Introduction, The PID Controller, State Space Models

Automatic Control, Basic Course, Lecture 1

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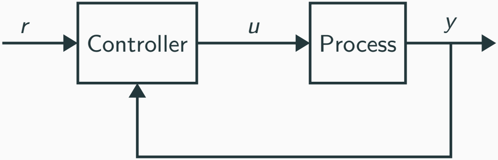
Content

Introduction

The Simple Feedback Loop



- Reference value r
- Control signal u
- Measured signal/output y

The problem: Design a controller such that the output follows the reference signal as good as possible

Find the Control Problem - 1

Find the Control Problem - 1

- Reference value - Desired temperature
- Control signal - E.g., power to the AC, amount of hot water to the radiators
- Measured value - The temperature in the room

Find the Control Problem - 2

Find the Control Problem - 2

- Reference value - Desired speed
- Control signal - Amount of gasoline to the engine
- Measured value - The speed of the car

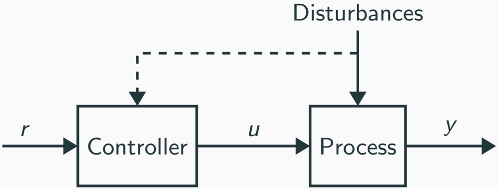
Find the Control Problem - 3

Find the Control Problem - 3

- Reference value - Number of bacterias
- Control signal - Food
- Measured value - E.g., the oxygen level in the tank

Feedforward

Some systems can operate well without feedback, i.e., in open loop.



Examples of open loop systems?

Feedforward vs. Feedback

Benefits with feedback:

- Stabilize unstable systems
- The speed of the system can be increased
- Less accurate model of the process is needed
- Disturbances can be compensated
- WARNING: Stable systems might become unstable with feedback

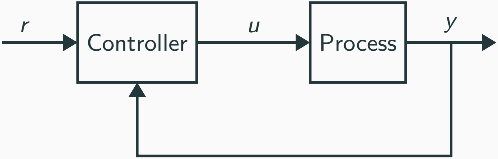
Feedforward and feedback are complementary approaches, and a good controller typically uses both.

The PID Controller

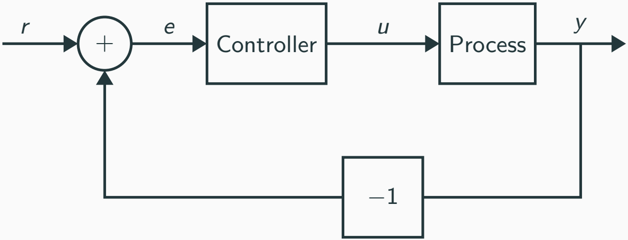
The Error

The input to the controller will be the error, i.e., the difference between the reference value and the measured value.

$$e = r - y$$

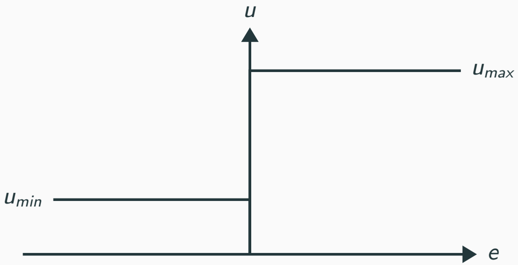


New block scheme:



On/Off Controller

$$u = \begin{cases} u_{max} & \text{if } e > 0 \\ u_{min} & \text{if } e < 0 \end{cases}$$



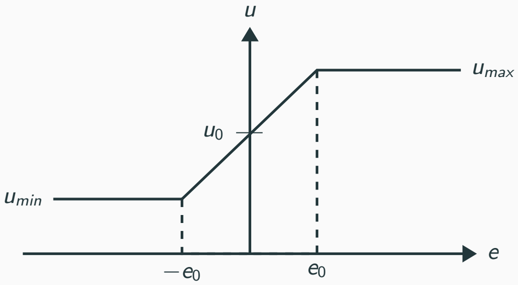
Usually not a good controller. Why?

The P Part

Idea: Decrease the controller gain for small control errors.

P-controller:

$$u = \begin{cases} u_{max} & \text{if } e > e_0 \\ u_0 + Ke & \text{if } -e_0 \leq e \leq e_0 \\ u_{min} & \text{if } e < -e_0 \end{cases}$$



The P Part

Idea: Decrease the controller gain for small control errors.

P-controller:

$$u = \begin{cases} u_{max} & \text{if } e > e_0 \\ u_0 + Ke & \text{if } -e_0 \le e \le e_0 \\ u_{min} & \text{if } e < -e_0 \end{cases}$$

The control error

$$e = \frac{u - u_0}{K}$$

To have $e = 0$ at stationarity, either:

- $u_0 = u$
- $K = \infty$

The P Part

Idea: Decrease the controller gain for small control errors.

P-controller:

$$u = \begin{cases} u_{max} & \text{if } e > e_0 \\ u_0 + Ke & \text{if } -e_0 \leq e \leq e_0 \\ u_{min} & \text{if } e < -e_0 \end{cases}$$

The control error

$$e = \frac{u - u_0}{K}$$

To have $e = 0$ at stationarity, either:

- $u_0 = u$ (What if u varies?)
- $K = \infty$ (On/off control)

The I Part

Idea: Adjust u_0 automatically to become u.

PI-controller:

$$u = K\left(\frac{1}{T_i}\int e(t)\mathrm{d}t + e\right)$$

Compared to the P-controller, now

$$u_0 = \frac{K}{T_i}\int e(t)\mathrm{d}t$$

At stationary $e = 0$ if and only if $r = y$.

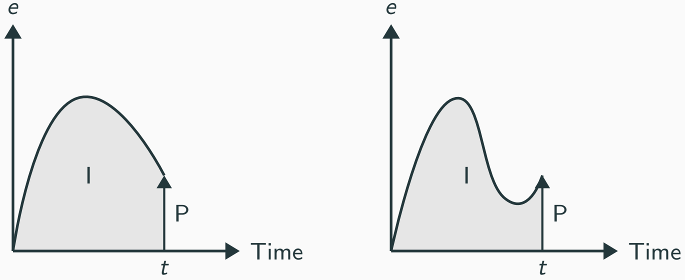
PI controller archives what we want, if performance requirements are not extensive.

The D Part

Idea: Speed up the PI-controller.

PID-controller:

$$u = K\left(e + \frac{1}{T_i}\int e(t)\mathrm{d}t + T_d\frac{\mathrm{d}e}{\mathrm{d}t}\right)$$



P acts on the current error, I acts on the past error, D acts on the "future" error

State Space Models

State Space Models

Linear dynamics can be described in the following form

$$\dot{x} = Ax + Bu$$
$$y = Cx\,(+Du)$$

Here $x \in \mathbb{R}^n$ is a vector with states. States can have a physical "interpretation", but not necessary.

In this course $u \in \mathbb{R}$ and $y \in \mathbb{R}$ will be scalars. (For MIMO systems, see Multivariable Control (FRTN10))

Example

Example

The position of a mass m controlled by a force u is described by

$$m\ddot{x} = u$$

where x is the position of the mass.

Introduce the states $x_1 = \dot{x}$ and $x_2 = x$ and write the system on state space form. Let the position be the output.

Dynamical Systems

	Continous Time	Discrete Time (sampled)
Linear	This course	Real-Time Systems (FRTN01)
Nonlinear	Nonlinear Control and Servo Systems (FRTN05)	

Next lecture: Nonlinear dynamics can be linearized.