Nonlinear Control (FRTN05)

Computer Exercise 2

Last updated: Spring of 2008

The following exercises should be solved with help of computer programs and simulation tools. Remember that computers can help to give insight, but you still need to understand the underlying theory.

1. Exercise 2.3 in the exercise book. Use the MATLAB-tool `pplane6.m` or `pplane7.m` which can be down-loaded from `http://www.control.lth.se/~FRTN05`. You may also try `dee`, `dfield` or `ictools`.

2. Consider the three systems

$$
\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 \end{cases}
\qquad
\begin{cases} \dot{x}_1 &= x_1^3 + x_2 \\ \dot{x}_2 &= -x_1 \end{cases}
\qquad
\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - x_2^3 \end{cases}
$$

   Verify that all three systems have the same linearization around the origin $(x_1, x_2) = (0, 0)$ (which is the only equilibrium). What can be said about the local behavior of the nonlinear systems around the origin based on the linearization for these particular systems? Use the MATLAB-tool `pplane6.m` or `pplane7.m` to simulate the system. Look at both the phase plane and the time-plots of the states for the different systems.

3. Use `maple` or Symbolic Toolbox in Matlab to solve the differential equation

$$
\dot{r}(t) = r(t)(1 - r^2(t))
$$
$$
r(0) = r_0
$$

   (taken from lecture 3) symbolically. Use the command `dsolve`. To get help in maple you write ?dsolve.
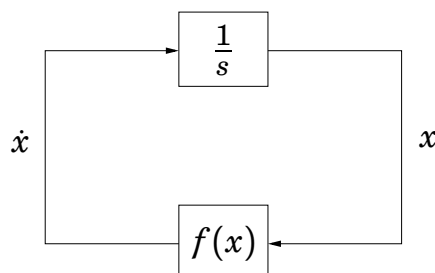
   Example (Matlab[1]):

   ```
   >> help dsolve
   >> solution = dsolve('Df =  f + sin(t)', 'f(pi/2) = 0')
   >> ezplot(solution); grid on;   % explain behavior
   ```

   Hint: Remember that a dynamical system on the form

$$
\dot{x}(t) = f(x(t))
$$

   can in Simulink be represented as

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[1]You need to have access to the Matlab/Symbolic Toolbox for this.

4. Exercise 1.16 in the exercise book. Try to evaluate a linear approximation of the Poincare' map numerically as is described in the lecture slides (try different solvers and specify the accuracy).