // Try both with and without this
end DoubleIntegrator2d;
```

Notice that the initial conditions are expressed in the Modelica model using the `start` attribute, whereas the terminal constraints are given in the `subject to` clause in the Optimica model.

(a) Read and understand the Modelica file (`DoubleIntegrator2d.mo`) describing the model, and the Optimica file (`DoubleIntegrator2d.op`). Solve the optimization by typing
> `optimicac DoubleIntegrator2d.op DoubleIntegrator2d.mo DoubleIntegrator2d`
> `./ampl DoubleIntegrator2d.run`

(b) Load the result-file into Matlab and plot the result. You can use the matlab script `DoubleIntegrator2d.m`.

(c) Plot the path constraint and check that the solution avoids the specified area in the XY-plane.

2. Consider the Van der Pol oscillator

$$\dot{x}_1 = (1 - x_2^2)x_1 - x_2 + pu$$
$$\dot{x}_2 = x_1$$

where p is a parameter with the nominal value 1;

(a) Create a Modelica model (.mo) for the dynamics of the Van der Pol oscillator.

(b) Assuming the initial conditions $(x_1(0), x_2(0)) = (0, 1)$ and the Lagrange cost function

$$J = \int_0^{10} 10x_1^2 + 10x_2^2 + u^2\,dt,$$

create an Optimica file (.op) containing the cost function and a suitable mesh specification.

(c) Solve the optimization problem problem and load the solution into Matlab. Try different choices of elements in the mesh and study how this affects the solution.

(d) Add the constraint

$$0 \le u \le 1$$

to the Optimica code and solve the problem assuming the same cost function and initial conditions as in **(b)**. Also check what happens if you decrease the upper bound on u.

(e) Compute the minimum-time optimal solution assuming the terminal constraints $(x_1(t_f), x_2(t_f)) = (0, 0)$

3. Consider the double pendulum as illustrated in figure 3.

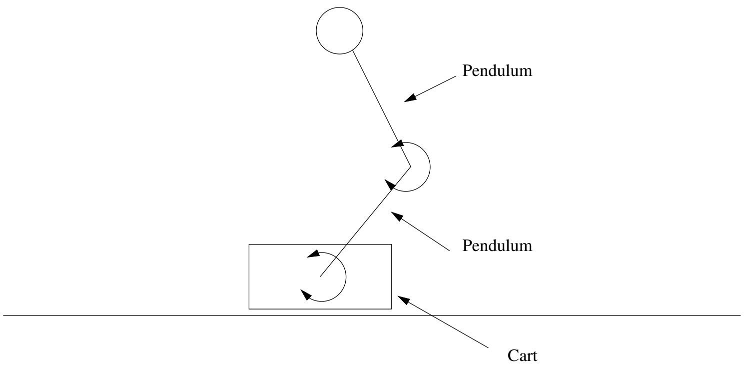

Figure 1 The double pendulum. Two pendula in series are attached to a moving cart.

The model is quite complicated, so the Modelica file is given to you in the file `doublepend.mo`.

(a) Compute the time–optimal solutions where all states goes to the origin, and the following constraints are respected.
   - The position of the cart (pos) is is in the range ± 3.
   - The control signal (u) is in the range ± 2

   Furthermore, there are restriction on how fast the control signal can be changed:
   - The derivative of the control signal (u_vel) is in the range ± 100.
   - The double derivative of the control signal (u_acc) is in the range ± 1000.

   You can use the file `doublepend.op`.

(b) You can use `doublepend.m` for animating the solution.