LUND
UNIVERSITY

Department of
AUTOMATIC CONTROL

Real-Time Systems

Exam April 21, 2017, hours: 14:00 – 19:00

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in **English or Swedish**. The total number of points is 25. The maximum number of points is specified for each subproblem.

Accepted aid

The textbooks Real-Time Control Systems and Computer Control: An Overview - Educational Version. Standard mathematical tables and authorized "Real-Time Systems Formula Sheet" and "Reglerteknik AK Formula Sheet". Pocket calculator.

Results

The result of the exam will become accessible through LADOK. The solutions will be available on WWW:
http://www.control.lth.se/course/FRTN01/

1. Consider the following task set that is executing under fixed-priority scheduling on a uniprocessor, using the standard notation.

Name	T_i	D_i	C_i	Priority
A	10	10	3	Low
B	4	4	1	Medium
C	6	6	1	High

Decide if the task set schedulable or not?

(1.5 p)

2. Given the system

$$x(k+1) = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} x(k) + \begin{pmatrix} d \\ e \end{pmatrix} u(k),$$

where a, b, c, d and e are scalars.

a. Deduce precisely what condition these coefficients need to satisfy for the system not to be reachable. (0.5 p)

b. Explain in words the reason why the system is unreachable in the following three cases:

- $e = 0$
- $b = 0$ and $d = 0$
- $b = 0$ and $c = a$

(1.5 p)

3. Consider the system given by

$$\dot{x}(t) = -x(t) + u(t - 2.6).$$

Sample the system using zero-order hold with sampling period $h = 1$. Write the resulting discrete-time system on state space form.

(2 p)

4. You have been given the following continuous controller from a friend.

$$U(s) = K(1 + \frac{1}{s^2} + bs)E(s)$$

The controller is a variant of a PID controller that has double integral action instead of normal integral action.

a. Discretize the controller using the Forward difference approximation (Euler's method) with $h = 1$. (1 p)

b. Is it possible to implement the resulting controller? If yes, provide the difference equation for the controller. If no, explain why it is not possible. (0.5 p)

c. How would the situation in the previous subproblem change if one added a first order low-pass filter on the derivative part of the controller? (0.5 p)

5. You have just been employed by a control company and your first task is to design a controller for a system which you are given the model of.

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.2 & 0 \\ -0.8 & 0.4 \end{bmatrix} x(k) + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(k) \end{aligned}$$

a. The company manager asks you to design a state feedback controller and to ensure that the poles of the closed loop system are located in $+0.2$ and -0.2, and the static gain from the reference signal to the output is equal to 1. Is it possible? If yes, provide the equation for the controller. If no, explain why. (2 p)

b. Design an observer on predictor form with both poles in 0.1. To get the full points you should also write down the equations for the observer. (2 p)

6. Three tasks are running concurrently in a system, using semaphores for synchronization. The code for the synchronization for each of the tasks is shown below. The dots indicate a sequence of instructions that are irrelevant for synchronization purposes. If two instructions are not interleaved with dots they are executed directly one after the other.

Task A
```
...
lock(s1);
lock(s3);
...
unlock(s1);
lock(s5);
...
unlock(s3);
unlock(s5);
...
```

Task B
```
...
lock(s3);
lock(s4);
lock(s2);
...
...
unlock(s4);
unlock(s2);
unlock(s3);
...
```

Task C
```
...
lock(s2);
lock(s4);
...
...
...
...
unlock(s4);
unlock(s2);
...
```

a. What is the problem with the current code and how can you modify the code for the tasks to solve the problem? (1 p)

b. Assuming that the priority ceiling protocol is used and that Task A has priority 5, Task B has priority 9 and Task C has priority 10 (the higher the number, the higher the priority of the process). Compute the ceiling for all the semaphores. Remember to justify your answer. (1 p)

7. You are working on the control system for a small autonomous racing car and you have been asked to write a simple controller with a very limited memory footprint. The processor that you are using only allows for a wordlength of 8 bits. Therefore you have to implement your controller using fixed-point arithmetic.

a. You know that you need a resolution of *at least* 0.1. Using the $Q\,m.n$ representation, what is your choice for m and n? (1 p)

b. Using the obtained format can you represent the number 14.28? If no, why? If yes, write the decimal and binary representation of the number. In case you have not solved the previous problem, use $Q\,2.5$ notation. (1 p)

c. If possible, encode the constant $k_p = 5.5$ with the given notation. If not possible, explain why. To get full score, if possible, you need to provide both the decimal and the binary representation of the constant. In case you have not solved the first subproblem, use a $Q\,2.5$ notation. (1 p)

8. You want to measure a signal with two frequency ranges of interest: $[f_0,\ 2f_0]$ and $[4f_0,\ 5f_0]$. There is also a disturbance at $3f_0$.

a. For what frequencies can you sample the system while still being able to measure the regions of interest. (1 p)

b. Now imagine you only care about the first frequency range, $[f_0,\ 2f_0]$. What sample rate can you use if you want to be sure that the disturbance will not be aliased into the frequency range of interest? (2 p)

9. The functions of the lunar lander JokRT are implemented in three different tasks: Task T1 manages the movement of the lander. Task T2 collects samples and task T3 manages the communication with Earth. The engineers working at the Automatic Control department wrote a superb real-time system for the first two tasks, managing to keep the needed worst-case computation down to 3 time units and 1 time unit per task, respectively. Analyzing the stability of the system, they found that T1 should be executed with a period of 5 time units. Also, the geologist working on the project determined that a new sample has to be collected every 10 time units.

a. The communication module (T3) was implemented last and its WCET turned out to be 4 time units. How often can the lunar module communicate with earth if the Earliest Deadline First Scheduler is used to schedule the tasks, i.e., what is the smallest possible period for T3 in order for the system to be schedulable? Assume that the context switch time is negligible, that the relative deadline of all tasks are equal to the periods, that only integer period values are supported, and that there are no interrupts.

(1 p)

b. When will the schedule repeat? If you have not solved the previous point, use a period and deadline of of 12 time units for Task T3.

(1 p)

10. Consider the following buffer system

```
class BufferSystem {

  Buffer BufferOne;
  Buffer BufferTwo;

  public synchronized void put(Object o) throws InterruptedException {
    while (bufferOne.isFull()) {
      wait();
    }
```

```
    bufferOne.put(o);
    notify();
  }

  public synchronized void move() throws InterruptedException {
    while (bufferOne.isEmpty() || bufferTwo.isFull()) {
      wait();
    }
    Object o = bufferOne.get();
    operateOnObject(o);
    bufferTwo.put(o);
    notify();
  }

  public synchronized Object get() throws InterruptedException {
    while (bufferTwo.isEmpty()) {
      wait();
    }
    Object o = bufferTwo.get();
    notify();
    return o;
  }

  private void operateOnObject(Object o) {
    ...
  }

  public BufferSystem () {
    ...
  }
}
```

There are three threads. One repeatedly calls the `put` method, the second repeatedly calls the `move` method, and the third repeatedly calls the `get` method. All threads run on the same processor core. The `Buffer` class only inserts and extracts objects, it does not contain any internal synchronization. The buffers may only contain maximum one object each.

a. There is an error in the implementation of the `BufferSystem` that may cause all the threads to sleep at the same time. Explain how this can happen. (1 p)

b. Make changes to the class `BufferSystem` so that this problem can not occur. (1 p)

11. A student from a (not so) well renowned university in Sweden is in charge of developing a control system for a train track. The goal is to prohibit collisions while ensuring that the train system does not reach a deadlock, is live, safe, and bounded.

The system has a single track and two trains, with a Petri net representation depicted in Figure 1.

With the current solution there is a risk for the two trains to collide. You are asked to help the student and develop a solution that prohibits collisions (while ensuring that the Petri net is live, deadlock-free, safe, and bounded).

Hint: it is okay to add both new nodes, and transitions, but you should strive to add as little as possible.

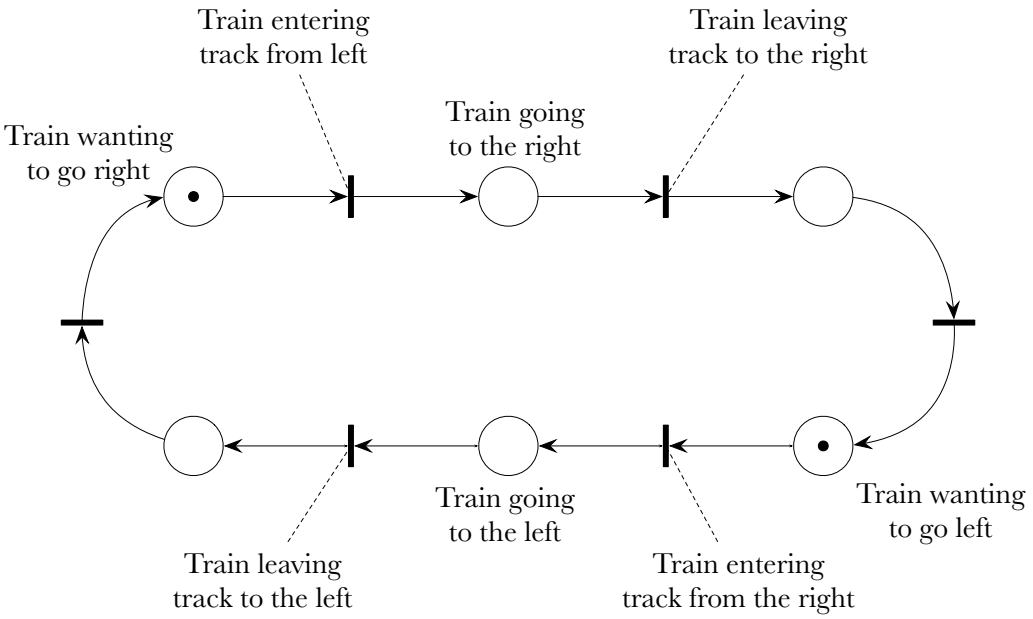

Figure 1

(1.5 p)