

Solutions Home Assignment 2

November 3, 2014

1. The transfer function is:

$$G = -0.16 \frac{e^{-0.18s}}{(0.1s + 1)^3} \quad (1)$$

An example of a state-space representation is given by the following. Introduce state variables in the frequency domain:

$$X_1(s) = \frac{1}{0.1s + 1} U(s) \quad (2)$$

$$X_2(s) = \frac{1}{0.1s + 1} X_1(s) \quad (3)$$

$$X_3(s) = \frac{1}{0.1s + 1} X_2(s) \quad (4)$$

$$Y(s) = -0.16e^{-0.18s} X_3(s) \quad (5)$$

$$(6)$$

Converting to time-domain yields the following differential equation system:

$$\dot{x} = Ax + Bu(t) \quad (7)$$

$$y(t) = Cx(t - 0.18) \quad (8)$$

where

$$A = \begin{bmatrix} -10 & 0 & 0 \\ 10 & -10 & 0 \\ 0 & 10 & -10 \end{bmatrix} \quad B = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

$$C = [0 \quad 0 \quad -0.16] \quad (10)$$

Matlab code to confirm the result:

```
sys = ss(A,B,C,[],'OutputDelay',0.18);  
tf(sys)
```

2. Using the Matlab commands **impz**, **step** and **bode**, plots are given according to Figure 1, 2 and 3. The physiological interpretations can be found in the captions of each figure.

3. Amplitude Margin

The phase of the system is:

$$\arg(G(iw)) = -0.18w - 3 \arctan(0.1w) \quad (11)$$

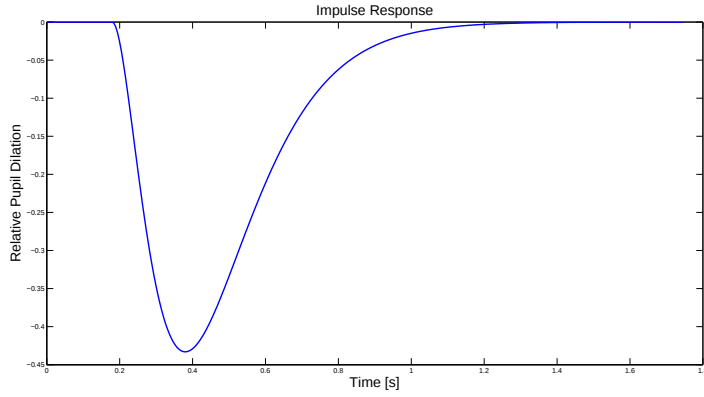


Figure 1 Impulse response: This plot shows how the pupil reacts to a sudden flash of light. After a brief excursion the pupil size returns to the size prior to the flash.

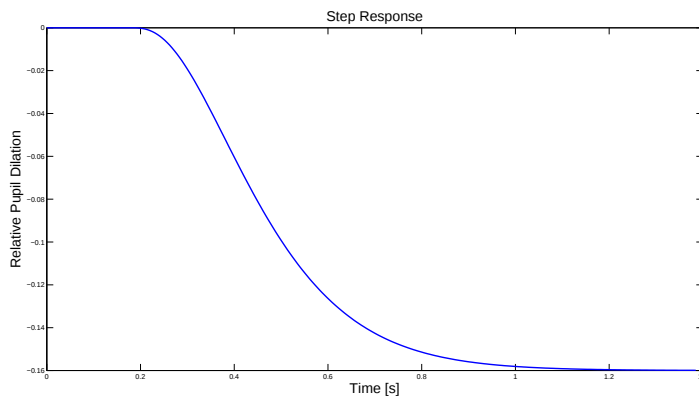


Figure 2 Step response: When the light intensity increase stays on, the pupil adapts accordingly.

Solving for the crossover frequency

$$\arg(G(iw_0)) = \pi \quad (12)$$

numerically gives:

$$w_0 \approx 7.1 \text{ rad/s} \quad (13)$$

The gain at this frequency is:

$$|G(iw_0)| = \frac{0.16}{(1 + 0.01w_0^2)^{3/2}} = 0.087 \quad (14)$$

The amplitude margin A_m is thus:

$$A_m = \frac{1}{|G(iw_0)|} = 11.5 \quad (15)$$

4. Simulink model

The closed-loop system becomes unstable at about $K = 11.6$. By looking at the response for the marginally unstable system the period can be determined to be around 0.9 seconds, which corresponds to a frequency of about 7 rad/s. Thus, the simulation results correspond well to the theoretical analysis.

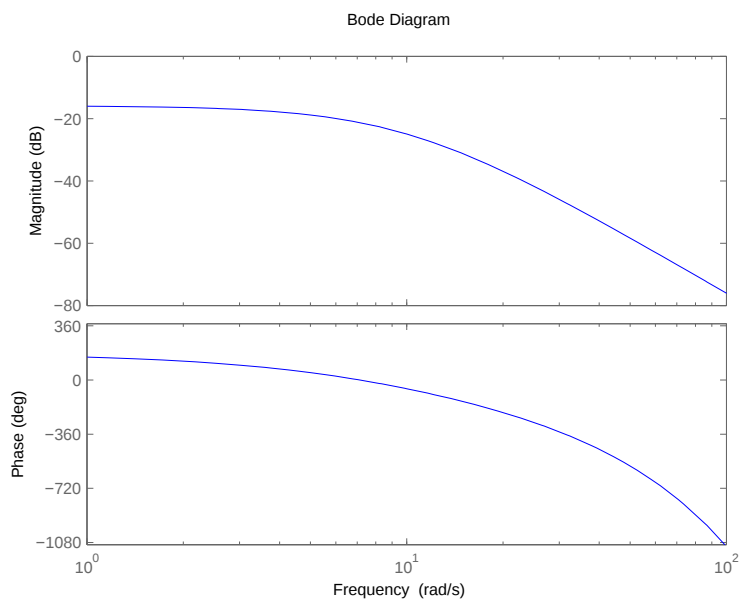


Figure 3 Bode diagram: The upper diagram depicts the amplification from input to output for different frequencies. The lower diagram shows how much the pupil variations for a specific frequency lag behind the input of the same frequency.