# Automatic Control, Basic Course

# Laboratory Exercises

Department of Automatic Control
Lund University

# Automatic Control – Basic Course

# Laboratory Exercise 1

# PID Control

Department of Automatic Control
Lunds tekniska högskola
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## 1. Introduction

The purpose of this laboratory exercise is to give insight into elementary concepts and principles in automatic control. We shall also get closer acquainted with the PID controller, the industrially most commonly occurring controller.

The lab process consists of a pump and two serially connected tanks. A PID controller is implemented in a PC and by means of this we shall control the water level in the tanks.

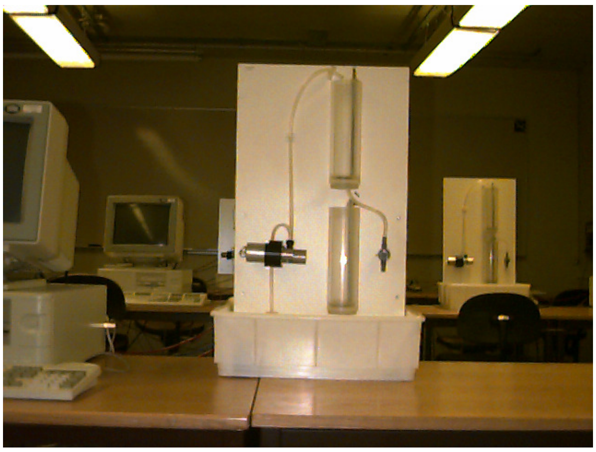


**Figure 1** Lab setup.

### Preparations

To get out as much as possible of the lab you shall know the following concepts:

- open and closed loop system
- block diagram
- reference value, process output, control signal
- stationary error

You shall also have read through this lab manual, including the appendix on the user interface.

# 2. Elementary Concepts

This section deals with important concepts in automatic control. We shall also acquaint ourselves with the properties of the process by manually controlling the water level in the tanks.

### What is Good Control?

The reason one wants to control a process is to have it behave in a preferred way. This could involve the process to become more exact, more reliable or more economic. In certain cases processes are unstable and good control is necessary to prevent them from breaking (which could cause large damage).

Good control, consequently, means different things for different applications. When it comes to the tank process in this lab, the following requirements on the control could be suitable:

- We obviously want the real tank level to coincide with our reference (so that process output = reference value).
- If the reference value is changed we want the tank level to adjust to the new reference fast and without large overshoots.
- The control ought to handle disturbances in the form of load disturbances, when the process is affected by an external signal, and measurement noise, when the measurement of the process output contains some sort of error or disturbance.
- Finally, we don't want the control signal to the pump to be too "jerky" because this causes unnecessary wear.

These properties are usually important in most applications. Can you think of any other requirements one could impose on good control?

### Examination of the Process

***Assignment 2.1*** Acquaint yourself with the lab equipment. How can we introduce load disturbances? Is there any measurement noise in the process and can we affect its extent? Mark the process and the controller together with control signal and process output in figure 2 below.

### Block Diagram Representation

To describe a control system it can often be suitable to use block diagrams. A block diagram is a schematic drawing of a system, where one has abstracted away all properties of the different subsystems, except those one is interested in. In this case we are interested in the relation between reference value, measured process output and control signal.

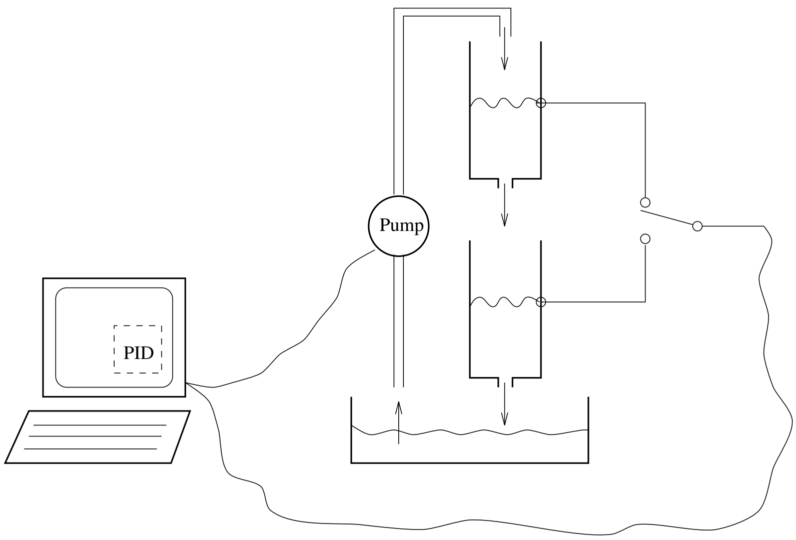



**Figure 2** Lab setup.

Aided by block diagrams, one can easier understand and analyze a process. It is of great importance to understand the relation between the real process and the block diagram.

***Assignment 2.2*** Draw a block diagram for the lab setup when a controller controls the level in one of the tanks. Mark the process, control signal and measured process output also here.

Convince yourself that you understand the relation between the components in the block diagram and the different parts of the real process.

## Open Loop Control

We differentiate between open loop control (program control, feed forward) and closed loop control (feedback). In open loop control, as opposed to closed loop control, the value of the control signal does not depend on the measured process output. The control signal is instead based on a model or tables similar to the one below. For the tank process open loop control means that we should control the tank level without knowing the present level.

Before we experiment with open loop control we first have to construct a simple model of the tank process. Log in according to your lab assistant's instructions. Set the controller to manual mode. You can now directly affect the control signal, yourself, (i.e. the voltage to the pump) and thereby the flow to the upper tank.

***Assignment 2.3*** Adjust the control signal so that the level in the upper tank settles at approximately 5 cm. Note the corresponding control signal in the table below.

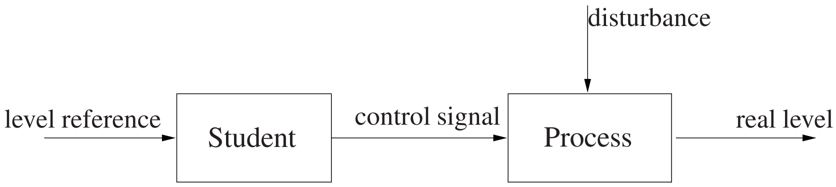
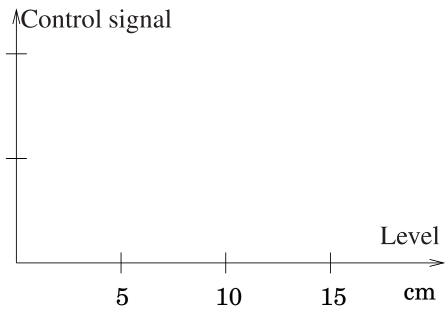



**Figure 3** Open loop system.

| Höjd | Control signal |
|---|---|
| 5 cm | |
| 10 cm | |
| 15 cm | |




Repeat the experiment for the levels 10 cm and 15 cm, respectively. Draw a diagram where the control signal is given as a function of the level. (Don't forget that the curve should pass through the origin!) Can you explain the shape of the curve? You may assume that the flow is proportional to the control signal.

***Assignment 2.4*** Adjust the control signal to the pump so that the level in the upper tank settles at 10 cm. Try, guided by your model from the previous assignment, to change the level by approximately 3 cm when your partner obscures the upper tank. What happens if your partner opens the valve without informing you?

### Closed Loop Control

Now you have access to the measured process output, i.e. the real tank level, and your visual impressions can be *fed back* to control the tank level, cf. figure 4.

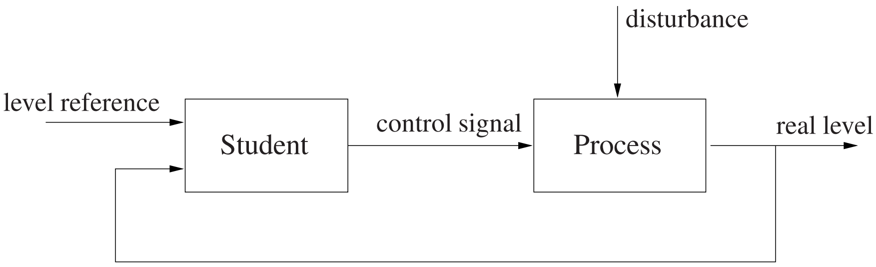



**Figure 4** Closed loop system.

***Assignment 2.5*** Again, try to change the level by 3 cm. What is limiting how fast you can change the level? Observe that you should still control the tank manually!

Next, try to keep the tank level constant while your partner generates load disturbances. What is preferable, open or closed loop control? Why?

In the remainder of the lab we stick to closed loop control.

**Comparison between the Upper and Lower Tanks**

We shall now study how control of the upper tank differs from control of the lower tank.

***Assignment 2.6*** Switch to the lower tank and repeat the experiments from assignment 2.5. Obscure the upper tank!

What is limiting the speed?

Which tank is easier to control? Why?

## 3. Control

We shall now use different controllers to control the levels in the tanks. A controller compares the reference value with the measured process output and computes an "appropriate" control signal.

**P-control**

To start with, we incorporate a proportional controller (P-controller). The control signal $u$ is calculated according to the following relation

$$u(t) = K\,(r(t) - y(t))$$

where $r$ is the reference value and $y$ is the measured process output. In our case this means that the voltage to the pump is proportional to the control error $e = r - y$. The constant $K$ is usually called the gain of the controller.

***Assignment 3.1*** We shall now examine how the properties of the controller depend on the gain $K$. Return to the upper tank and set the reference value to approximately 8 cm prior to each experiment.

Examine how well the tank level follows changes in the reference value. Start with $K = 10$. Increase the reference value by 3 cm. Wait until the level is constant and subsequently reset the reference value. Is there a difference in behavior between positive and negative change in reference value?

Repeat the experiment with $K = 3$ and $K = 30$. How do control error and speed depend on the gain $K$?

Increase $K$ to 40 and repeat the above changes in reference value. Does the result differ from what we obtained with $K = 30$? Explain!

Study how the system behaves when load disturbances are introduced. Generate both step disturbances (in the upper tank) by means of the valve and impulse disturbances by pouring water directly into the upper tank. How does the behavior change when $K$ is varied?

How is the system affected by measurement noise? Vary the gain $K$ and study especially the appearance of the control signal. Give a reasonable value for $K$.

***Assignment 3.2*** Now experiment with P-control of the lower tank. Repeat the experiments of assignment 3.1. Try for example $K = 3,\ 10,\ 30$.

***Assignment 3.3*** Discuss the difference between P-control of the upper and lower tank. Are the results satisfactional? Any problems with the control? Give reasonable values of $K$ for both cases. What constitutes an upper limit on $K$, for the two cases, respectively?

***To Think About*** How could one estimate a reasonable start value for $K$ if it was not given?

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How could one modify the control law of the P-controller so that the stationary error vanishes?

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**PI-control**

A problem with P-control is, as we have seen, that one can end up with a persisting control error. To counteract this, it is natural to increase the control signal as long as the reference value is smaller than the process output. A way to do this is to let the control signal depend also on the integral of the control error. In a PI-controller, the control signal $u$ is calculated according to the relation

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right)$$

where $e$ is the control error, $e = r - y$. The voltage to the pump is now given as the sum of two terms. The first consists of a constant $K$ times the control error and this term is usually called the P-part of the controller (cf. P-controller). The second term is given by a constant $K/T_i$ times the integral of the control error. This part of the sum is consequently called the I-part (integral part) of the controller, and it changes as long as the measured process output differs from the reference value, cf. figure 5. $T_i$ is called the integral time because it has the dimension time. Observe that $T_i$ does *not* influence the integration limits.

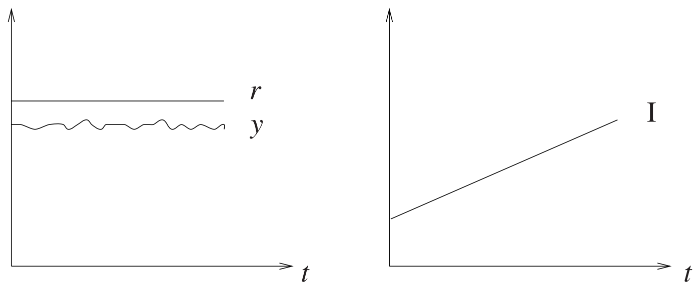



**Figure 5** The I-part is changed as long as there is a control error.

If the control signal $u$ saturates (reaches its max- or min value) and there is a persisting control error $e$, the integral part could impose a problem. It continues to grow and wants to "go at it even harder" despite that the maximal control action is already issued. When the control error has vanished and it is time to crank down the control signal, it remains on its maximum because the integral has grown and obtained a too large value. This phenomenon, which can result in large overshoots or even instability, is known as integrator wind-up. The lab software therefore has a so called anti wind-up protection shceme, counteracting this.

***Assignment 3.4*** Experiment with PI-control of the upper tank. Vary the integral time $T_i$ and study how the responses to reference value changes and load disturbances are affected. Set $K = 10$ and change $T_i$ from 50 down to 5.

***Assignment 3.5*** Experiment with different values on $K$ and $T_i$. Give a suitable setting for a PI-controller of the upper tank. Which are the pros / cons compared to P-control?

***Assignment 3.6*** Try PI-control of the lower tank. Can you find suitable values of $K$ and $T_i$?

**PID-control**

Sometimes additional information about the process is required to obtain good control. For example the derivative of the control error gives an estimate of future values of the error, see figure 6. By letting the control signal depend also on the derivative of the control error, one obtains a control signal which increases when the error increases and decreases when the error decreases. This results in “smoother” control as one approaches the reference value. If we extend the PI-controller to include derivative action, we obtain a PID-controller where the control signal $u$ is given by

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right)$$

The output of the controller now consists of a P-part, an I-part and a D-part $(KT_d\frac{de}{dt})$. $T_d$ is called the derivative time of the controller. It can be interpreted as a prediction horizon, see figure 6.

***Assignment 3.7*** First try to control the upper tank with a PID-controller. Start with the best values found for $K$ and $T_i$ when experimenting with PI-control of the upper tank. Does control performance increase or decrease when adding the D-part? Explanation?

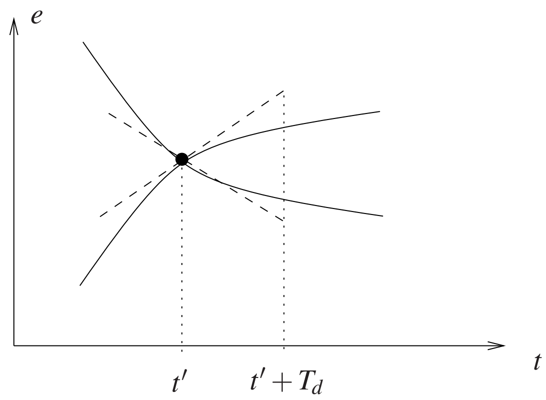



**Figure 6** By means of the derivative part one tries to estimate future values of the error.

***Assignment 3.8*** Try to find a good PID-setting for level control of the lower tank. Start with the best values of $K$ and $T_i$ found for PI-control of the lower tank. Examine the influence of the D-part by varying $T_d$ from 5 to 50. Conclusion?

# 4. Tuning Methods

We have now seen how the P-, I- and D-parts affect the behavior of the control system. This is of course of great importance, but when tuning the controller one also wants to know what initial values of $K$, $T_i$ and $T_d$ should be chosen in order to avoid an all too lengthy tuning process. If dealing with a slow process, one could need to wait for hours, or even days, to evaluate wether the control works satisfactory.

***Model Based Controller Design*** If we have access to a mathematical model of the process, we can exploit it to calculate the controller parameters. This is usually called model based controller design and is treated in lab 2.

***Experimental Methods*** A different way to obtain controller parameters is to conduct simple experiments to gain knowledge of the process dynamics (behavior). Subsequently, known rules of thumb are used to tune the controller. The experimental methods do not guarantee good controller settings but often give reasonable initial values for the controller parameters. Today there exists a large number of different experimental methods for tuning PID-parameters. The perhaps most known, but not necessarily best, are the Ziegler-Nichols methods.

***Auto Tuning*** Today some commercial PID-controllers have built in tuning functions for automatic controller tuning. These functions are often based on some experimental method, cf. the above section.

## 5. Summary

***Assignment 5.1*** Summarize the most important differences between open loop control (feed forward, program control) and closed loop control (feedback).

***Assignment 5.2*** Discuss pros and cons of P-, PI- and PID-control of the upper and lower tank, respectively. Especially, answer the following questions and fill out the below table.

How is the control performance affected if the gain $K$ is small / large? (How is the answer affected by reference value changes and load disturbances? How is the control signal affected? How is the stationary error affected?)

How is the control performance affected if the integral time $T_i$ is small / big?

How is the control performance affected if the derivative time $T_d$ is small / big? Difference between the upper and lower tank?

**Table of suitable controller settings (bring this to lab 2!)**

| | övre tank | Lower tank |
|---|---|---|
| P | K = | K = |
| PI | K =<br>$T_i$ = | K =<br>$T_i$ = |
| PID | K =<br>$T_i$ =<br>$T_d$ = | K=<br>$T_i$ =<br>$T_d$ = |

## User Interface for Labs 1 and 2

Here follows a short description of the user interface of the software which is used during the tank labs. The interface consists of two windows: the “Process window” and the “Controller window”.

### The Process Window

This window gives an overview of the lab setup and shows how the various process objects are interconnected, see figure 7. To the right of the center line, real world objects are shown. We find for example a picture of the pump and animations of the water tanks together with blocks corresponding to the level sensors. To the left of the center line are the objects which have been implemented in the computer. Most important is the PID controller, but here are also different controls and switches. On the centerline, which constitutes a border between computer and reality, we find blocks which represent D/A- and A/D converters. These convert signals in Volts to digital numbers and vice versa (10 V corresponds to the digital number 1).

By moving the cursor to locations in the window, where there are measurable entities, (electric conductor, tanks with water levels and outflows, etc.) one can see their present values in the “Probe box”, bottom right.

By using the mouse and keyboard the following operations can be carried out:

**Manual / PID.** By clicking on the upper switch one chooses between manual- and PID-control of the pump. The current control mode is indicated by the window title and the routing of the virtual wires.

**Upper / lower tank.** By clicking on the lower switch, one can choose wether the controller should control the upper or lower tank, i.e. if the process output

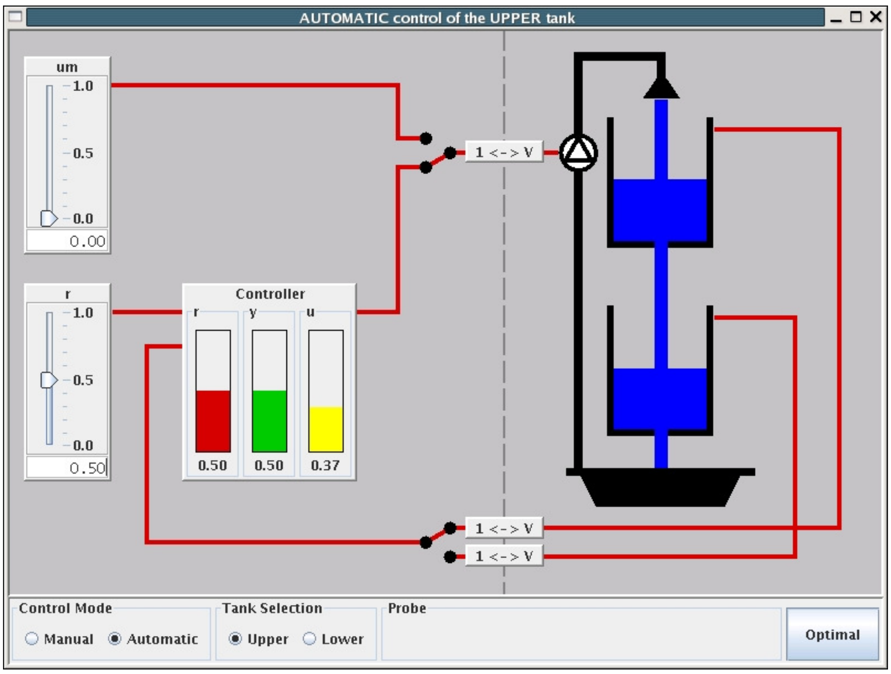



**Figure 7** Process View.

measurement should be taken from the upper or lower tank. Also current tank selection is indicated by the window title and the routing of the virtual wires.

**Reference value.** The control marked $r$ is used to set the reference value (between 0 and 1). The value is changed by pulling the triangle to the desired position with the mouse. Alternatively, one can click in the box where the present value is shown and enter a new value.

**Manual control.** The control marked $u_m$ is used to control the pump when it is driven manually. The value is changed in the same way as the reference value, cf. the above item.

**Optimal.** This function only works when controlling the lower tank. A time optimal controller is used to change the level in the lower tank to the reference value. The function can be used fast ”reset” of the process between two experiments.

### The Controller Window

This window shows the interconnections within the controller. Additionally, reference value and control signal are shown in two diagrams. At the upper left a block diagram of the controller is shown. By clicking the different blocks one can activate the P-, I- and D-parts independent of each other. In figure 8 the P- and I-parts are active, and we have a PI controller. The control marked $r$ is used as before to set the reference value. At the lower left there are three controls for changing the controller parameters $K$, $T_i$ and $T_d$. Also the title of this window indicates wether the upper or lower tank is chosen and wether the pump is controlled manually or by the controller.

To the right, two plot windows are shown. In the upper the reference value $r$ is shown, while the lower shows the control signal $u$ together with its components $P$, $I$ and $D$. The length of the time axis corresponds to 100 seconds when the upper tank is chosen and 400 seconds when the lower tank is chosen. Observe that the upper plot can be frozen using the button “Freeze Plot”.

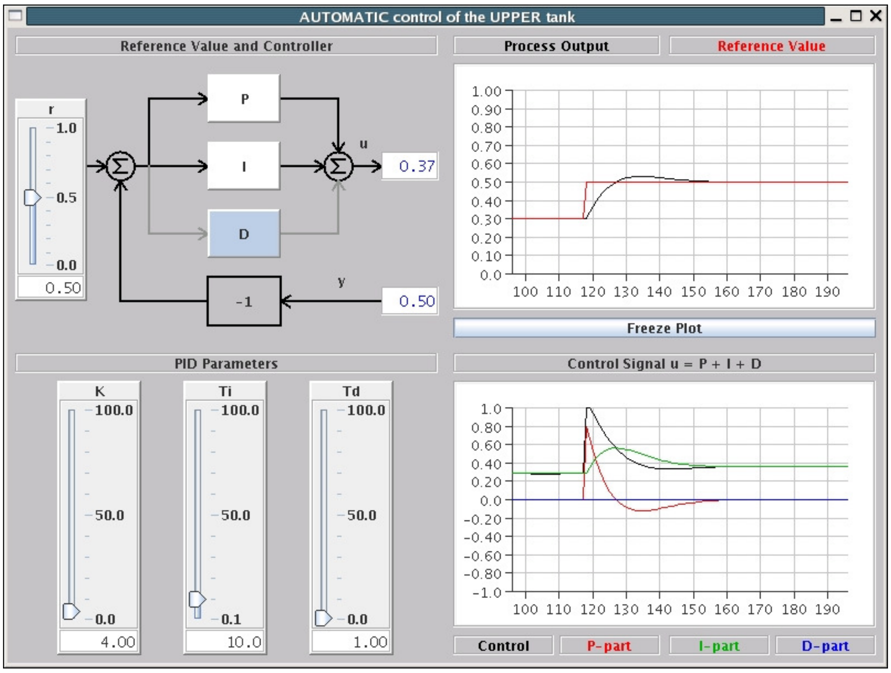



**Figure 8** Controller View.

# Automatic Control – Basic Course

# Laboratory Exercise 2

# Model construction and calculation of PID Controller

Department of Automatic Control
Lunds tekniska högskola
Latest updated December 2006

## 1. Introduction

Lab exercise 1 gave practical experience and insight into PID control. However, we lacked systematic methods for choosing the controller parameters. The purpose of this lab is to show how one can construct a mathematical model for the process one wants to control and how to calculate suitable controller settings using this model.

The lab is conducted on the same tank system which was used in lab 1, see figure 1.

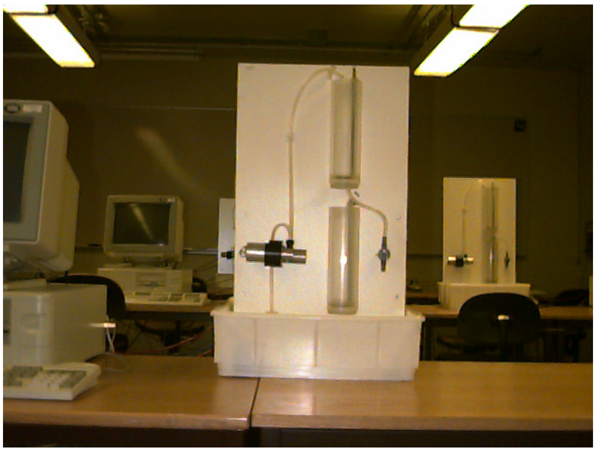


**Figure 1** Lab setup.

### Preparations

To get out as much as possible of the lab it is important that you master the concepts of linearization, transfer function, characteristic polynomial and pole placement.

You shall have read through this lab manual. You shall also have worked through the preparatory assignments 2.1, 2.2, 2.3, 2.5, 2.6, 3.1 and 3.5. Cooperation is allowed (and encouraged). Observe that assignments 3.1 and 3.5 are done on an exercise session.

The lab stars with a written test, where two randomly chosen review questions shall be answered. **Both questions must be somewhat correctly answered for you to do the lab. Additionally, you must be able to account for your solutions of the preparatory assignments.** The review questions are found on page 17.

Don't forget to bring the lab manual from lab 1 also to this lab in order to compare your results.

# 2. Model Construction

In this section we shall deduce a mathematical model for the tank system, starting out with physical principles and construction data. The obtained mathematical model shall then be verified through a couple of experiments.

***Assignment 2.1 (Preparation)*** Give the differential equations which describe how the level in the upper and lower tank, respectively, depend on time. An approximate relation between outflow speed $v(t)$ and tank level $h(t)$ in a tank is given by Toricelli's law:

$$v(t) = \sqrt{2gh(t)}$$

The dynamics in the hoses and the motor can be neglected. Let $A_1$ and $A_2$ represent the cross sections of the tanks, $a_1$ and $a_2$ the areas of their outflows, respectively. Assume that the flow $q$ from the pump is proportional to the motor voltage $u$ with $k$ being the constant of proportionality.

***Assignment 2.2 (Preparation)*** Show that if the tanks have the same cross section, $A_1 = A_2 = A$, we can write the model as

$$\begin{aligned} \frac{dh_1(t)}{dt} &= -\gamma_1\sqrt{2gh_1(t)} + \beta u(t) \\ \frac{dh_2(t)}{dt} &= \gamma_1\sqrt{2gh_1(t)} - \gamma_2\sqrt{2gh_2(t)} \end{aligned} \tag{1}$$

where $\beta = k/A$, $\gamma_1 = a_1/A$ and $\gamma_2 = a_2/A$.

Calculate theoretical values for the parameters $\beta$, $\gamma_1$ and $\gamma_2$ from the below construction data. Insert your answers into the below table.

| | |
|---|---|
| The cross sections of the tanks: | $A_1 = A_2 = 2.8 \cdot 10^{-3}$ m$^2$ |
| The outflow areas of the tanks: | $a_1 = a_2 = 7 \cdot 10^{-6}$ m$^2$ |
| Constant of proportionality for the pump: | $k = 2.7 \cdot 10^{-6}$ m$^3$/s/V |

***Assignment 2.3 (Preparation)*** In practice all the tank processes do not have exactly the same construction data. Additionally, their properties are changed over time – the holes are sedimented, the pumps are worn, etc. The theoretical parameter values are therefore not always totally reliable. The real values can, however, be estimated through a few simple experiments:

- $\beta$ can be estimated by blocking the outflow of the lower tank, setting a constant pump voltage and then measuring how long time it takes for the water to rise to a certain level.
- $\gamma_1$ and $\gamma_2$ can be measured by setting a constant pump voltage, waiting until the system reaches an equilibrium and then reading the stationary levels $h_1^0$ and $h_2^0$.

Starting out with equation (1), show how one can calculate experimental values of first $\beta$, then $\gamma_1$ and finally $\gamma_2$ using the above experiments.

***Assignment 2.4*** Log in according to your lab assistant's instructions. Execute the experiments and calculations according to assignment 2.3 to determine experimental values of $\beta$, $\gamma_1$ and $\gamma_2$. Insert the results in the below table.

| | Theoretical values | Experimental values |
|---|---|---|
| $\beta$ | | |
| $\gamma_1$ | | |
| $\gamma_2$ | | |

Check that the experimental values coincide fairly well with the theoretical ones. You shall preferably base your controller design on the experimental values.

***Assignment 2.5 (Preparation)*** Linearize the system (1) about an arbitrary equilibrium $(h_1^0, h_2^0)$. (During the lab we will use $h_1^0 = 10$ cm, $h_2^0 = 10$ cm)

***Units and Unit Conversions*** Although unit conversions are in principle simple, they often lead to errors. The problem is especially severe for a control engineer, who often works with many different units within the system. Most often one works with physical units during model construction. Later, when walking over to control, it is necessary to involve converters and conversion constants.

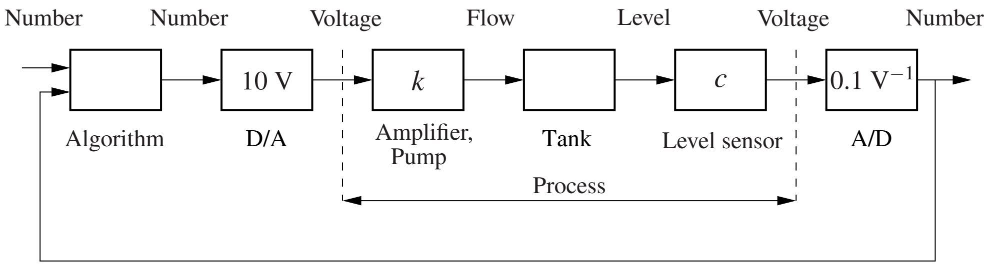



**Figure 2** Block diagram for the process with unit converters.

Figure 2 shows a block diagram of the process and controller with all involved converters. It is not obvious where the border between process and controller shall be drawn. It is, however, according to custom to choose the border so that the process inputs and outputs are of same units. With this convention the transfer function of the controller becomes unit-less. In our case the process in- and outputs will be of the units Volt (V). However, note that the control algorithm in the computer works with numbers (i.e. unit-less) because the A/D- and D/A converters contain a conversion factor of 10 V.

***Assignment 2.6 (Preparation)*** Introduce the two measurement signals

$$y_1(t) = c \cdot h_1(t)$$
$$y_2(t) = c \cdot h_2(t)$$

where the level sensors have the proportionality constants $c = 50$ V/m. Show that the linearized system from assignment 2.5 can be described by the following transfer function:

$$\Delta Y_1(s) = \frac{p\tau_1}{1+s\tau_1}\Delta U(s) \tag{2}$$
$$\Delta Y_2(s) = \frac{p\tau_2}{(1+s\tau_1)(1+s\tau_2)}\Delta U(s) \tag{3}$$

Determine the parameters $p$, $\tau_1$ and $\tau_2$ as functions of the process parameters $\beta$, $\gamma_1$, $\gamma_2$, $k$, $c$ and the the working point $h_1^0$, $h_2^0$.

# 3. Calculation of Controller Settings

In this section we shall calculate the controller settings for control of the upper and lower tank, respectively. We start out with the mathematical models obtained in the previous section. The controllers will be tested together with the tanksystem.

Controller parameters will depend on the specifications which we want the closed loop system to fulfill. A specification can have different forms; in this case the poles of the closed loop system shall be given as specification. By suitably placing the poles, one can achieve wanted speed and damping of the closed loop system.

In this lab we will work with PI- and PID-controllers. By choosing the PI(D)-parameters suitably, we can obtain a pre-specified characteristic polynomial (denominator polynomial) for the closed loop system, see figure 3.

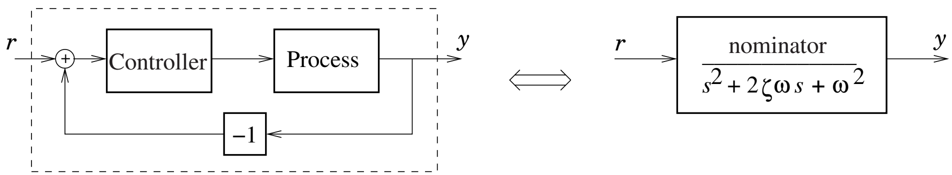



**Figure 3** The closed loop system is specified by a desired characteristic polynomial.

## Control of the Upper Tank

***Assignment 3.1 (Preparation)*** Use the model (2) from assignment 2.6 to design a PI-controller,

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right) \quad \Leftrightarrow \quad U(s) = K\left(1+\frac{1}{sT_i}\right)E(s)$$

for control of the upper tank. Choose the controller parameters such that the closed loop system gets a relative damping $\zeta$ and an undamped natural frequency $\omega$, i.e. such that the closed loop system gets a characteristic polynomial of the form

$$s^2 + 2\zeta\omega s + \omega^2$$

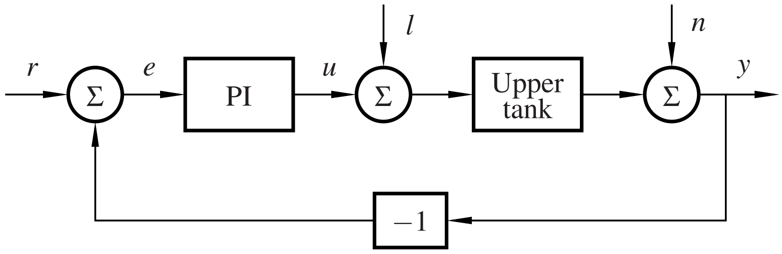



**Figure 4** block diagram for control of the upper tank.

In the answer $K$ and $T_i$ shall be expressed in the process parameters $p$ and $\tau_1$ together with the design parameters $\omega$ and $\zeta$.

***Poles and Zeros*** We shall begin by investigating control of the upper tank. A block diagram of the closed loop system is shown in figure 4. It is marked in the block diagram where load disturbances, $l$, and measurement noise, $n$, enter. The transfer functions from reference to output ($G_{yr}$), from load disturbances to output ($G_{yl}$) and from measurement noise to output ($G_{yn}$) are given below.

$$
\begin{aligned}
G_{yr} &= \frac{pK(s+\frac{1}{T_i})}{s^2+s(\frac{1}{\tau_1}+pK)+\frac{pK}{T_i}} \\
G_{yl} &= \frac{sp}{s^2+s(\frac{1}{\tau_1}+pK)+\frac{pK}{T_i}} \\
G_{yn} &= \frac{s(s+\frac{1}{\tau_1})}{s^2+s(\frac{1}{\tau_1}+pK)+\frac{pK}{T_i}}
\end{aligned}
$$

A step load disturbance $l$ corresponds to opening the side valve of the upper tank. The measurement disturbances $n$ can model measurement noise or a constant offset error in the level sensor of the tank.

The three transfer functions have the same denominator polynomial, whereas their nominator polynomials differ. As the controller parameters are changed, the poles of the system will move. In the transfer function from reference value to output, $G_{yr}$, also the zeros of the system will move. The zeros of systems $G_{yl}$ and $G_{yn}$ are unaffected by the controller parameters. If we want to determine how the pole placement affects the system, we shall mainly study the response to load disturbances. If we want to see the combined effect of poles and zeros we shall study the the response to a change in reference value.

***Assignment 3.2*** Fix $\zeta$ to 1 and vary $\omega$ according to the below table. Assume that the stationary level is $h_1^0 = 10$ cm and calculate the parameters $K$ and $T_i$ of a PI-controller, for every value of $\omega$. This can be done using the MATLAB script `calcpi` according to the following example (insert your estimated values of `beta`, `gamma1` and `gamma2`):

```
>> beta = ... ;
>> gamma1 = ... ;
>> gamma2 = ... ;
>> omega = 0.1;
```

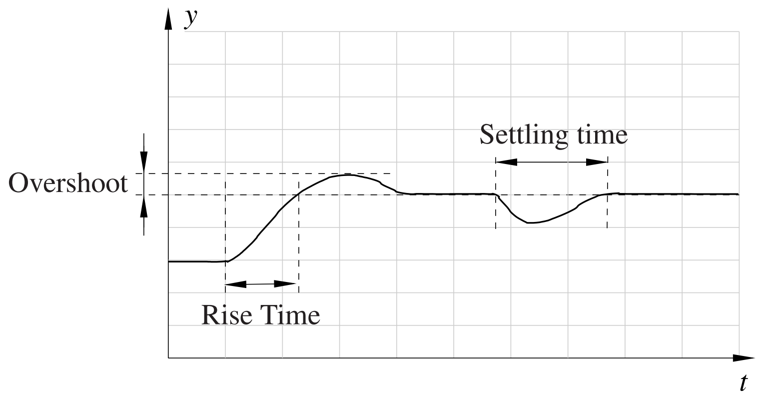



**Figure 5** The definition of rise time and overshoot when changing reference value and settling time of a load disturbance.

```
>> zeta = 1;
>> calcpi
K =
    3.6856
Ti =
   18.2071
```

Also view the script by typing

```
>> type calcpi
```

and compare the calculations with your preparatory assignments.

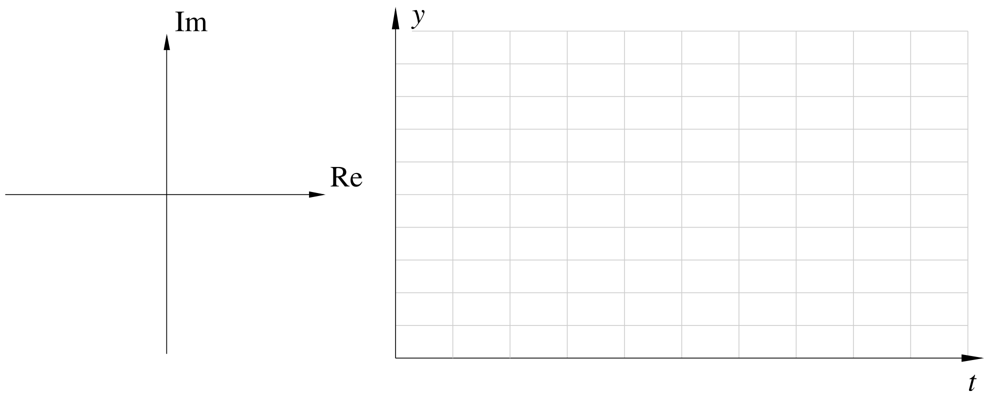
Try the controllers on the upper tank and investigate the responses to reference value changes and load disturbances. Draw the responses in the below time diagrams (cf. figure 5). Also insert the location of the poles in the pole-zero plots and compare with the shape of the responses; especially observe their speed.

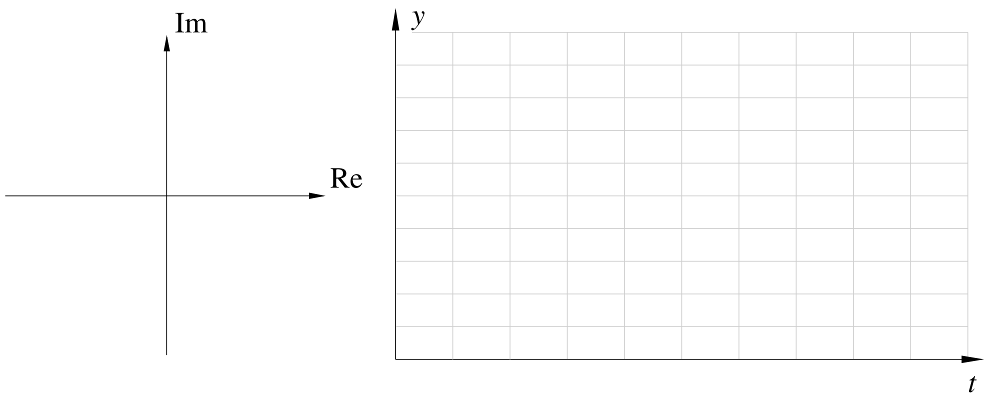
Carry out the experiments as follows:

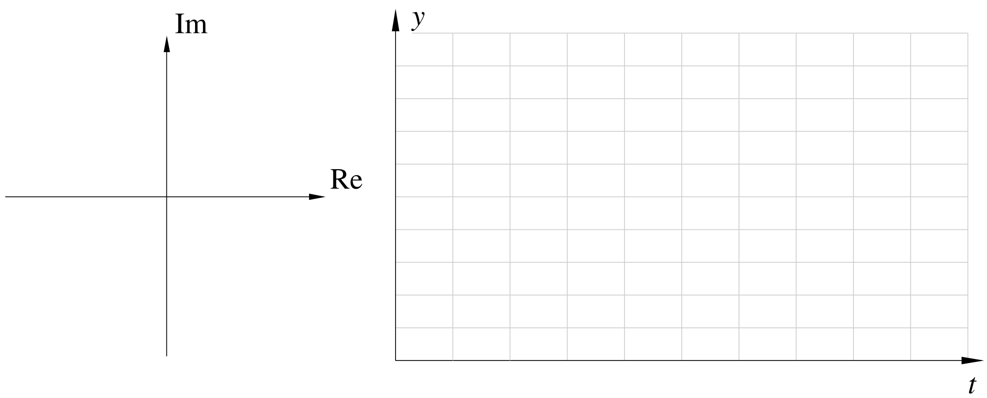
1. Make sure that the interface is set to **PI**-control of the **upper** tank.
2. Make sure that the side valve of the upper tank is closed.
3. Enter the controller parameters $K$ and $T_i$.
4. Enter the reference value 6 cm ($r = 0.3$) and wait until all signals have become stationary.
5. Issue a reference value change to 10 cm ($r = 0.5$) and draw its response. Enter the rise time and the size of the overshoot (see figure 5) in the table and also wether the control signal saturates (i.e. reaches its max value) and for how long.
6. When the system has anew reached a stationary state, open the side valve and draw the response to the load disturbance. Enter the settling time for the load disturbance in the table.

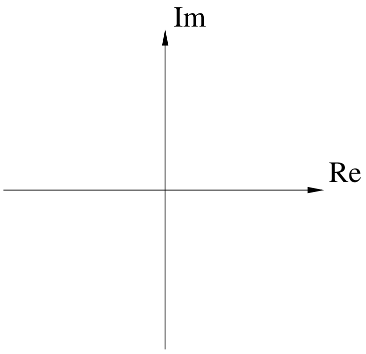
Finally, also try the controller for the upper tank which you ended up with in lab 1. (Fill out the last row of the table.)

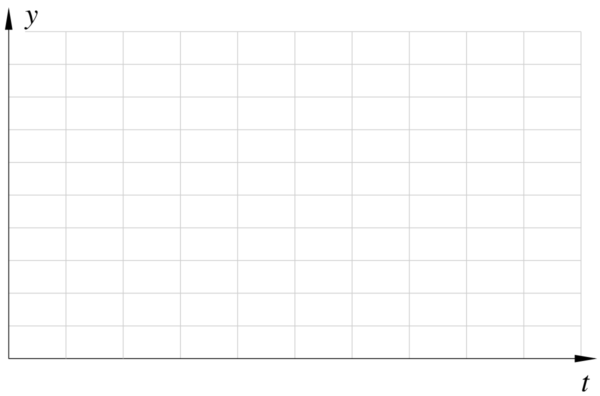
| | | | | Change in reference value | | | Load disturbance |
|---|---|---|---|---|---|---|---|
| $\omega$ | $\zeta$ | $K$ | $T_i$ | Rise time | Overshoot | Saturation | Settling time |
| 0.1 | 1 | | | | | | |
| 0.2 | 1 | | | | | | |
| 0.5 | 1 | | | | | | |
| | | | | | | | |



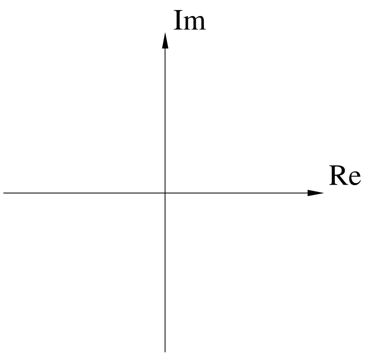


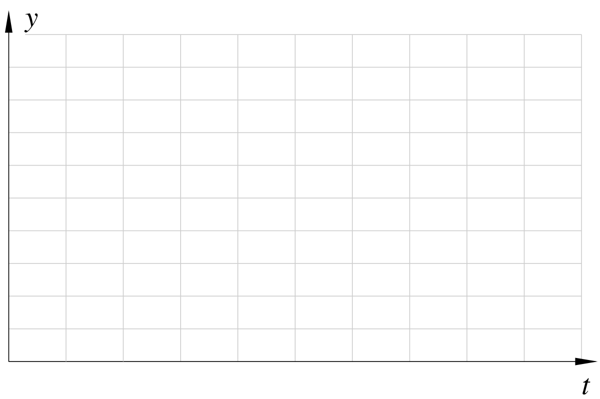



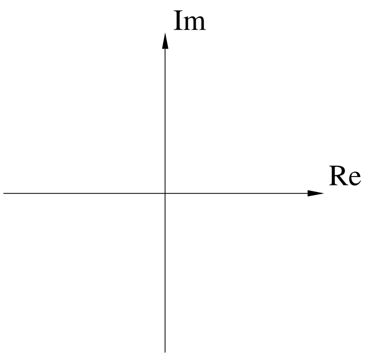

***Assignment 3.3*** Now fix $\omega$ to 0.2 and instead vary $\zeta$ according to the below table. Calculate the controller parameters $K$ and $T_i$ using MATLAB in the same way as previously. Try the controllers on the upper tank and investigate the responses to changes in reference value and load disturbances. Draw the responses in the time diagrams below. Also draw the locations of the poles in the pole-zero plots and compare with the shape of the responses; especially observe their damping.

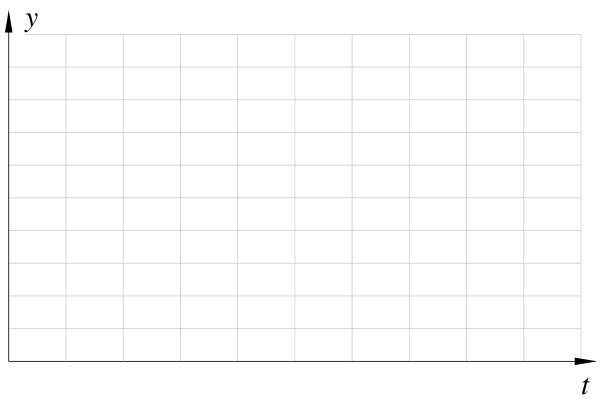
Carry out the experiments in the same way as in the previous assignment.

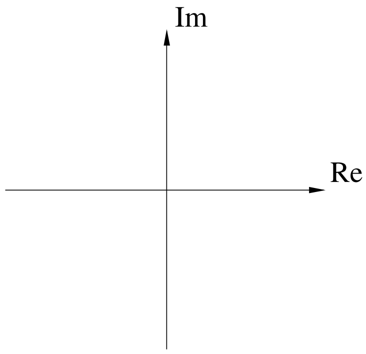
| | | | | Change in reference value | | | Load disturbance |
|---|---|---|---|---|---|---|---|
| $\omega$ | $\zeta$ | $K$ | $T_i$ | Rise time | Overshoot | Saturation | Settling time |
| 0.2 | 0.7 | | | | | | |
| 0.2 | 0.4 | | | | | | |
| 0.2 | 0.1 | | | | | | |

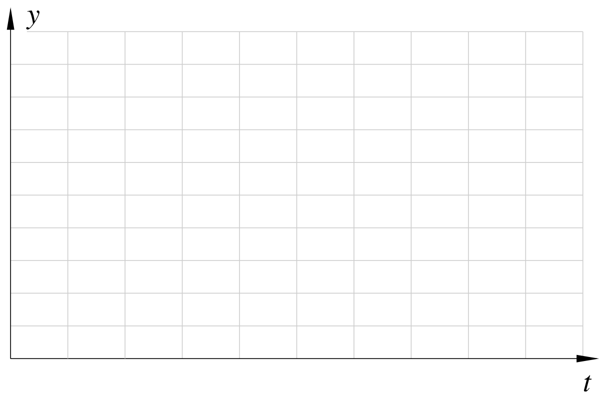





***Assignment 3.4 (Extra)*** Use one of the controllers calculated in assignment 3.3. Decrease the gain $K$ to one tenth of its calculated value. How will the step response change? Use the controller and try to explain the result.

$$K_{oldl} =$$
$$K_{new} =$$

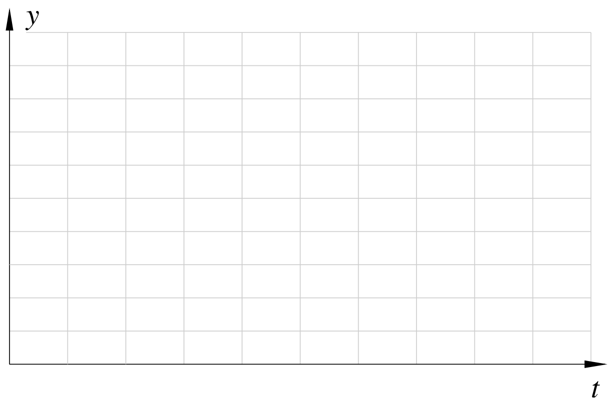



**Figure 6** Step response for $K_{old}$ and $K_{new}$.

*Hint:* From assignment 3.1 we can obtain the relation

$$\omega = \sqrt{\frac{Kp}{T_i}}$$
$$\zeta = \frac{Kp + \frac{1}{\tau_1}}{2\omega} \approx \frac{\sqrt{KpT_i}}{2}$$

---

---

---

## Control of the Lower Tank

***Assignment 3.5 (Preparation)*** Use the model (3) from assignment 2.6 in order t to design a PID-controller,

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right) \quad \Leftrightarrow \quad U(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right)E(s)$$

to control the level in the lower tank. Choose controller parameters such that the closed loop system gets the characteristic polynomial

$$(s+\alpha\omega)(s^2+2\zeta\omega s+\omega^2)$$

In the answer $K$, $T_i$ and $T_d$ shall be given in the process parameters $p$, $\tau_1$ and $\tau_2$ together with the design parameters $\omega$, $\zeta$ and $\alpha$.

***Poles and Zeros*** We shall now investigate control of the lower tank. A block diagram of the closed loop system is shown in figure 7. It is marked in the block diagram where load disturbances, $l_1$, $l_2$, and measurement noise, $n$, can enter. The transfer functions from reference to output ($G_{yr}$), from load disturbances to output ($G_{yl_1}$, $G_{yl_2}$) and from measurement noise to output ($G_{yn}$) are given below.

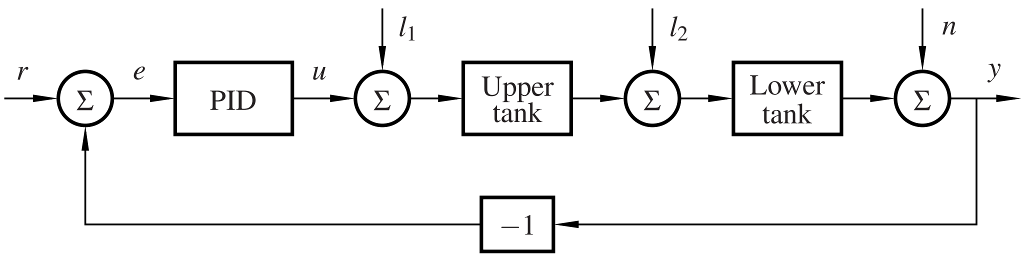



**Figure 7** Block diagram for control of the lower tank.

$$G_{yr} = \frac{Kp(s^2\frac{T_d}{\tau_1} + s\frac{1}{\tau_1} + \frac{1}{T_i\tau_1})}{s^3 + s^2(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{pKT_d}{\tau_1}) + s(\frac{1}{\tau_1\tau_2} + \frac{pK}{\tau_1}) + \frac{pK}{T_i\tau_1}}$$

$$G_{yl_1} = \frac{s\frac{p}{\tau_1}}{s^3 + s^2(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{pKT_d}{\tau_1}) + s(\frac{1}{\tau_1\tau_2} + \frac{pK}{\tau_1}) + \frac{pK}{T_i\tau_1}}$$

$$G_{yl_2} = \frac{s\frac{1}{\tau_1}(s + \frac{1}{\tau_1})}{s^3 + s^2(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{pKT_d}{\tau_1}) + s(\frac{1}{\tau_1\tau_2} + \frac{pK}{\tau_1}) + \frac{pK}{T_i\tau_1}}$$

$$G_{yr} = \frac{s(\frac{1}{\tau_1\tau_2} + s(\frac{1}{\tau_2} + \frac{1}{\tau_1}) + s^2)}{s^3 + s^2(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{pKT_d}{\tau_1}) + s(\frac{1}{\tau_1\tau_2} + \frac{pK}{\tau_1}) + \frac{pK}{T_i\tau_1}}$$

A step load disturbance $l_1$ corresponds to opening the side valve. The load disturbance $l_2$ corresponds to an extra inflow to the lower tank, whereas measurement disturbances $n$ can model measurement noise or constant measurement errors in the level sensor of the tank.

The four transfer functions have the same denominator polynomial, whereas the nominator polynomials differ. As the controller parameters are changed, the poles of the system will move. In the transfer function from reference value to output, $G_{yr}$, the zeros of the system will also move. The zeros of the systems $G_{yl_1}$, $G_{yl_2}$ and $G_{yn}$ are unaffected by the controller parameters. If we wish to investigate how the location of the poles influence the system, we shall mainly study the response to load disturbances. If we want to investigate the combined effect of poles and zeros, we may study the response to changes in reference value.

***Assignment 3.6*** Fix $\zeta$ to 0.7, $\alpha$ to 1 and vary $\omega$ according to the below table. Assume that the stationary level is $h_2^0 = 10$ cm and calculate the parameters for a PID -controller using MATLAB and the script `calcpid` as below:

```
>> omega = 0.04;
>> zeta = 0.7;
>> alpha = 1;
>> calcpid
K =
    3.9726
Ti =
   54.9773
Td =
   17.0928
```

Also view the script by typing

```
>> type calcpid
```

and compare the calculations with the ones in your preparation assignments.

Try the controller on the lower tank and investigate the response to changes in reference value and load disturbances. Draw the responses in the below time diagrams. Also enter the locations of the poles in the pole-zero plots and compare with the properties of the responses; especially observe their speed.

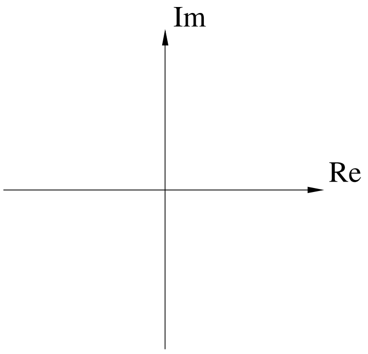
Carry out the experiments as follows

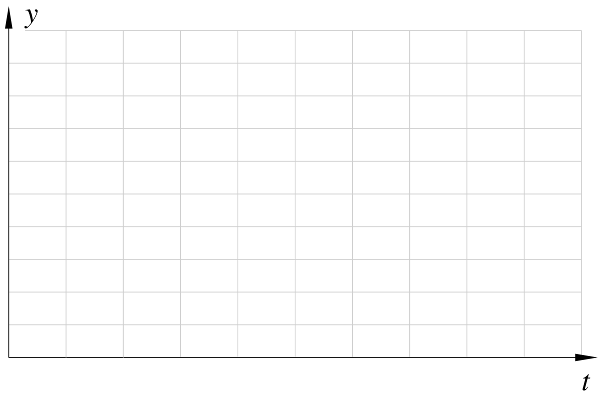
1. Make sure that the interface is set to **PID**-control of the **lower** tank.
2. Make sure that the side valve of the upper tank is closed.
3. Set the controller parameters $K$, $T_i$ and $T_d$.
4. Set the reference value at 6 cm ($r = 0.3$) and wait until all signals have become stationary. (The ”Optimal” button could be used for fast reset.)
5. Issue a change in reference value to 10 cm ($r = 0.5$) and draw its response. Enter the rise time and overshoot corresponding in the table. Also write down wether the control signal saturates, and for how long.
6. When the system is anew stationary, open the side valve and draw the response to the load disturbance. Enter the settling time for the load disturbance in the table.

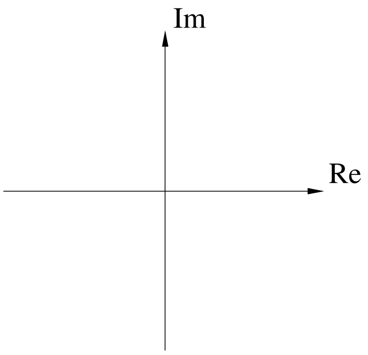
Finally, try the controller for the lower tank which you ended up with in lab 1. (Fill out the last row of the table.)

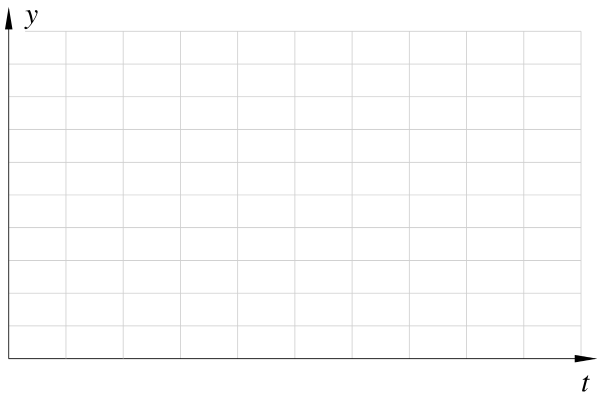
N.B.! These experiments take quite some time to perform. Preferably work with the summary in chapter 4 during that time.

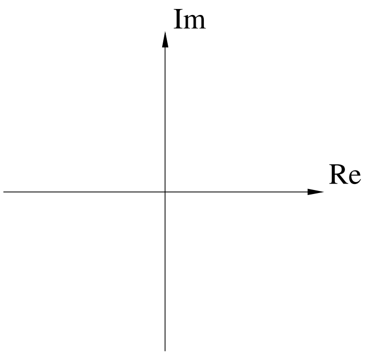
| | | | | | | Change in reference value | | | Load disturbance |
|---|---|---|---|---|---|---|---|---|---|
| $\omega$ | $\zeta$ | $\alpha$ | $K$ | $T_i$ | $T_d$ | Rise time | Overshoot | Saturation | Settling time |
| 0.035 | 0.7 | 1 | | | | | | | |
| 0.05 | 0.7 | 1 | | | | | | | |
| 0.1 | 0.7 | 1 | | | | | | | |
| | | | | | | | | | |

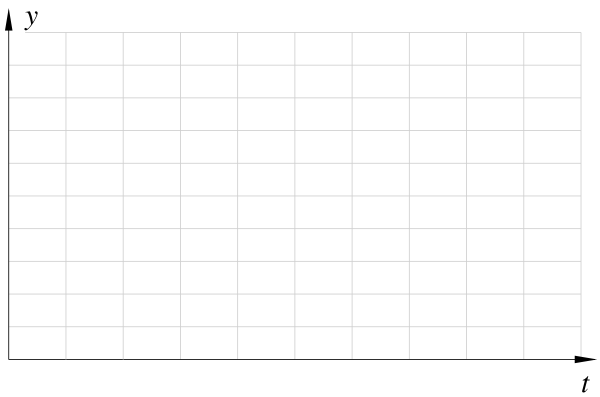


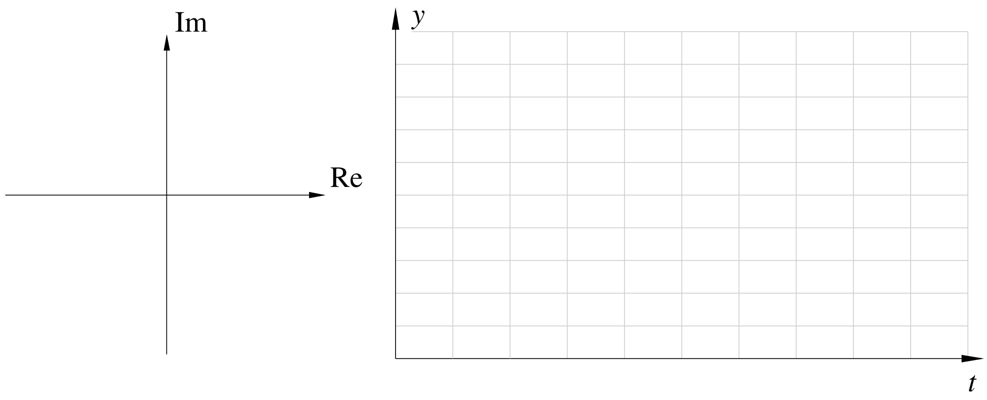



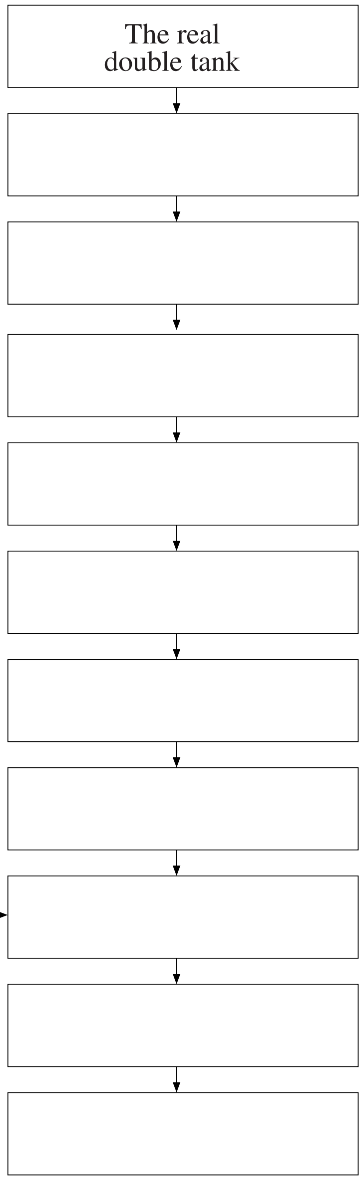



## 4. Summary

This summary intends to illustrate the workflow used in controller design and to post relevant questions which you shall be able to answer after finishing the experiments. The lab assistant will go through your summary before you pass this lab.

***Assignment 4.1*** Enter the stages you have gone through before and during the lab in the empty boxes of the below figure, in correct order. (Observe that the parameter estimation experiments are excluded. Where would they fit in?)

- Closed loop transfer function $Y(s) = \frac{G_p G_r}{1+G_p G_r} R(s)$
- Physical modeling
- Nonlinear differential equation
  $\dot{x} = f(x,u)$
- Linear differential equation
  $\dot{x} = ax + bu$
- Linearization
- Laplace transform
- Specification as pole placement
  $s^2 + 2\zeta\omega s + \omega^2 = 0$
- Test on process
- Evaluation
- Expressions for controller parameters
  $K = \ldots,\ T_i = \ldots,$
- Transfer functions
  $Y(s) = G(s)U(s)$




***Assignment 4.2*** Give at least two limitations of the real process which are not captured by the mathematical model (1).

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***Assignment 4.2*** During-PI control of the upper tank, how are the poles of the closed loop system changed when the parameter $\omega$ is increased? How does this affect responses to changes in reference and load disturbances?

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How do $K$ and $T_i$ change when $\omega$ is increased? Why don't we try $\omega = 5$ rad/s?

***Assignment 4.3*** During PI control of the upper tank, how are the poles of the closed loop system changed if the parameter $\zeta$ is decreased? How does it affect the responses to changes in reference value and load disturbances, respectively? How would the step response look in case we chose $\zeta = 0$?

***Assignment 4.4*** Why don't we use the D-part when controlling the upper tank?

***Assignment 4.5*** During PID-control of the lower tank, how many poles does the closed loop system have?

How are the poles of the closed loop system changed if the parameter $\omega$ is increased? What effect does this have to the responses to changes in reference value and load disturbances, respectively?

How is $K$, $T_i$ and $T_d$ changed when $\omega$ is increased? Why don't we try $\omega = 1$ rad/s?

***Assignment 4.7*** Enter your recommendations for suitable controller parameters in the below table. Compare with the parameters you ended up with in lab 1.

| | övre tank | Lower tank |
|---|---|---|
| P | K = | K = |
| PI | K =<br>$T_i$= | K =<br>$T_i$= |
| PID | K =<br>$T_i$=<br>$T_d$= | K=<br>$T_i$=<br>$T_d$= |

## Review Questions for Lab 2

1. Determine all stationary points $(x^0, u^0, y^0)$ for the system

$$\frac{dx}{dt} = -a\sqrt{x} + bu$$
$$y = cx$$

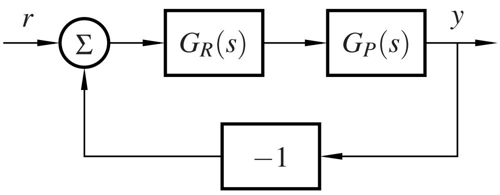
2. Linearize the system

$$\frac{dx}{dt} = -a\sqrt{x} + bu$$
$$y = cx$$

   about the stationary point $(x^0, u^0, y^0)$.

3. Write down the transfer function for a
   (a) P-controller
   (b) PI-controller
   (c) PID-controller

4. Determine the closed loop transfer function for the open loop




   when $G_R(s) = K$ and $G_P(s) = \frac{1}{1+sT}$

5. In second order systems it is common to talk about two parameters

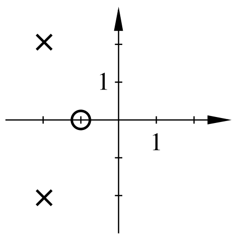
$$\zeta \qquad \text{(relative damping)}$$
$$\omega \qquad \text{(natural frequency)}$$

   Illustrate how these parameters define the location of the poles in a pole zero plot.

6. The transfer function of a system can be written as

$$G(s) = K\frac{Q(s)}{P(s)}$$

   Observe the pole-zero plot of the system (figure to the right) and determine $Q(s)$ and $P(s)$, respectively.



# Automatic Control – Basic Course

# Laboratory Exercise 3

# Control of a Flexible Servo

Department of Automatic Control
Lund University
November 2009

## 1. Introduction

In previous labs we have studied a process, which has been relatively simple. We have been able to control it satisfactory using a simple PID-controller. In this lab we will examine a process which is a bit more complicated and which requires a more advanced controller. The controller which we will use is based on state feedback and state estimation.

**Preparations**

- Read this manual carefully.
- Review the lectures on state feedback, Kalman filtering, and output feedback. At the beginning of the lab you should be able to answer the following questions:
  - What does state feedback mean? Explain in words!
  - Why is an observer often used in connection with state feedback? Explain in words!
  - Draw a block diagram that shows how an observer can be used in connection with state feedback.
- Solve the preparatory assignments 4.2, 5.2, and 6.1.
- Study the MATLAB scripts `define_process.m`, `design1.m`, `design2.m` and `design3.m` which are found in the appendix. Relate their content to the assignments in the manual.

## 2. The Process

A picture of the flexible servo to be controlled is shown in figure 1. The process consists of two masses which are interconnected by a spring. A conceptual drawing of the process is shown in figure 2. The mass at one end of the spring can be moved by a motor. We call this end the motor end and the other end the load end. Note that of the two dampers $d_1$ and $d_2$, only $d_2$ is present as a discrete part of the real process. Damping at other locations in the process can, however, be added and modeled according to figure 2.

The purpose is to control the position $p_2$ of the mass on the load side. In the lab we will assume that only $p_2$ is measurable. The remaining states will be estimated by a Kalman filter.

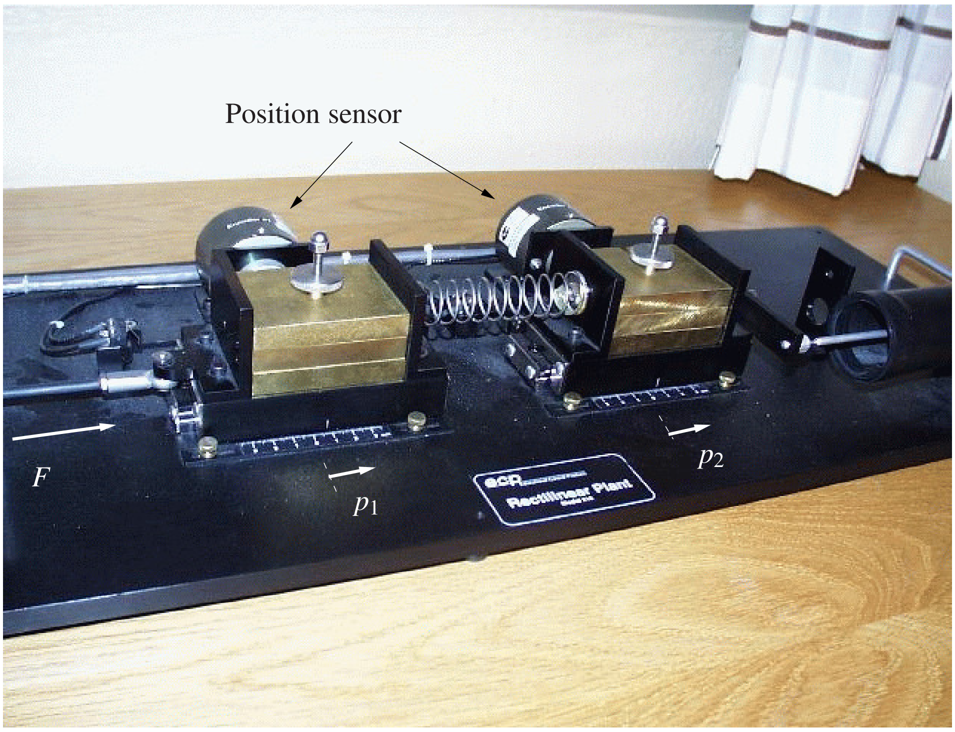



**Figure 1** The flexible servo.

### A Linear Model of the Process

The two masses are $m_1$ and $m_2$. The interconnecting spring has the spring constant $k$. The damping of the masses are $d_1$ and $d_2$, respectively.

The first mass is driven by a brush-less DC motor which is driven by a current-feedback amplifier. Motor and amplifier dynamics are neglected. The force of the motor becomes proportional to the input voltage $u$ of the amplifier according to

$$F = k_m u$$

A force balance gives the following dynamic model:

$$m_1 \frac{d^2 p_1}{dt^2} = -d_1 \frac{dp_1}{dt} - k(p_1 - p_2) + F$$

$$m_2 \frac{d^2 p_2}{dt^2} = -d_2 \frac{dp_2}{dt} + k(p_1 - p_2)$$

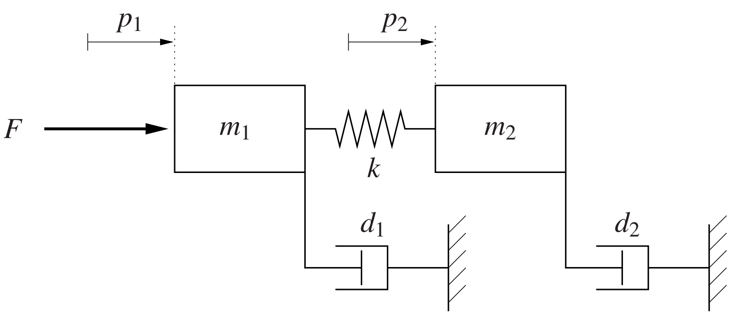



**Figure 2** Conceptual drawing of the process.

Introduce the state vector $x = \begin{pmatrix} p_1 & \dot{p}_1 & p_2 & \dot{p}_2 \end{pmatrix}^T$. The process can then be expressed in state space form as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & -\frac{d_1}{m_1} & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & -\frac{d_2}{m_2} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \frac{k_m}{m_1} \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & k_y & 0 \end{pmatrix}$$

For the real lab process the following values of constants and coefficients have been measured and estimated

$$\begin{aligned} m_1 &= 2.29 \text{ kg} \\ m_2 &= 2.044 \text{ kg} \\ d_1 &= 3.12 \text{ N/m/s} \\ d_2 &= 3.73 \text{ N/m/s} \\ k &= 400 \text{ N/m} \\ k_m &= 2.96 \text{ N/V} \\ k_y &= 280 \text{ V/m} \end{aligned}$$

**Analysis of the Process**

***Assignment 2.1*** Log in and start MATLAB according to the instructions from your lab assistant. Define the process `Gp` by executing the script `define_process.m`:

```
>> define_process
```

Calculate the poles of the system by typing

```
>> pole(Gp)
```

Where are the poles located? Is the system stable? Asymptotically stable? Simulate the impulse response of the process:

```
>> impulse(Gp,5)
```

Does the behavior agree with the stability analysis?

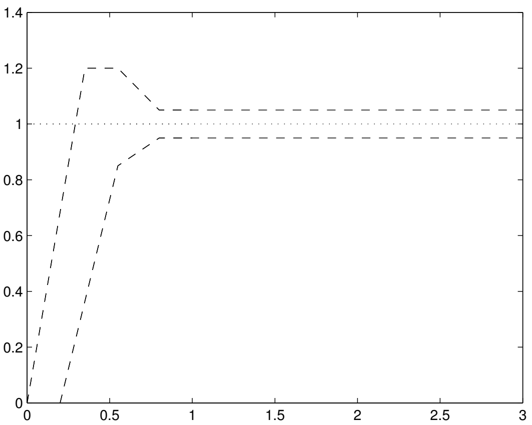



**Figure 3** The step response should stay within the marked region.

***Assignment 2.2*** Draw the Bode plot of the process

```
>> bode(Gp)
>> grid on
```

Note the resonance peak in the amplitude curve. At what frequency is it located? What approximate relation holds between the location of the resonance peak and the locations of the poles? Estimate the natural frequency by again studing the impulse response of the process. Does it coincide with that of the model?

## 3. Performance Specifications

The performance specifications are the requirements which the controlled system should fulfill. In this case we have chosen to specify the closed loop system in the time domain. We wish to have a well damped step response with a rise time between 0.2 and 0.4 seconds, see figure 3. At the same time, the magnitude of the control signal should never exceed 10, since this could damage the equipment.

The oscillative properties of the process make it impossible to fulfill the specifications with a PID-controller. An attempt using a PD-controller is shown in figure 4. The step response is fast enough, but the controller cannot damp out the oscillations.

## 4. State Feedback

To be able to change the process dynamics arbitrarily we make use of state feedback. First we verify that the system is controllable.

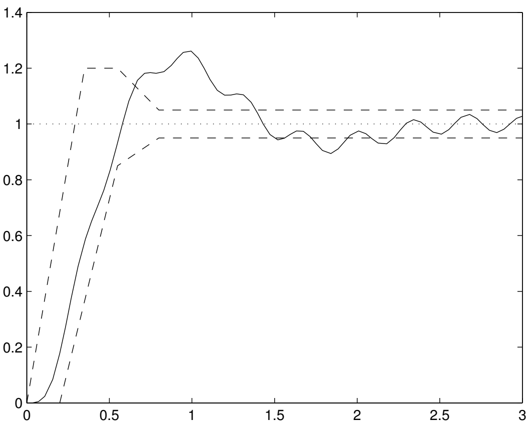



**Figure 4** Step response using a PD controller with $K = 0.07$ and $T_d = 0.1$.

***Assignment 4.1*** Calculate the rank (i.e. the number of linearly independent columns) of the system's controllability matrix:

```
>> Wc = [ ... ]
>> rank(Wc)
```

(Note that the matrices `A` and `B` are already defined in the workspace.)

What is the rank? Is the system controllable?

---

---

If we start out under the assumption that the entire state vector $x$ can be measured, the following control law can be used

$$u = -Lx + l_r r \tag{2}$$

Here $L$ is a row vector, $r$ is the reference value and $l_r$ is a scalar, see figure 5.

***Assignment 4.2 (Preparation)*** Show that the closed loop system can be written in the form

$$\begin{aligned}\dot{x} &= A_{cl}\,x + B_{cl}\,r \\ y &= C_{cl}\,x\end{aligned}$$

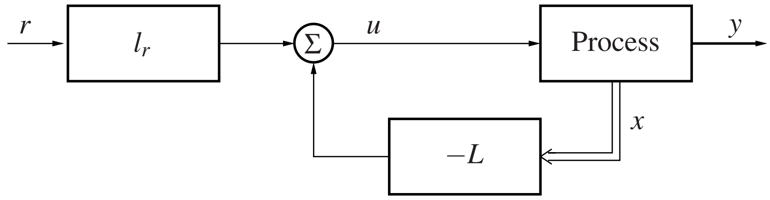



**Figure 5** State feedback.

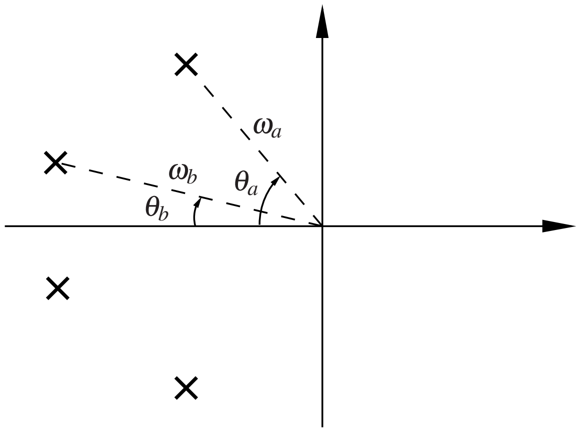



**Figure 6** Pole placement according to the characteristic polynomial $\left(s^2+2\zeta_a\omega_a s+\omega_a^2\right)\left(s^2+2\zeta_b\omega_b s+\omega_b^2\right)$, where $\zeta_a=\cos\theta_a$ och $\zeta_b=\cos\theta_b$.

when the control law (2) is used on the process. (1). What are $A_{cl}$, $B_{cl}$ and $C_{cl}$? How should $l_r$ be chosen to obtain unit static gain from $r$ to $y$? ◇

By means of state feedback we can place the poles of the closed loop system arbitrarily. Practically, there are, however, limitations, e.g. limitations on the control signal. Somewhat simplified one can say that, the further a pole is moved from its original location, the more control action will be required. In our case, we want to move the poles further into the left half plane, to make the closed-loop system fast and well-damped. For strongly oscillatory process poles, it is usually a good idea to only change their relative damping.

The desired pole placement can be expressed using a forth order characteristic polynomial (see figure 6):

$$\left(s^2+2\zeta_a\omega_a s+\omega_a^2\right)\left(s^2+2\zeta_b\omega_b s+\omega_b^2\right)$$

Given a pole placement, the feedback vector $L$ is easily computed using the command `place` (see `design1.m`).

***Assignment 4.3*** Edit the script `design1.m` and insert suitable values of $\omega_a$, $\zeta_a$, $\omega_b$ and $\zeta_b$. Then calculate the controller in MATLAB by typing:

```
>> design1
```

Open the Simulink model `model1.mdl` by typing

```
>> model1
```

Simulate the closed loop system and see if it fulfills the specification on the step response. Click on “Plot against specifications” after a simulation to compare the results to the performance specifications. Change the design parameters and repeat the procedure until a desired behavior is obtained. What is a suitable pole placement for the state feedback?

# 5. Observer

In practice, we cannot measure all states of the process, but only its output $y$. Instead we use a model of the process and feed the model with the same input as the real process. The difference between the outputs of the model and real process is used to correct the state of the model so that it converges to the state of the process. Such a device is called an *observer* or a *Kalman filter*.

The observer is described by

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + K\big(y - C\hat{x}\big) \tag{3}$$

where $\hat{x}$ denotes the estimated states. The column vector $K$ can be chosen such that the states of the observer converge to the states of the process arbitrarily fast, given that the system is observable.

***Assignment 5.1*** Calculate the rank of the observability matrix for the system:

```
>> Wo = [ ... ]
>> rank(Wo)
```

What is the rank? Is the system observable?

---

---

Using the Kalman filter we can establish feedback from the estimated states instead of the real states, see figure 7. The new control law becomes

$$u = -L\hat{x} + l_r r \tag{4}$$

***Assignment 5.2 (Preparation)*** Starting out with (3) and (4), show that the controller based on state feedback from the estimated states can be written in the form

$$\begin{aligned} \frac{d\hat{x}}{dt} &= A_R\hat{x} + B_{R_y}y + B_{R_r}r \\ u &= C_R\hat{x} + D_{R_y}y + D_{R_r}r \end{aligned}$$

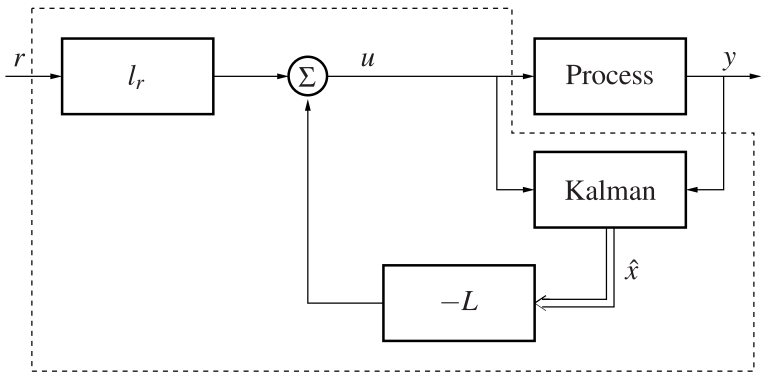



**Figure 7** Feedback from estimated states. The controller consists of the blocks within the dashed line.

What are $A_R$, $B_{R_y}$, $B_{R_r}$, $C_R$, $D_{R_y}$ and $D_{R_r}$? ◇

Since the process has four states, the observer will also have four states. We specify the poles of the observer according to the following characteristic polynomial:

$$\left(s^2+2\zeta_c\omega_c s+\omega_c^2\right)\left(s^2+2\zeta_d\omega_d s+\omega_d^2\right)$$

A suitable choice of poles depend, among other things, on the amount of measurement noise, the size of modeling inaccuracies and wether the initial condition is known. Fast poles mean high amplification of measurement noise, whereas slow poles give slow convergence of the estimate. As starting point, a rule of thumb stating that the observer poles should be 1.5-2 times faster than the state feedback, could be used.

***Assignment 5.3*** Edit the script `design2.m` and enter the values of $\omega_a$, $\zeta_a$, $\omega_b$ and $\zeta_b$ from Section 4. Then enter some suitable values for $\omega_c$, $\zeta_c$, $\omega_d$ and $\zeta_d$. Calculate the entire controller (state feedback + observer) by executing the script

```
>> design2
```

Then open the Simulink model `model2.mdl` by typing

```
>> model2
```

Simulate the closed loop system (using the “Simulated Process”) and see if it fulfills the specifications. Change the design parameters and iterate the procedure until desired behavior is obtained. (If necessary, also change the pole placement for the state feedback.) What is a suitable pole placement for the observer?

---

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***Assignment 5.4*** Draw the Bode plot of the controller by typing

```
>> bode(Gr)
```

What gain does the controller have for low frequencies? What does this mean to the controller’s ability to suppress constant load disturbances?

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***Assignment 5.5*** Try the controller on the “Real Process” (double click on the process block to toggle between simulated and “real” process). The “real” process has an additional motor time constant, measurement noise, and nonlinear friction added to the dynamics.

How does the “real” step response differ from the simulated one?

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# 6. Integral Action

To eliminate stationary errors due to friction, integral action is introduced in the controller, see figure 7. The integrator is introduced as an extra state $x_i$ according to

$$x_i = \int (r - y)\,dt$$
$$\dot{x}_i = r - y = r - Cx \tag{5}$$

If the extended state vector

$$x_e = \begin{pmatrix} x \\ x_i \end{pmatrix}$$

is introduced, the extended system (i.e. the process and the integrator) can be written

$$\begin{aligned}\dot{x}_e &= \underbrace{\begin{pmatrix} A & 0 \\ -C & 0 \end{pmatrix}}_{A_e} x_e + \underbrace{\begin{pmatrix} B \\ 0 \end{pmatrix}}_{B_e} u + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B_r} r \\ y &= \underbrace{\begin{pmatrix} C & 0 \end{pmatrix}}_{C_e} x_e\end{aligned}$$

If we, for the moment, reassume that the entire state vector is measurable, we can establish feedback from both the states of the process and the integral state according to

$$u = -Lx - l_i x_i + l_r r = -L_e x_e + l_r r$$

where

$$L_e = \begin{pmatrix} L & l_i \end{pmatrix}$$

The closed loop system becomes

$$\begin{aligned}\dot{x}_e &= (A_e - B_e L_e)x_e + (B_e l_r + B_r)r \\ y &= C_e x_e\end{aligned}$$

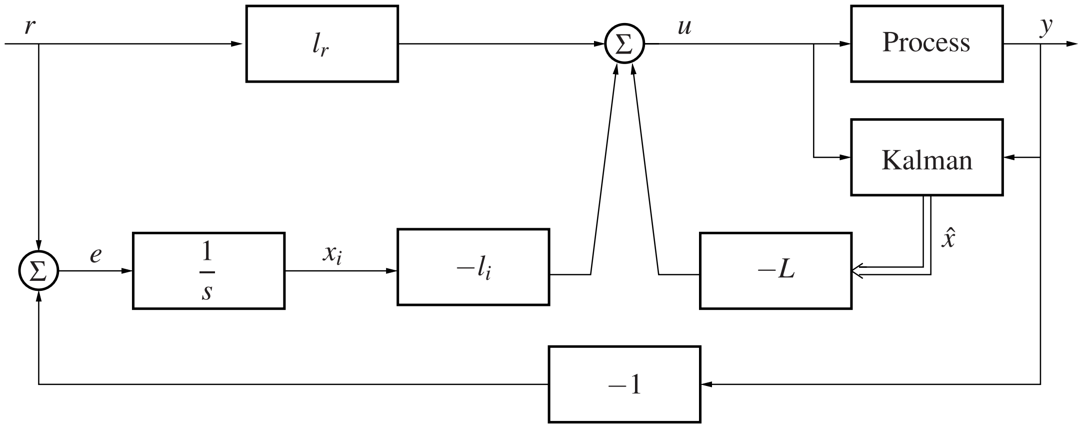


**Figure 8** Feedback from estimated states with integral action.

Because the extended system has five states, the poles of the state feedback are now specified using a fifth order characteristic polynomial:

$$\left(s^2+2\zeta_a\omega_a s+\omega_a^2\right)\left(s^2+2\zeta_b\omega_b s+\omega_b^2\right)\left(s+\omega_e\right)$$

As before, we cannot measure the states of the process. Consequently, feedback is established from the estimated states and the integrator according to the control law

$$u = -L\hat{x} - l_i x_i + l_r r \tag{6}$$

***Assignment 6.1 (Preparation)*** Starting out with (3), (5) and (6), show that a controller with integral action based on state feedback from estimated states can be written in the form

$$\begin{pmatrix} \frac{d\hat{x}}{dt} \\ \frac{dx_i}{dt} \end{pmatrix} = \underbrace{\begin{pmatrix} * & * \\ * & * \end{pmatrix}}_{A_R} \begin{pmatrix} \hat{x} \\ x_i \end{pmatrix} + \underbrace{\begin{pmatrix} * \\ * \end{pmatrix}}_{B_{Ry}} y + \underbrace{\begin{pmatrix} * \\ * \end{pmatrix}}_{B_{Rr}} r$$

$$u = \underbrace{\begin{pmatrix} * & * \end{pmatrix}}_{C_R} \begin{pmatrix} \hat{x} \\ x_i \end{pmatrix} + \underbrace{\begin{pmatrix} * \end{pmatrix}}_{D_{Ry}} y + \underbrace{\begin{pmatrix} * \end{pmatrix}}_{D_{Rr}} r$$

What are $A_R$, $B_{R_y}$, $B_{R_r}$, $C_R$, $D_{R_y}$ and $D_{R_r}$? ◇

***Assignment 6.2*** Edit the script `design3.m` and insert your values on $\omega_a$, $\zeta_a$, $\omega_b$, $\zeta_b$, $\omega_c$, $\zeta_c$, $\omega_d$ and $\zeta_d$. Also insert suitable values of $\omega_e$. Calculate the entire controller (state feedback + integrator + observer) by executing the script

```
>> design3
```

Open the Simulink model `model3.mdl` by typing

```
>> model3
```

Simulate the closed loop system and see wether it fulfills the specifications. Change the design parameters and iterate the procedure until the specifications are fulfilled. What is a suitable pole placement?

---

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***Assignment 6.3*** Draw the Bode plot of the controller using

```
>> bode(Gr)
```

How can it seen that the controller has integral action?

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***Assignment 6.4*** Try the controller on the "real" process. How do the results differ from Assignment 5.5.?

When integral action is introduced, the term $l_r r$ is no longer needed in the control law to obtain the correct static gain – this is handled by the integrator. Instead $l_r$ can be chosen to trim the step response at reference value changes. As seen in (6), $l_r \neq 0$ means a direct connection between reference value and control signal. A value $l_r > 0$ can also be used to give the process an extra "push" at a reference value step. (This could be especially useful when controlling the "real" process, which has friction.)

***Assignment 6.6*** What value does $l_r$ have now? Change the value of `lr` in `design3.m` and conduct new experiments on the "real" process. What is a suitable value for $l_r$? What happens if $l_r$ is negative?

## 7. Summary

This summary is intended to review relevant questions which you should be able to answer after finishing the experimental part. The lab assistant will go through your summary before you can pass the lab.

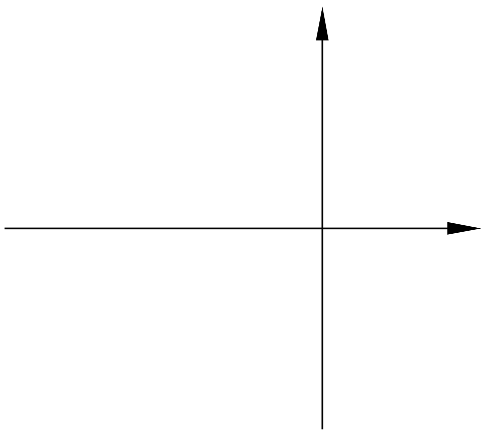
***Assignment 8.1*** The flexible servo is a strongly resonant process. How can this be seen in its Bode plot and pole-zero diagram, respectively?

***Assignment 8.2*** How can state feedback be used if all states are not measurable?

***Assignment 8.3*** When using state feedback from estimated states (Section 5), how many poles does the closed loop system have?

***Assignment 8.4*** How many states did the controller with integral action (Section 6) contain? Which?

***Assignment 8.5*** Draw *all* poles of the closed-loop system when using the final controller with integral action (Section 6) in the pole zero plot below:



***Assignment 8.6*** How can it be seen in the Bode plot of a controller whether it has integral action or not?

# A. MATLAB Scripts

### define_process.m

```
% Create a linear model of the process

m1 = 2.29; m2 = 2.044;                % masses
d1 = 3.12; d2 = 3.73;                 % damping constants
k = 400;                              % spring constant
km = 2.96;                            % motor constant
ky = 280;                             % measurement constant

A = [0 1 0 0; -k/m1 -d1/m1 k/m1 0; 0 0 0 1; k/m2 0 -k/m2 -d2/m2];
B = [0; km/m1; 0; 0];
C = [0 0 ky 0];
D = 0;

Gp = ss(A,B,C,D);                     % create state space model of the process
```

### design1.m — Calculation of controller based on pure state feedback

```
% Design of state feedback

omegaa = ...;                         % speed of one pole pair
zetaa =  ...;                         % damping of one pole pair
omegab = ...;                         % speed of the other pole pair
zetab =  ...;                         % damping of the other pole pair

pc = conv([1 2*omegaa*zetaa omegaa^2],[1 2*omegab*zetab omegab^2]);
poles1 = roots(pc);

L  = place(A,B,poles1);               % compute the state feedback vector L

lr = 1/(C*inv(-A+B*L)*B);             % compute lr such that the static gain
                                      % from r->y becomes 1
```

### design2.m — Calculation of controller based on state feedback from observer

```
% Design of state feedback

omegaa = ...;                         % speed of one pole pair
zetaa =  ...;                         % damping of one pole pair
omegab = ...;                         % speed of the other pole pair
zetab =  ...;                         % damping of the other pole pair

pc = conv([1 2*omegaa*zetaa omegaa^2],[1 2*omegab*zetab omegab^2]);
poles1 = roots(pc);

L  = place(A,B,poles1);               % compute the state feedback vector L
lr = 1/(C*inv(-A+B*L)*B);             % compute lr such that the static gain
                                      % from r->y becomes 1
% Design of Observer

omegac = ...;                         % speed of one pole pair
zetac =  ...;                         % damping of one pole pair
omegad = ...;                         % speed of the other pole pair
zetad =  ...;                         % damping of the other pole pair

po = conv([1 2*omegac*zetac omegac^2],[1 2*omegad*zetad omegad^2]);
poles2 = roots(po);
```

```
K = place(A',C',poles2)';              % compute the Kalman gain K

% Computation of controller (observer + state feedback)

AR = A-B*L-K*C;
BRy = K;
BRr = B*lr;
CR = -L;
DRy = 0;
DRr = lr;

Gr = -ss(AR, BRy, CR, DRy);            % transfer function from -y to u
```

### design3.m — Calculation of controller based on state feedback from observer with integral action

```
% Design of state feedback with integral action

Ae = [A zeros(4,1); -C 0];             % A-matrix for the extended system
Be = [B; 0];                           % B-matrix for the extended system

omegaa = ...;                          % speed of one pole pair
zetaa =  ...;                          % damping of one pole pair
omegab = ...;                          % speed of the other pole pair
zetab =  ...;                          % damping of the other pole pair
omegae = ...;                          % speed of the fifth pole

pc = conv([1 2*omegaa*zetaa omegaa^2],[1 2*omegab*zetab omegab^2]);
pc = conv(pc, [1 omegae]);
poles1 = roots(pc);

Le  = place(Ae,Be,poles1);             % compute the state feedback vector Le
L = Le(1:4);
li = Le(5);
lr = 0;                                % direct term from reference value

% Design of observer

omegac = ...;                          % speed of one pole pair
zetac =  ...;                          % damping of one pole pair
omegad = ...;                          % speed of the other pole pair
zetad =  ...;                          % damping of the other pole pair

po = conv([1 2*omegac*zetac omegac^2],[1 2*omegad*zetad omegad^2]);
poles2 = roots(po);

K = place(A',C',poles2)';              % compute the Kalman gain K

% Computation of controller (observer + state feedback with integral action)

AR = [A-B*L-K*C -B*li; zeros(1,4) 0];
BRy = [K; -1];
BRr = [B*lr; 1];
CR = [-L -li];
DRy = 0;
DRr = lr;

Gr = -ss(AR,BRy,CR,DRy);               % transfer function from -y to u
```