Math Repetition for Automatic Control, Basic Course

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Complex numbers

1.

a. What is the real part $Re(z)$ and the imaginary part $Im(z)$ of the complex number

$$z = -2 + 3i$$

b. Mark the number $z = 2 + 4i$ in the complex plane.

c. Mark the number $z = -1 + i$ in the complex plane, and indicate its magnitude and argument.

d. Compute the magnitude, $|z|$, and argument, $\arg(z)$, of $z = -1 + i$.

e. Write $z = -1 + i$ in polar coordiantes.

f. What is the real and imaginary parts of $z = 3e^{\pi i}$

2.

a. Compute $|e^{\omega i}|$, where ω is a real number.

b. Compute $\arg(e^{\omega i})$, where ω is a real number.

c. Compute $|-2(-1+2i)(-4-3i)|$

d. Compute $\arg(-2(-1+2i)(-4-3i))$

e. Compute $|\frac{2e^{-5i}(2-i)^2}{2i+3}|$

f. Compute $\arg(\frac{2e^{-5i}(2-i)^2}{2i+3})$

Second order polynomial equations

3. Solve $x^2 - x + 4 = 0$

4. Solve $3x^2 + 2x + 1 = 0$

Partial fractions expansion

5. Expand

$$f(x) = \frac{1}{(x+1)(x+2)}$$

in partial fractions. That is express $f(x)$ as

$$f(x) = \frac{a}{x+1} + \frac{b}{x+2}$$

where a and b are constants.

6. Expand

$$f(x) = \frac{3x+11}{(x+1)(x-3)(x+2)}$$

in partial fractions.

7. Expand

$$f(x) = \frac{2}{x^2+3x+2}$$

in partial fractions.

Matrices

8.

a. Compute the product of the matrices A and B.

$$A \cdot B = \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ 4 & -5 \end{pmatrix}$$

b. Compute the product of the matrices A and B.

$$A \cdot B = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \end{pmatrix}$$

c. Compute the product of the matrices A and B.

$$A \cdot B = \begin{pmatrix} -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

9. Compute the determinant of the matrix

$$A = \begin{pmatrix} -2 & 4 \\ 1 & 0 \end{pmatrix}$$

10. Invert the matrix A

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

11.

a. Compute the eigenvalues of matrix A in problem 10.

b. Compute the eigenvalues of the matrix

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

12.

a. Consider the following system of equations

$$\begin{aligned} 5x_1 + 3x_2 &= 7 \\ 2x_1 - x_2 &= 0 \end{aligned}$$

Express the system as $Ax = B$, where A is a matrix, B a vector, and x is given by:

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

b. Express the system of equations below as $Ax = B$

$$\begin{aligned} x_1 + x_3 &= 0 \\ x_2 - x_3 &= 1 \\ x_1 + x_2 &= 2 \end{aligned}$$

Taylor series expansion

13.

a. Write the Taylor series expanson of $f(x) = x^2$ at $x = 2$. Neglect terms of order 2 and above.

b. Write the Taylor series expanson of $f(x, u) = 5\sqrt{3x} + \sin(u)$ at $x = 3, u = \pi$. Neglect terms of order 2 and above.