LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

Exam in Systems Engineering/Process Control

2017-06-02

Points and grading

All answers must include a clear motivation. Answers may be given in English or Swedish. The total number of points is 20 for Systems Engineering and 25 for Process Control. The maximum number of points is specified for each subproblem. Preliminary grading scales:

Systems Engineering:		Process control:	
Grade 3:	10 points	Grade 3:	12 points
4:	14 points	4:	17 points
5:	17 points	5:	21 points

Accepted aid

Authorized *Formelsamling i reglerteknik / Collection of Formulae*. Standard mathematical tables like TEFYMA. Pocket calculator.

Results

The solutions will be posted on the course home page, and the results will be transferred to LADOK. Date and location for display of the corrected exams will be posted on the course home page.

1. Figure 1 shows step responses of six systems. Match them to the following six transfer functions. (3 p)

$$G1 = \frac{10}{s+10}e^{-s} \qquad G2 = \frac{4}{s^2+1.2s+4}$$

$$G3 = \frac{1}{s(s+1)} \qquad G4 = \frac{4}{s^2+2.4s+4}$$

$$G5 = \frac{4}{s^2-3.6s+4} \qquad G6 = \frac{1}{s+1}$$

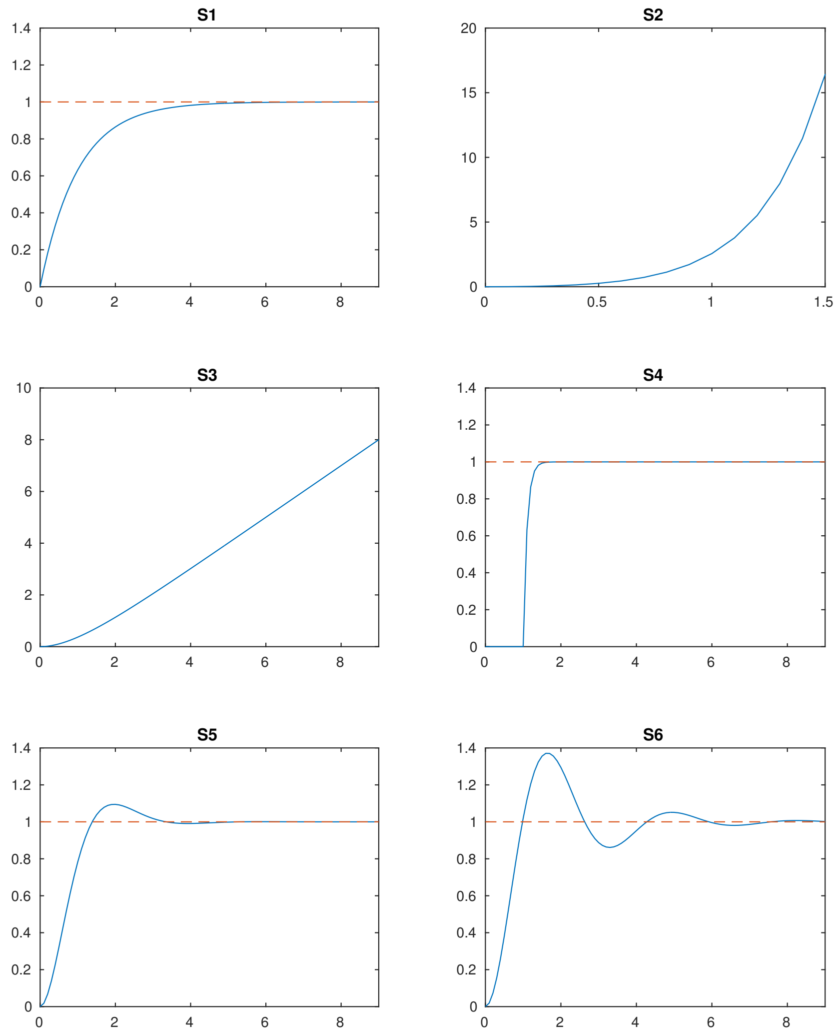

Figure 1 Step responses for the six transfer functions in Problem 1

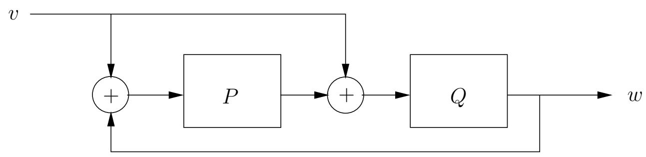

Figure 2 Block diagram for Problem 2 with two block components.

2. Compute the transfer function from v to w for the block diagram in Figure 2.

(2 p)

3. Figure 3 depicts some characteristics of three different second order systems, all of the form

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} e^{-sL}$$

but with different parameters ζ, ω_0, and L. Combine each Nyquist plot (i)-(iii), with a bode diagram (A)-(C), a step response (1)-(3), and a singularity diagram (I)-(III) in Figure 3 with clear motivations.

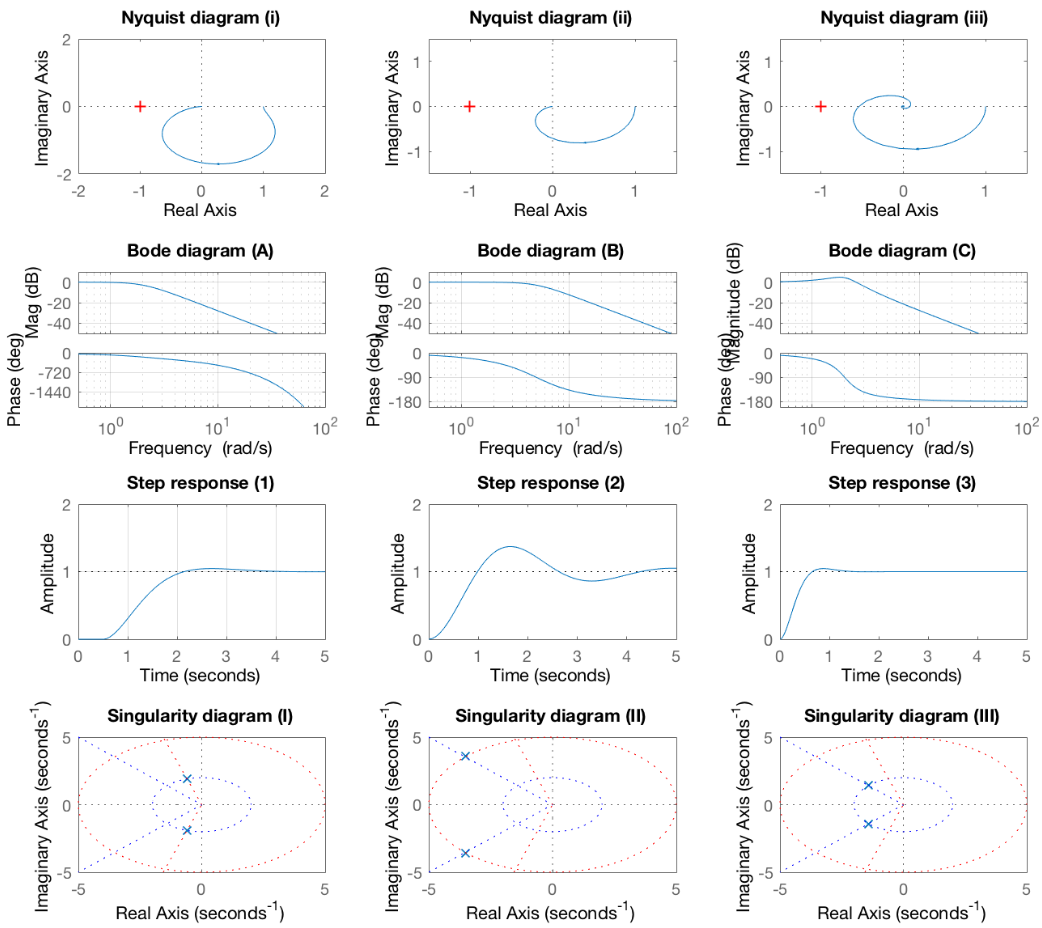

Figure 3 Characteristics of the three second order transfer functions in Problem 3.

(4 p)

4. Consider the system

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ -1 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 1 \end{pmatrix} x$$

a. Determine the transfer function from u to y. (1.5 p)

b. Determine the poles of the system. Is the system unstable, marginally stable or asymptotically stable? (1.5 p)

5. Consider a system with an input signal $u(t)$ and a measurement signal $y(t)$. Let

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{(s+1)^2}$$

be the transfer function from the input $U(s) = \mathscr{L}\{u(t)\}$ to the output $Y(s) = \mathscr{L}\{y(t)\}$. Using the final value theorem,

a. Compute the output $y(t)$ as $t \to \infty$ if the input is a unit step. (1 p)

b. Compute the output $y(t)$ as $t \to \infty$ if the input is a unit ramp. (1 p)

c. Compute the difference $d(t) = u(t) - y(t)$ as $t \to \infty$ if the input is a unit ramp. (1 p)

6. The ideal pendulum of length l $[m]$ in Figure 4 is governed by the nonlinear dynamical state equations

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -(g/l)\sin(x_1)$$

where x_1 is the angle, x_2 is the angular velocity, and g $[m/s^2]$ denotes the constant of gravity.

a. Find *all* stationary points, $\mathbf{x}^0 = [x_1^0, x_2^0]^T$, of the system. (1 p)

b. Give a physical explanation of the location of the stationary points. (1 p)

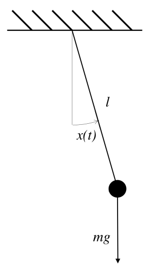

Figure 4 Pendulum in Problem 6.

c. Linearize the system around an arbitrary stationary point $\mathbf{x}^0 = [x_1^0, x_2^0]^T$ by introducing the variables $\Delta x_1 = x_1 - x_1^0$, $\Delta x_2 = x_2 - x_2^0$. (1 p)

d. Decide for each linearized system if it is stable, marginally stable, or unstable. Give a physical explanation of the result. (2 p)

7. **Only for Process Control:** Consider the multivariable system

$$G(s) = \begin{pmatrix} \frac{2e^{-s}}{s+4} & \frac{e^{-s}}{(s+1)(s+5)} \\ \frac{1}{2s+5} & \frac{3}{s+5} \end{pmatrix}$$

The system should be controlled using two PID controllers.

a. Calculate the relative gain array, RGA, for the system. (1 p)

b. Determine how should the inputs and outputs be paired? Comment on the interaction. (1 p)

c. Find a decoupling matrix that decouples the system dynamics in stationarity and gives the corresponding decoupled system static gains of one. Is the decoupler realizable? (1 p)

8. **Only for Process Control:** Consider a first order system

$$\dot{y}(t) = -ay(t) + bu(t)$$

with $a > 0$ and $b > 0$.

a. Discretize the system using *forward Euler* with sampling time h. State the resulting difference equation. (0.5 p)

b. Find the range of $h > 0$, possibly as a function of a and b, such that the discretized system in **a.** is stable. (0.5 p)

c. Discretize the system using *backward Euler* with sampling time h. State the resulting difference equation. (0.5 p)

d. Find the range of $h > 0$, possibly as a function of a and b, such that the discretized system in **c.** is stable. (0.5 p)