LUND INSTITUTE OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

Automatic Control, Basic Course (FRT010)
Exam 2013-12-07

Points and grades

All answers must include a clear motivation. The total number of points is 25. The maximum number of points is specified for each subproblem.

Grade 3: at least 12 points
4: at least 17 points
5: at least 22 points

Accepted aid

Mathematical collections of formulae (e.g. TEFYMA), 'Collections of formulae in automatic control', and calculators that are not programmed in advance.

Results

You should write a personal code on your cover sheet. When the exams have been corrected, the results will be presented on the course web page and you can check your grade using your code.

The corrected exams will be displayed at December 8, 13:00, inside the entrance of the International College Building.

1. Consider the system

$$\dot{x} = \begin{pmatrix} -7 & 2 \\ 0 & 4 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

a. Determine the transfer function from u to y. (1 p)

b. Determine the poles of the system. Is the system unstable, (marginally) stable or asymptotically stable? (1 p)

2. Consider the system of nonlinear differential equations below:

$$\dot{x}_1 = -2x_1 + u^2$$
$$\dot{x}_2 = -x_1 + \sqrt{x_2}$$

a. Assume $u^0 = 1$ and find the stationary points of the system. (1 p)

b. Linearize the system around the stationary point from **a.** If you did not solve **a.**, use the point $(u^0, x_1^0, x_2^0) = (1, 0.1, 0.2)$. (1.5 p)

3. In Lab 3 of this course, a flexible servo process was studied. The process consists of two masses which are interconnected by a spring. A sketch of this process is given in Figure 1. The mass at one end of the spring can be moved by a motor, that can give a maximum voltage of 10 V. The linear model of the process has four states; the position and the velocity of each of the two masses.

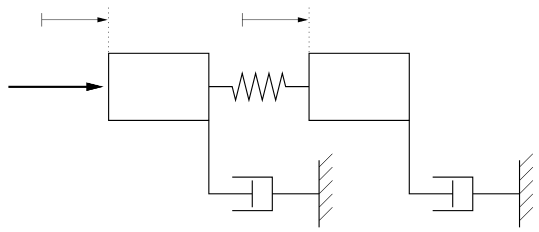

Figure 1 Flexible servo process in Problem 3.

In the lab, state feedback was used to place the closed-loop poles of the system. Consider the set of step and load disturbance responses displayed in Figure 2, which were obtained using state feedback directly from the states, without the use of a Kalman filter.

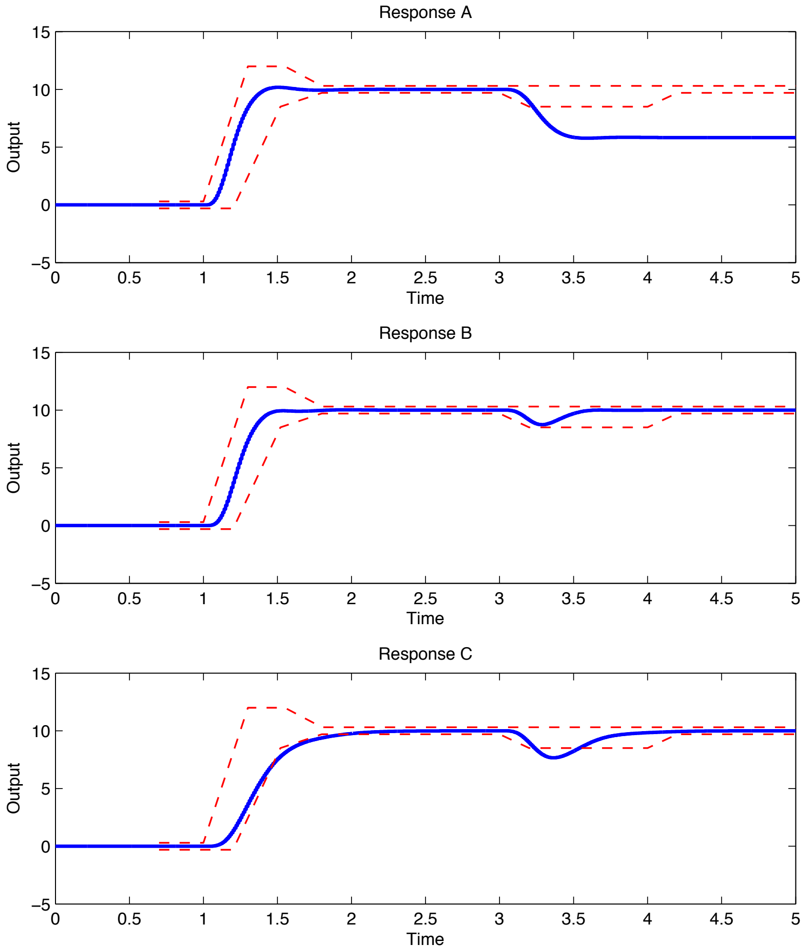

Figure 2 Step responses and load disturbance responses in Problem 3.

a. Pair the responses (A–C) together with the three sets of closed-loop poles (1–3) below, describing the closed loop behavior from reference r to output y. Provide a short motivation for each pair. (1.5 p)

1.	**2.**	**3.**
$-17.1+8.3i$	$-17.1+8.3i$	$-17.1+8.3i$
$-17.1-8.3i$	$-17.1-8.3i$	$-17.1-8.3i$
$-14.4+7.0i$	$-14.4+7.0i$	$-14.4+7.0i$
$-14.4-7.0i$	$-14.4-7.0i$	$-14.4-7.0i$
-25	-5	

b. Calculate ω_0 and the relative damping ζ for the pole-pairs $-17.1 \pm 8.3i$ and $-14.4 \pm 7.0i$. (1 p)

c. One of the responses in Figure 2 show significantly worse load disturbance rejection than the other two. How would this change if a larger value of ω_0 for all the poles were to be chosen? Motivate your answer. (1 p)

d. What is in practice the limiting factor to choosing ω_0, when doing pole-placement for the flexible servo process in Lab 3? (1 p)

4. Engineer Wang Fang wants to examine a process section of a plant for producing baijiu. She did frequency response experiments on the process and obtained the Bode plot in Figure 3.

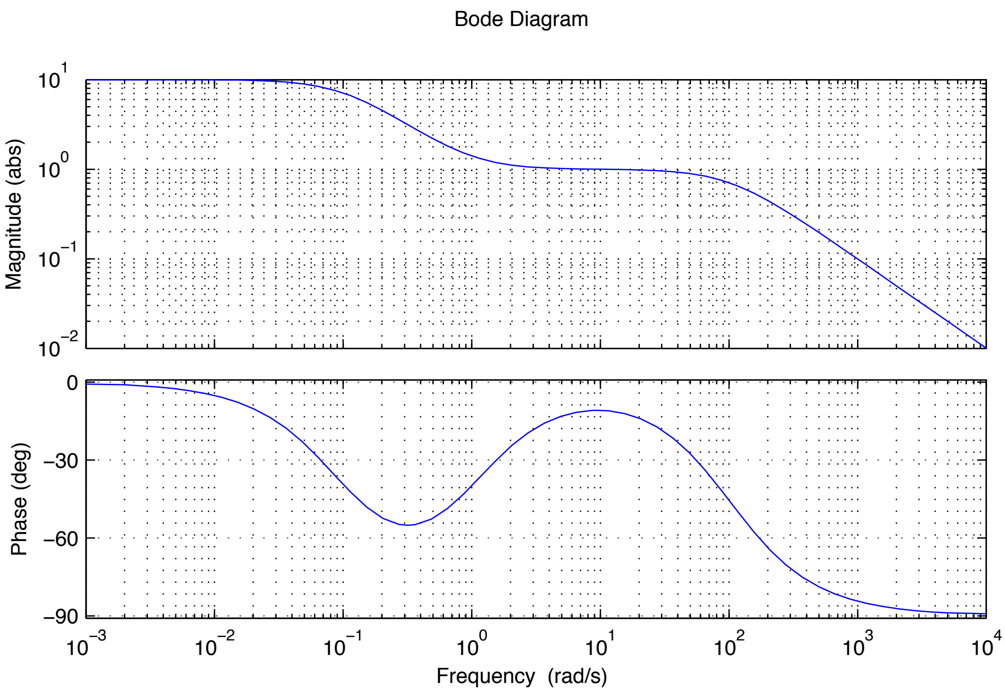

Figure 3 Bode plot in Problem 4.

a. What is the transfer function $G(s)$ of the system? (2 p)

b. What is the process output $y(t)$ when all transients have disappeared if a sinusoid $u(t) = 0.1\sin(10t)$ is applied to the system? (1 p)

c. The process in Figure 3 is to be controlled by a P controller with $K = 2$. What is the phase margin of the system using this controller? (1 p)

5. The system for temperature control in the exam room can be described by

$$\dot{x} = \begin{pmatrix} -1 & 1 \\ 0 & -4 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

with the control law $u = -\begin{pmatrix} 3 & 2 \end{pmatrix} x + 4r$.

a. Where are the closed-loop poles located? (1 p)

b. If we can not directly measure both x_1 and x_2, but only the output signal y, we need to estimate the states. Design an observer such that the poles of the estimation error dynamics will be placed in -8. (2 p)

c. The control system is not fast enough for the customer, who thus wants to move both poles to -8. Is this possible? Why/why not? (1 p)

6. The 'Ball and beam' process is shown in Figure 4. For small angular variations, the process can be described by the transfer functions

$G_\varphi(s) = \frac{4.5}{s}$ from the motor voltage, u, to the beam angle, φ, and

$G_z(s) = \frac{10}{s^2}$ from the beam angle to the position of the ball, z.

We want to control the ball on the beam by controlling the beam angle with a P controller, and the position of the ball by a PID controller. Suggest a suitable controller structure and draw a block diagram of the system. (2 p)

Figure 4 'Ball and beam' process i Problem 6.

7. A motor for an electric moped should be industrially manufactured, but the specifications for the motors from the different subcontractors are a bit different. The Nyquist plots for each of the three motors are shown in Figure 5. Suppose that the motors should be controlled by a P controller, $u(t) = Ke(t)$. Which is the greatest positive K that will give stable closed-loop systems for all three models? (2 p)

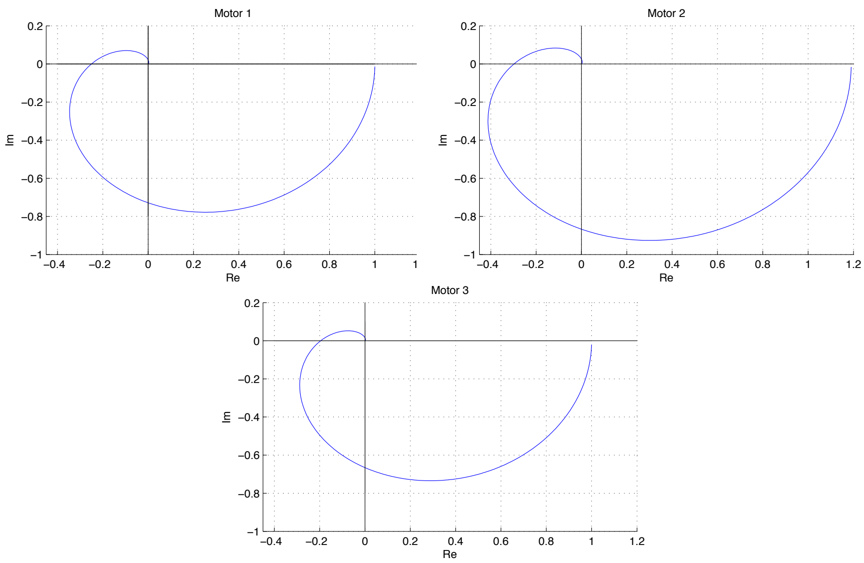

Figure 5 Nyquist curves for the three motors in problem 7.

8. Consider the system in Figure 6, that describes velocity control of the Shanghai Maglev Train. The process transfer function is given by

$$G_P(s) = \frac{4}{s(s+2)}.$$

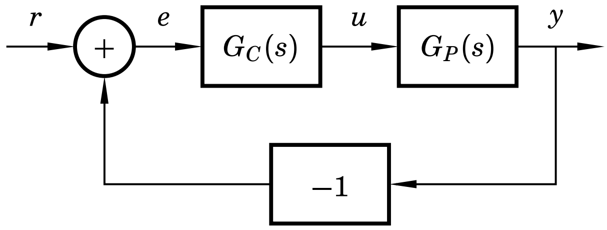

Figure 6 The feedback loop in problem 8.

a. Assume that the system is controlled by a proportional controller with a gain of 1. Show that the stationary control error is equal to zero when the reference signal, r, is a unit step. (1 p)

b. With the proportional controller from subproblem **a.**, the stationary error is equal to 0.5 when the reference is a unit ramp. Design a filter that gives a stationary error of 0.1 at unit ramp disturbances in r. The filter should not give any major change in robustness. (3 p)