LUND INSTITUTE
OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

# Automatic Control, Basic Course (FRT010)
## Exam 2012-12-08

**Points and grades**

All answers must include a clear motivation. The total number of points is 25. The maximum number of points is specified for each subproblem.

Grade 3: at least 12 points
4: at least 17 points
5: at least 22 points

**Accepted aid**

Mathematical collections of formulae (e.g. TEFYMA), 'Collections of formulae in automatic control', and calculators that are not programmed in advance.

**Results**

You should write a personal code on your cover sheet. When the exams have been corrected, the results will be presented on the course web page and you can check your grade using your code.

The corrected exams will be displayed Mon Jan 14, 12:15–12:45 in Automatic Control Lab C at the ground floor of LTH M-building in Lund. This information is posted on the course web page.

**1.** Consider the system

$$\dot{x} = \begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

**a.** Determine the transfer function from $u$ to $y$. (1 p)

**b.** Determine the poles of the system. Is the system unstable, (marginally) stable or asymptotically stable? (1.5 p)

**2.** Express the transfer function from $A$ to $B$ in Figure 1 in terms of the block transfer functions $P$ and $Q$.

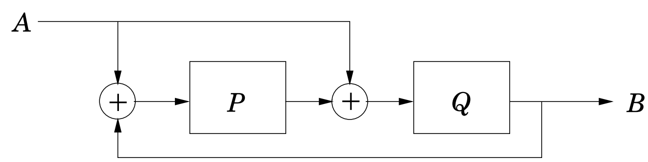


**Figure 1** The system in Problem 2 with two block components.

(2 p)

**3.** The Bode plot of a loop transfer function is shown in Figure 2. Determine the cross-over frequency $\omega_c$, the $-180°$ phase shift frequency $\omega_0$ as well as the amplitude margin $A_m$ and phase margin $\varphi_m$ which a negative feedback connection of the system would result in. Do not forget to explain how you obtained the reported values. (2 p)

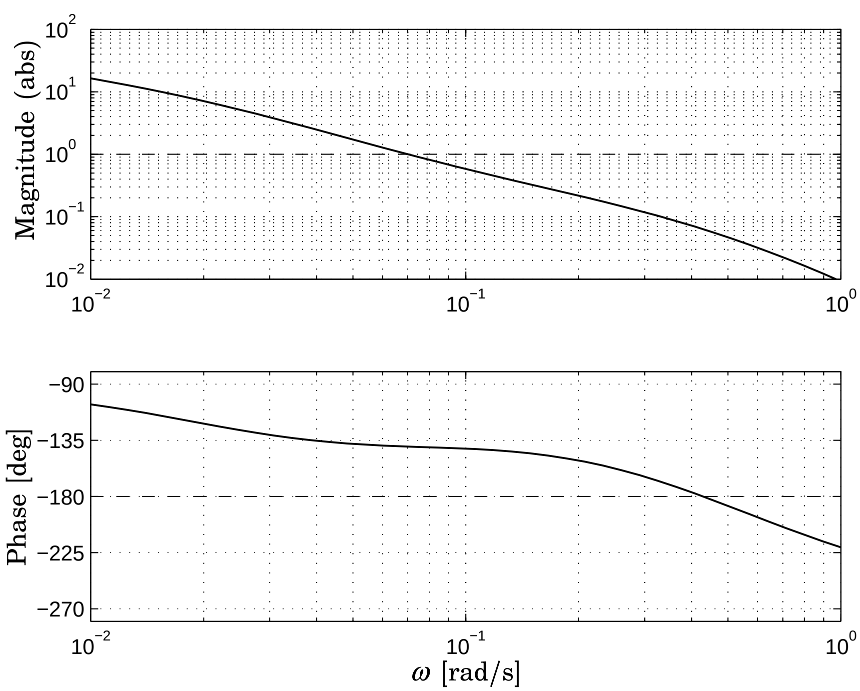


**Figure 2** The loop transfer function Bode plot from Problem 3.

**4.** Below, four systems are presented by their differential equations. Their step responses from $u$ to $y$ are shown in Figure 3, together with the step responses of two unrelated systems. For each system (1–4), indicate which is the corresponding step response ($A$–$F$). Do not forget to motivate your choices.

$$\begin{aligned}
1 &: \ \ddot{y} + 0.2\dot{y} + y = u \\
2 &: \ \dot{y} + 0.5y = u \\
3 &: \ \ddot{y} + 0.8\dot{y} + y = u \\
4 &: \ 4\dot{y} + y = 2u
\end{aligned}$$

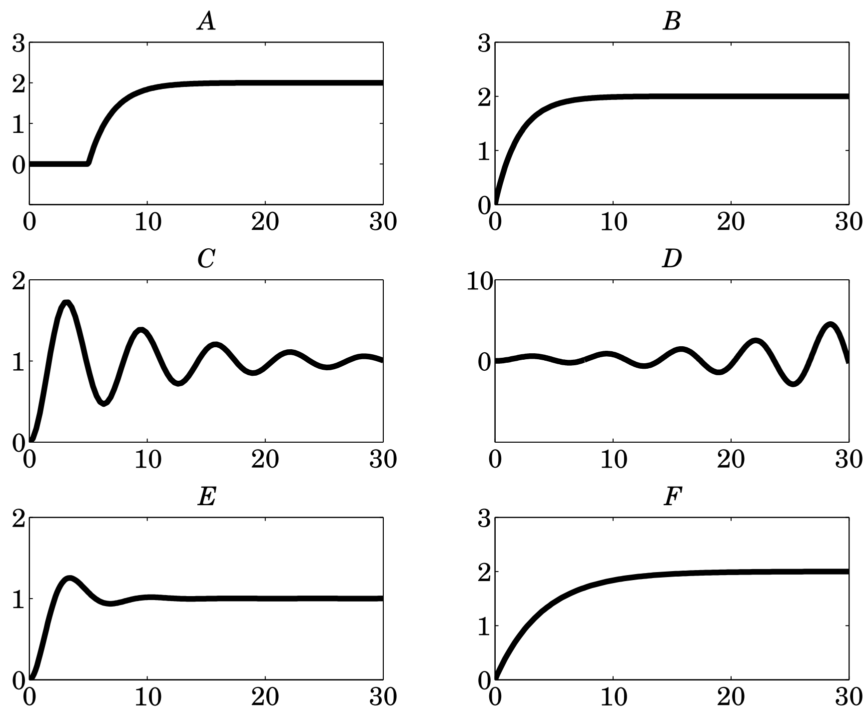


**Figure 3** Step responses of six systems from Problem 4.

(3 p)

**5.** Design a state feedback controller

$$u = -Lx + l_r r$$

for the system

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$
$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} x$$

such that the resulting closed-loop system has its poles in $s = -1$ and $s = -2$. Determine $l_r$ so that $y = r$ in stationarity. (3 p)

**6.** A schematic sketch of a sail boat with mass $m$ is shown in Figure 4. For a given wind direction, the propelling force $F_p$ is modelled proportional to the projection of the sail onto the direction of travel, with proportionality constant $\alpha$ such that $F_p = \alpha d \sin\theta$. There is also a drag force $F_d$ from the water, which is quadratically proportional to the boat speed $v$ with proportionality constant $\beta$ such that $F_d = \beta v^2$. The described behavior is captured by

$$m\dot{v} = \alpha d \sin\theta - \beta v^2$$

where we assume $0 \le \theta \le 90°$ and $v \ge 0$.

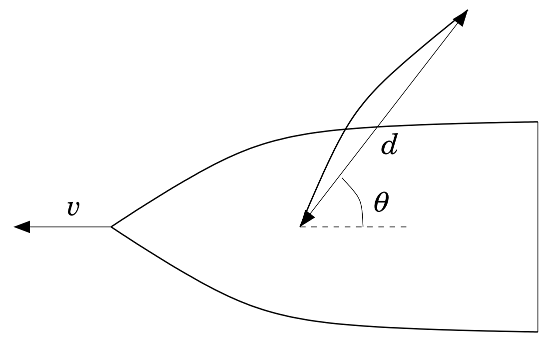


**Figure 4** A schematic sketch of the sail boat in Problem 6.

Our aim is to design a closed-loop controller for the speed $v$. Speed measurements are available from the onboard GPS and the sail angle $\theta$ can be instantaneously changed by means of a controlled electric motor.

**a.** What stationary speed will the boat travel with for a given sail angle $0 \le \theta \le 90°$? Which stationary speeds are possible? (1.5 p)

**b.** The parameters for our boat are $m = 1000$ kg, $d = 4$ m, $\alpha = 10$ kg/s$^2$ and $\beta = 0.2$ kg/m. We wish to design a controller for a nominal speed of $v_0 = 10$ m/s. Linearize the system around the corresponding operating point. (2 p)

**c.** Draw a block diagram of the control system, showing the following components and signals: controller $C$, process $P$, reference $R$, measurement signal $Y$, control error $E$, control signal $U$ and load disturbance $L$. (1 p)

**d.** Waves can be viewed as a load disturbance $L$ added to the input $\Delta\theta$ of the linearized system. Write down the transfer function from load disturbance $L$ to measurement $Y = \Delta v$.

*It is enough to give the answer as an expression in C and P; you do not have to insert the expression for P obtained in subproblem* **b.** (1 p)

**e.** Determine the gain $K$ of a P controller so that load disturbance steps are eliminated from the measurement signal with a time constant of 50 s.

*If you did not solve subproblem* **b.**, *assume* $P(s) = 4/(100s + 1)$. (2 p)

**7 a.** Show that the initial derivative (slope) of the step response is non-zero for a first order system

$$G_1(s) = \frac{b}{s+a}$$

but zero for a second order system

$$G_2(s) = \frac{e}{(s+c)(s+d)}$$

Use the initial value theorem, or another method of your own choice.

(2 p)

**b.** The step response of the system

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

is shown in the Figure 5. Clearly the system has two state variables and a step response with $\dot{y}(0) \neq 0$. Explain how this is possible.

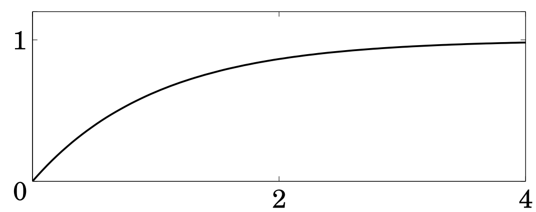


**Figure 5** The step response in Problem 7b.

(2 p)

**c.** Is it possible to construct a linear observer (Kalman filter) for the system in subproblem **b.**? Remember to motivate your answer. (1 p)