



LUND INSTITUTE
OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

Automatic Control, Basic Course (FRT010)

Exam 2011-12-10

Points and grades

All answers must include a clear motivation. The total number of points is 25. The maximum number of points is specified for each subproblem.

Grade 3: at least 12 points
4: at least 17 points
5: at least 22 points

Accepted aid

Mathematical collections of formulae (e.g. TEFYMA), 'Collections of formulae in automatic control', and calculators that are not programmed in advance.

Results

You should write a personal code on your cover sheet. When the exams have been corrected, the results will be presented on the course web page and you can check your grade using your code. The corrected exams will also be displayed at a time and location which will be announced later by e-mail.

1. Consider the system

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

Determine the transfer function from u to y . (1 p)

2. Recall the flexible servo process that was studied in Lab 3 of the course. A sketch of the process is given in figure 1.

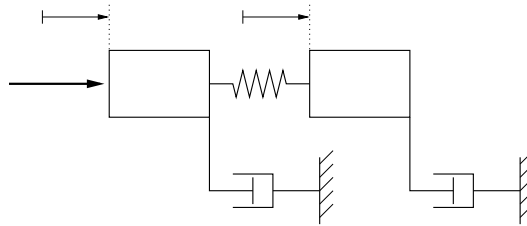


Figure 1 Flexible servo process in problem 2.

The process can be described by the dynamic model:

$$m_1 \ddot{p}_1 = -d_1 \dot{p}_1 - k(p_1 - p_2) + F$$

$$m_2 \ddot{p}_2 = -d_2 \dot{p}_2 + k(p_1 - p_2)$$

The force F is proportional to the input voltage u of the amplifier according to $F = k_m u$. We want to control the position of mass 2, p_2 . One sensor is available, which measures p_2 directly. The objective is to use state feedback to control the system.

- a. Introduce the state vector $x = \begin{pmatrix} p_1 & \dot{p}_1 & p_2 & \dot{p}_2 \end{pmatrix}^T$ and write the system in state-space form. (1 p)
- b. How many inputs, outputs, and state variables does the system have? (0.5 p)
- c. At the lab, a Kalman filter was designed for the process. Why? (0.5 p)
- d. Some students did three experiments on the process using different controllers, but unfortunately, they mixed up their results. Help them to pair the Bode magnitude diagrams of the three controllers in figure 2 with the three step responses in figure 3, and the closed-loop poles of the system with state feedback for each experiment that are given below. The poles of the Kalman filter are the same for all three experiments. (3 p)

As always, your answers must include a clear motivation.

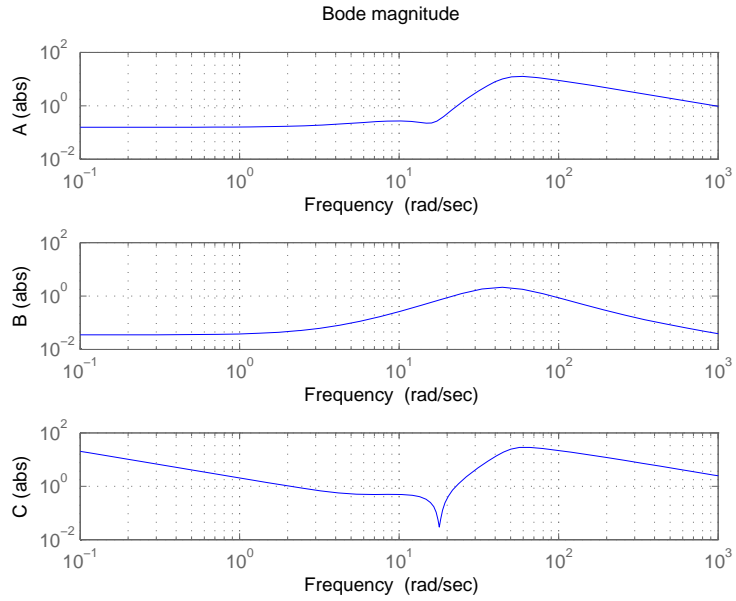


Figure 2 Bode magnitude plots of $G_R(s)$, where $u = G_R(s)y$ is the controller (state feedback with Kalman filter) in problem 2 d).

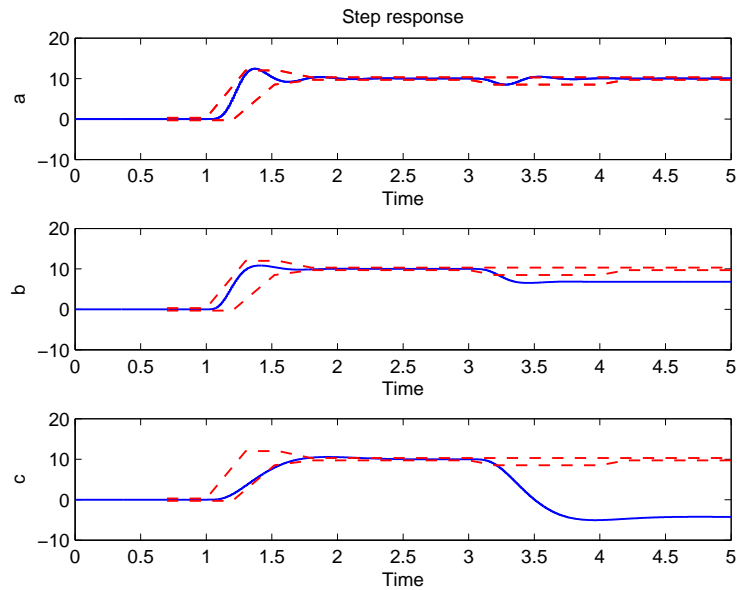


Figure 3 Closed-loop step responses of the system with controller in problem 2 d).

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poles_1 =          poles_2 =          poles_3 =
-13.4350 +13.4350i  -9.5000 +16.4545i  -13.4350 +13.4350i
-13.4350 -13.4350i  -9.5000 -16.4545i  -13.4350 -13.4350i
-9.8995 + 9.8995i   -7.0000 +12.1244i  -3.5355 + 3.5355i
-9.8995 - 9.8995i   -7.0000 -12.1244i  -3.5355 - 3.5355i
                    -22.0000

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3. Consider the process model

$$G_P(s) = \frac{2}{s(s+1)^2}$$

and the feedback system shown in figure 4.

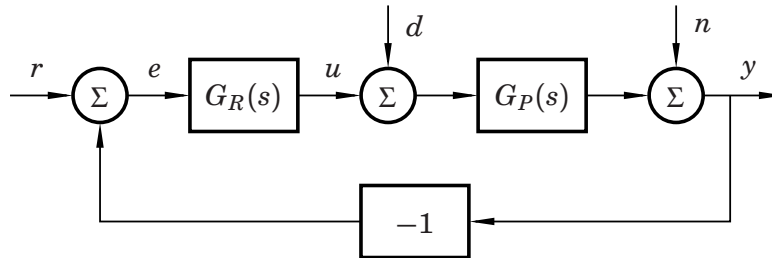


Figure 4 The feedback system in problem 3.

- a. Construct a feedback system, as in figure 4, with a constant controller $G_R(s) = K$ such that the system has a phase margin of 45° . (2 p)
- b. The four transfer functions that characterize the feedback system in figure 4 are called “The Gang of Four”. One interpretation of these transfer functions is as the transfer functions from:
- i) reference r to output y
 - ii) measurement noise n to output y
 - iii) load disturbance d to output y
 - iv) reference r to control signal u

Determine “The Gang of Four” for the process in this problem, using the controller $G_R(s) = K$ from a).

If you did not solve subproblem a), you can use $K = 0.5$. (2 p)

- c. The transfer function given in the problem is a model of a real process. Experiments on the real process showed that there is a delay between the controller and the process. How large delay can be tolerated, before the closed loop becomes unstable, if the controller $G_R(s) = K$ from a) is used?

If you did not solve subproblem a), you can use $K = 0.5$. (1 p)

4. A person that rides a bike in Hangzhou can be viewed as a feedback system, where the person compares his/her actual speed to the desired speed, and controls the speed using the pedals and the brake.

In addition, the person may also compensate in advance for a steep slope of the road ahead by pedalling faster. What is this second kind of control strategy called? Draw a block diagram that describes the system. (2 p)

5. Consider a double tank process, where x_1 is the level of the upper tank, and x_2 the level of the lower tank. A state-space representation of the process is given by

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} x$$

All states are measurable, and the process is controlled using state feedback $u = -Lx + l_r r$, with $L = \begin{pmatrix} 2 & 3 \end{pmatrix}$ and $l_r = 6$.

- For the double tank process, can the poles of the closed-loop system be placed arbitrarily using state feedback? (1 p)
- Write down the closed-loop system on state-space form. (1 p)
- What are the numerical values of Q and W in Figure 5? (1 p)

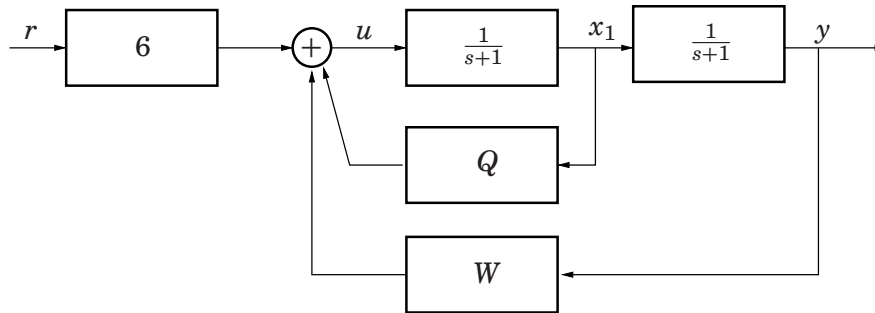


Figure 5 Block diagram for problem 5.

- Compute the stationary error for a unit reference step (i.e. when $r(t) = 1$). (2 p)
- Why could it be beneficial to include integral action in the controller in this case? (1 p)

6. The Nyquist curve of a system is given in figure 6. The system is stable, i.e. has no poles in the right half plane.

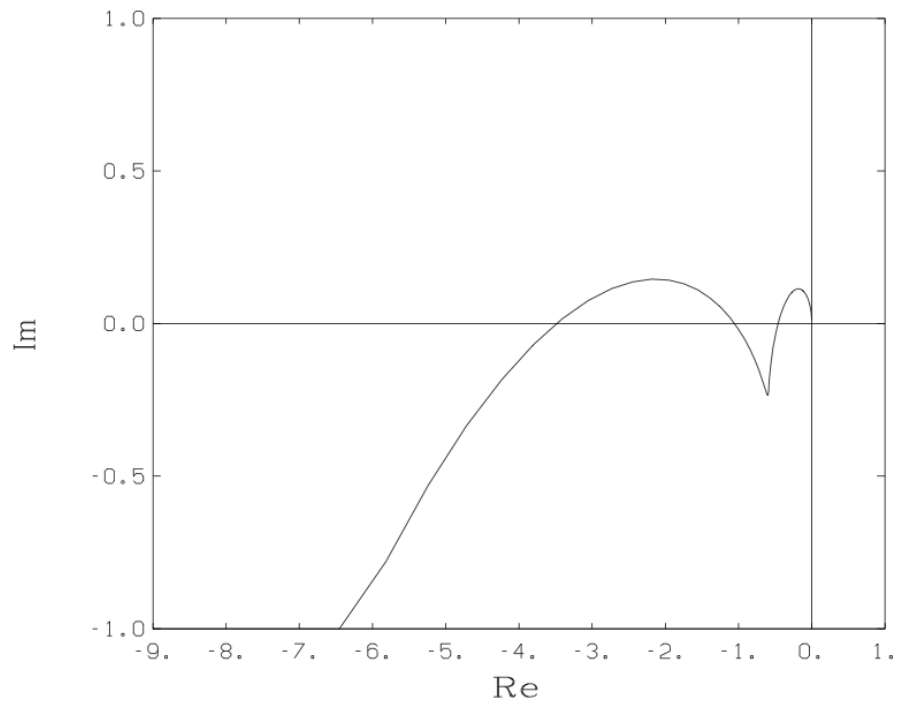


Figure 6 Nyquist curve in problem 6.

Assume that the system is subject to proportional feedback $u = K(r - y)$. Which values of the gain K result in a stable closed-loop system? (2 p)

7. In figure 7, four Bode magnitude plots are shown.

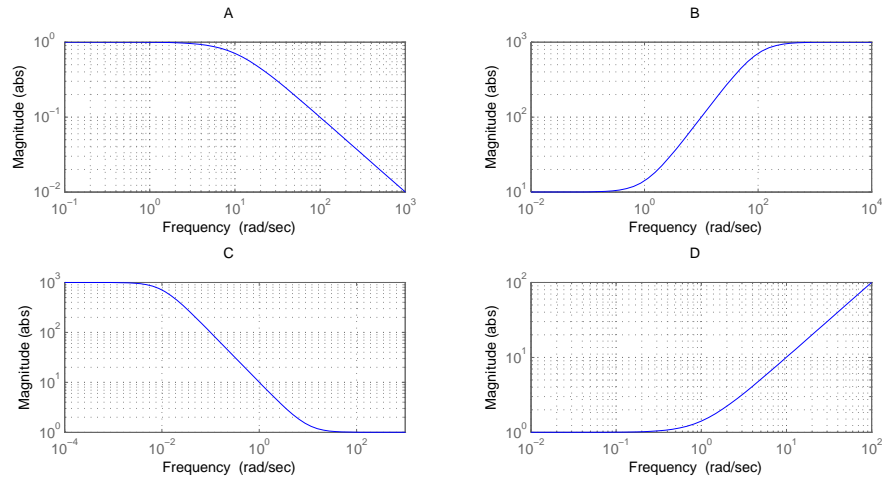


Figure 7 Bode magnitude plots for problem 7.

- a. Which of the magnitude plots corresponds to a lead compensator, and which corresponds to a lag compensator (with finite a , M , K_K , b , N)? (1 p)

Lead compensator

$$G_K(s) = K_K N \frac{s+b}{s+bN} = K_K \frac{1+s/b}{1+s/(bN)}$$

Lag compensator

$$G_K(s) = \frac{s+a}{s+a/M} = M \frac{1+s/a}{1+sM/a}$$

- b. Name at least two reasons to use a lead compensation link. (1 p)
- c. Let $N \rightarrow \infty$ in the lead compensator transfer function. What is the interpretation of the compensator in terms of the PID controller? Identify the PID parameters K , T_i and T_d for the PID controller as functions of K_K and b . You can use either parallel or series form of the PID controller. (2 p)