

# Nonlinear Control (FRTN05)

## Computer Exercise 1

Last updated: Spring of 2008

Consider the nonlinear system

$$\dot{x} = -x + x^3 + u \quad (1)$$

1. Build a simulation model in Simulink for system (1). Simulate it for different initial conditions  $x(t=0)$  when you don't use any control ( $u=0$ ). Determine via simulation in which region of initial values  $x(0)$  the solutions  $x(t) \rightarrow 0$  as  $t \rightarrow \infty$ .
2. Which are the equilibrium points of the system if  $u=0$ ? What equilibrium points are there if  $u=1$ ?<sup>1</sup>
3. Sketch a diagram of the  $x$ -axis and the corresponding directions of  $\frac{dx}{dt}$  when  $u=0$ . What does it show of the local stability properties of the equilibrium points found in (2)?
4. Suggest a feedback control law  $u = u(x)$  which makes the origin  $x=0$  a globally stable equilibrium point.
5. Assume now that you have bounded control action,  $u \in [-10, 10]$ . Try your controller from (4) realized as  $u = \text{sat}_{10}(\dots)$ , that is, you have a saturation at  $\pm 10$  after your control law from (4). What is the largest range of stability? Simulate and compare with your analysis.

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<sup>1</sup>You may use the Matlab command `roots`.