Lund University

Department of
AUTOMATIC CONTROL

Real-Time Systems

Exam May 31st, 2018, hours: 08:00–13:00

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in **English or Swedish**. The total number of points is 25. The maximum number of points is specified for each subproblem.

Accepted aid

The textbooks Real-Time Control Systems and Computer Control: An Overview - Educational Version. Standard mathematical tables, authorized "Real-Time Systems Formula Sheet", and authorized "Reglerteknik AK Formula Sheet". Pocket calculator.

Results

The result of the exam will become accessible through LADOK. The solutions will be available on WWW:
http://www.control.lth.se/course/FRTN01/

1. Consider the continuous-time unstable plant P:

$$\dot{x} = x + 2u$$
$$y = x$$

a. ZOH-sample the system with sample period $h = 1$. What are the poles and zeros of the sampled system $\hat{P}$? (1 p)

b. The sampled plant $\hat{P}$ is to be stabilized using a proportional feedback controller with gain K. For what values of K is the closed loop system stable? (1 p)

c. Assume that feedback control of the sampled system $\hat{P}$ is performed with $K = 1$. The reference r of the closed loop system is changed as a step of size 2 at time $t = 0$. Will the output y of the closed loop system converge? If yes, what will it converge to? (1 p)

2. You want to schedule the three following tasks

Name	T_i	D_i	C_i
Controller	10	10	6
Plotter	15	15	3
User interface	15	15	2

a. Is the task set schedulable under an Earliest Deadline First (EDF) policy? (1 p)

b. Your friend comes with the suggestion of always giving the controller highest priority, while letting the other two processes have an EDF policy between them. Is the task set schedulable under this scheduling policy? (1 p)

3. Consider a discrete-time control law $C(z)$ with poles $z_{p_1} = 0.72$, $z_{p_2} = 0.91$ and zeros $z_{z_1} = 0.15$, $z_{z_2} = 0.23$. Its direct form realization is given by

$$C(z) = \frac{z^2 - 0.38z + 0.0345}{z^2 - 1.63z + 0.6552}.$$

Assume that this control law is to be implemented in a micro-controller using fixed point arithmetic in the *Q4.3* format, i.e. using 8-bit word length.

a. Convert the controller parameters to a fixed point representation, using the *Q4.3* format. (0.5 p)

b. Calculate the locations of the distorted poles and zeros of $C(z)$, caused by the round-off errors from the fixed point representations of the controller parameters. (1 p)

c. Assume that the number of fractional bits can not be increased. Suggest an improvement to the implementation that reduces the controller distortion. (0.5 p)

4. Your colleague is checking for errors in some sensors and would like to print the sensor measurements periodically. To save some time your colleague attempts to print measurements from all the sensors concurrently in a terminal window by running several instances of the following Java class:

```java
public class PrintMeasurement extends Thread {
  private int h;
  private int channel;
  private AnalogIn yChan;

  public PrintMeasurement(int h, int channel) {
    this.h = h;
    this.channel = channel;
    try {
      this.yChan = new AnalogIn(channel);
    } catch (Exception e) {
      System.out.println(e);
      return;
    }
  }

  public void run() {
    long duration;
    long t = System.currentTimeMillis();
    double y;
    while (true) {
      y = yChan.get();
      synchronized(this) {
        System.out.print("Channel"+channel+" : "+y);
        System.out.print(", ");
      }
      t = t + h;
      duration = t - System.currentTimeMillis();
      if (duration > 0) {
        try {
          sleep(duration);
        } catch (InterruptedException e) {
          e.printStackTrace();
        }
      }
    }
  }
}
```

a. Your colleague occasionally gets strange printouts, but does not understand why. Explain what is wrong with your colleague's implementation of `PrintMeasurement`. (1 p)

b. Explain how to do a correct implementation which removes the strange printouts. (1 p)

5. You are working with a high-speed drill in a factory, which runs at a constant speed between 4000 and 5000 revolutions per minute. A sensor measures the rotational angle of the drill bit, which is then used by a feedback controller to keep the drill speed constant. The measurement signal is sampled periodically, with a period of $h = 20$ ms.

a. When looking at the frequency spectrum of the sampled measurement signal from the drill, you are surprised to observe a large peak at 20 Hz. What is the most likely cause of this problem? Give a suggestion for how to solve this problem. (1 p)

b. What was the exact frequency of the rotation of the drill in **a.**? (1 p)

6. An inertial measurement unit (IMU) is an electronic sensor device containing both an accelerometer and a gyroscope. Both accelerometer and gyroscope can be used to measure the angular orientation of the IMU, but each sensor has its own drawback. The accelerometer provides measurements which are corrupted by high-frequency noise, while the gyroscope's measurements are subject to low-frequency noise. To counter-act the drawbacks of each sensor, it is common to combine the measurements in a *complementary filter*.

Assume we are interested in measuring the angle α of the IMU. The complementary filter then produces an estimate $\hat{\alpha}$ of the angle according to:

$$\hat{\alpha}(s) = \frac{1}{sT+1}\alpha_1(s) + \frac{sT}{sT+1}\alpha_2(s),$$

where $T > 0$ is a constant and α_1, α_2 are two different measurements of α.

a. Which one of α_1 and α_2 should be the angle measurement from the gyroscope and accelerometer respectively? Motivate your answer! (1 p)

b. In order to implement the filter we need to compute a discrete-time approximation of it. Use the forward difference approximation to discretize the complementary filter, and determine the corresponding difference equation where $\hat{\alpha}$ is computed from α_1 and α_2. (1.5 p)

c. Implement the complementary filter using the pseudo-code skeleton below. Insert your code in the places indicated by "...". Full points require an implementation where the latency between reading the measurements and transmitting the estimate is minimized. Make sure that all variables are properly declared. (1.5 p)

```
private double alpha_hat = 0, alpha_1 = 0, alpha_2 = 0;
private double h = 0.01, T = 0.33;
...
while(true) {
  alpha_1 = readAlpha1();
  alpha_2 = readAlpha2();
  ...
  transmitEstimate(alpha_hat);
  ...
  sleep();
}
```

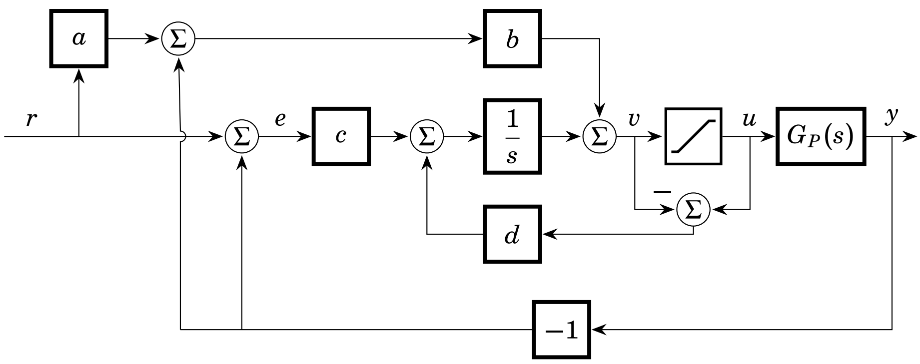

Figure 1 The control structure in Problem 7.

7. Consider the block diagram in Figure 1. It shows a very common control structure with some practical extensions.

a. What control structure is shown in Figure 1? Explain the roles of the design parameters a, b, c, d. (1 p)

b. Write pseudo-code that implements the control structure shown in Figure 1. Discretize the control algorithm (using e.g. forward difference) with sample period h and keep the same notation a, b, c, d for the design parameters. Assume that the measurement signal y is available from `getY()`, reference r from `getR()`, control signal u is set using `setU(u)` and saturated between `u_min` and `u_max` . For full points the code should be written such that the input-output latency is minimized.

```
// Code executed in the Regul thread

time = getCurrentTime();

// Declare variables with state here (if any)

while (true) {
    y = getY();
    r = getR();

    // Insert your controller implementation here

    time = time + h;
    waitUntil(time)
}
```

(1.5 p)

8. Consider the following discrete-time system:

$$x(k+1) = \begin{pmatrix} 1 & -0.25 \\ 1 & 0 \end{pmatrix} x(k) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(k)$$
$$y(k) = \begin{pmatrix} 0 & 1 \end{pmatrix} x(k)$$

The system has a double-pole in 0.5 and has been obtained through ZOH-sampling of a continuous-time system using the sampling interval 0.1.

a. Calculate a state feedback controller $u(k) = -Lx(k)$ so that the closed loop poles are twice as fast as the corresponding open-loop continuous-time poles. If you do not understand where to put the poles then choose a double pole in 0.3. (2 p)

b. Calculate an observer with direct term (also known as an observer on filter form or a reduced order observer) so that the dynamics for the estimation error has both its poles in 0.2. In order to get full points you should also write down the equations for the observer and the reconstruction error. (1.5 p)

c. Design a two-degree-of-freedom controller with a linear model and feedforward generator. Let the model be the open-loop model modified using state feedback so that the resulting model has the same poles as the ones used in **a.** and static gain equal to 1. Draw the resulting block diagram including the process, the observer, and the model+feedforward generator. It is not sufficient to answer with a reference to some figure in a textbook. You must draw the block diagram yourself. (1 p)

9. You are tasked with implementing the discrete-event logic of a coffee machine. The machine makes coffee by grinding coffee beans and boiling water in parallel. The coffee beans are ground several times depending on how finely ground coffee the user wants. When both tasks are done, the water is poured over the ground beans through a filter into a cup.

a. Your first assignment is to draw a GRAFCET diagram of the task of grinding coffee beans. The task is as follows:

1. Beans are added to the grinder by opening a small hatch for a fixed amount of time.
2. The grinder is activated for a fixed amount of time.
3. If the coffee beans should be ground more times, then return to 2. Else, the task is finished.

In your implementation you have access to the following variables and signals:

OpenHatch Boolean input signal, which keeps the hatch open while true.

Grind Boolean input signal, which keeps the grinder turned on when true.

OpenHatchTime Number of scan cycles the hatch should remain open.

GrindTime Number of scan cycles the grinder should be turned on.

Counter Integer variable which keeps track on number of grinds performed.

NGrinds Integer value from the user which specifies the number of grinds that should be done.

(1.5 p)

b. Your second assignment is to draw a GRAFCET diagram of the complete coffee-making process. The full process is as follows:

1. The coffee machine waits in an idle state for a user to press the start button.
2. When the start button is pressed, the machine starts to grind coffee beans and boil water in parallel.
3. The water is ready when the temperature has reached $100°C$.
4. When both coffee beans and water are ready, the machine pours the water over the ground beans through a filter.
5. The pouring stops when the machine senses a fixed level has been reached.
6. When finished pouring, the machine returns to the idle state.

In addition to the variables and signals in **a.**, you have access to:

Heat Boolean input signal, which keeps the heater turned on while true.

Pour Boolean input signal, which pours the water over the filter while true.

Start Boolean measurement signal, which becomes true when the user presses the start button.

LevelReached Boolean measurement signal, which is true when the prescribed coffee level has been reached.

Temperature Real valued measurement signal of water temperature in degrees Celsius.

(1.5 p)

Hint: The implementation designed in **a.** can be used as a macro block called `GrindBeans` in this assignment. If you did not complete **a.** you can still assume that you have a functioning macro block.