LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

Exam in Systems Engineering/Process Control

2016-05-31

Points and grading

All answers must include a clear motivation. Answers may be given in English or Swedish. The total number of points is 20 for Systems Engineering and 25 for Process Control. The maximum number of points is specified for each subproblem. Preliminary grading scales:

Systems Engineering:		Process control:	
Grade 3:	10 points	Grade 3:	12 points
4:	14 points	4:	17 points
5:	17 points	5:	21 points

Accepted aid

Authorized *Formelsamling i reglerteknik / Collection of Formulae*. Standard mathematical tables like TEFYMA. Pocket calculator.

Results

The solutions will be posted on the course home page, and the results will be transferred to LADOK. Date and location for display of the corrected exams will be posted on the course home page.

1. Pair the following transfer functions with the corresponding step responses given in Figure 1 or state that the corresponding plot is not there. Motivate your answers!

(3 p)

$$G_1(s) = \frac{1}{1+5s}, \qquad G_2(s) = \frac{e^{-5s}}{1+50s}, \qquad G_3(s) = \frac{e^{-5s}}{(1+5s)^2},$$
$$G_4(s) = \frac{5e^{-5s}}{(1+5s)^2}, \qquad G_5(s) = \frac{e^{-5s}}{1-5s}, \qquad G_6(s) = \frac{1}{(s^2+0.5s+1)},$$
$$G_7(s) = \frac{5}{s^2+0.5s+1} \qquad G_8(s) = \frac{5e^{-5s}}{5+s}, \qquad G_9(s) = \frac{1}{(1+5s)^2}.$$

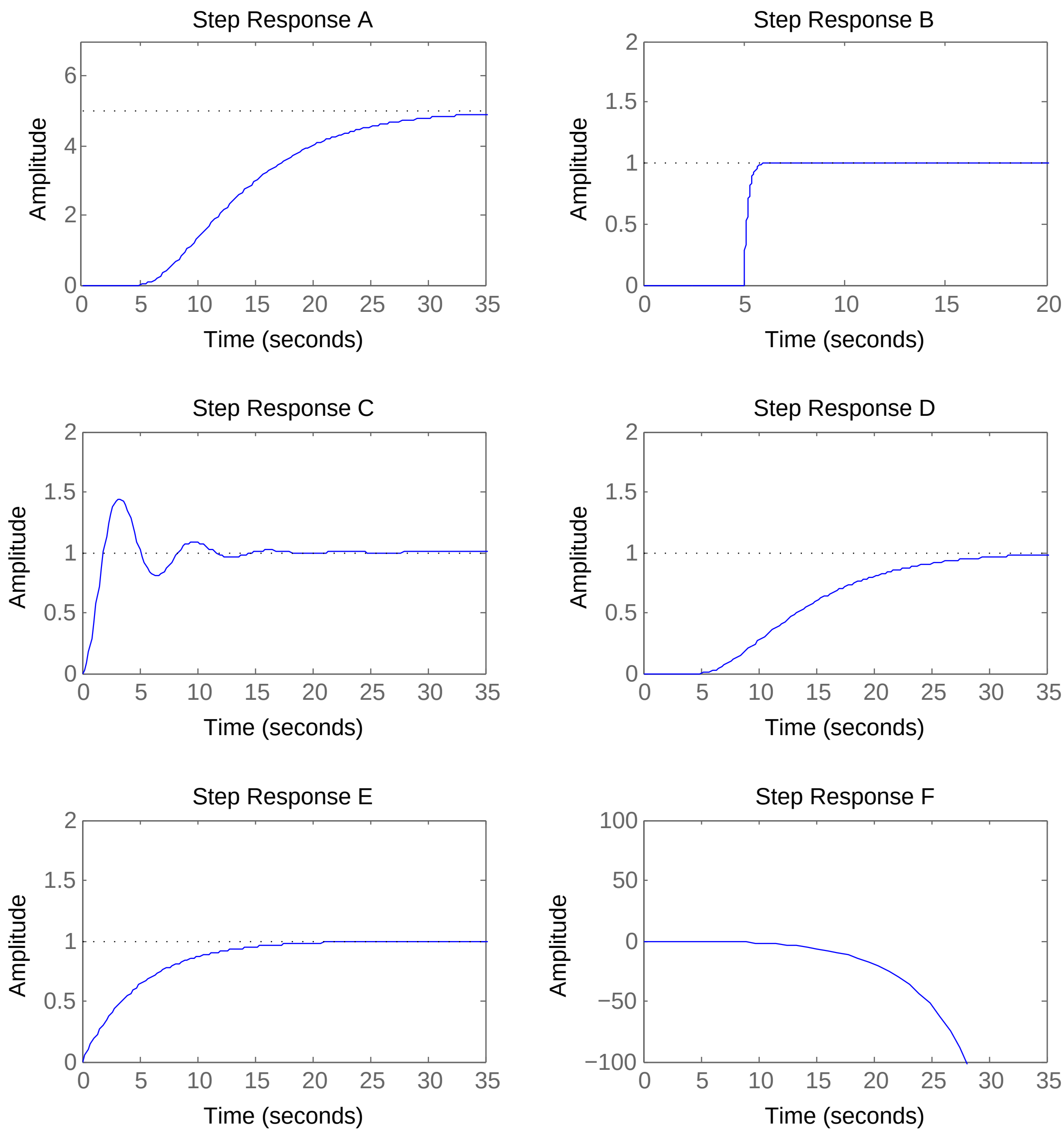

Figure 1 Step responses for some of the transfer functions in Problem 1.

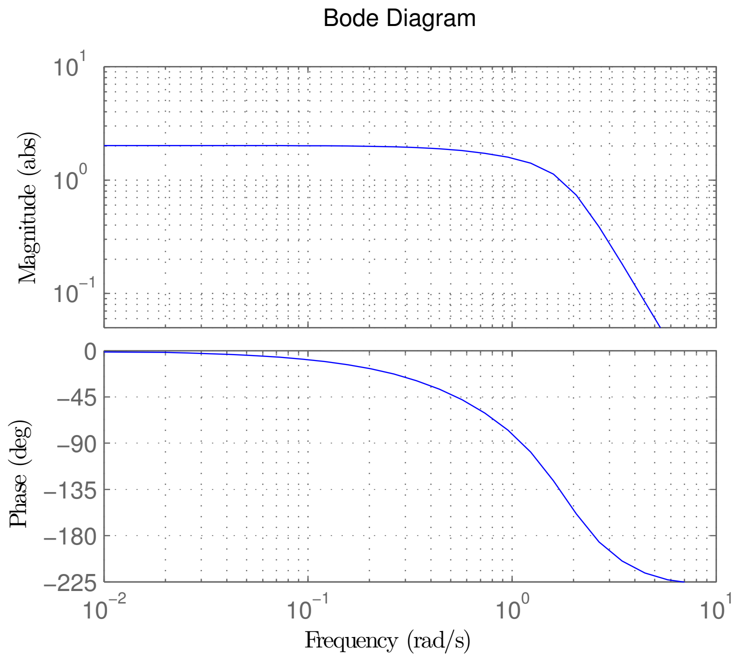

Figure 2 The Bode diagram for process P in Problem 3.

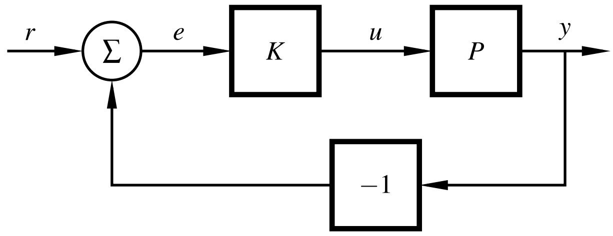

Figure 3 The closed loop system in Problem 3.

2. The following differential equation is given

$$\alpha\ddot{y} + \beta\dot{y} + \gamma y = \delta u.$$

a. Write the system on state-space form using states $x_1 = y$ and $x_2 = \dot{y}$. (1 p)

b. Give the transfer function of the system from u to y. (1 p)

c. What conditions on the parameters α, β, γ and δ are required for the system to be asymptotically stable? (1 p)

3. The Bode diagram of a process P is shown in Figure 2.

a. Determine the static gain of P. (0.5 p)

b. Determine the amplitude margin. (0.5 p)

c. The process P is controlled by a P-controller with gain K according to Figure 3. Suppose that a unit step change in the reference r is performed. Determine the steady state error $e(\infty)$ as a function of K. (1.5 p)

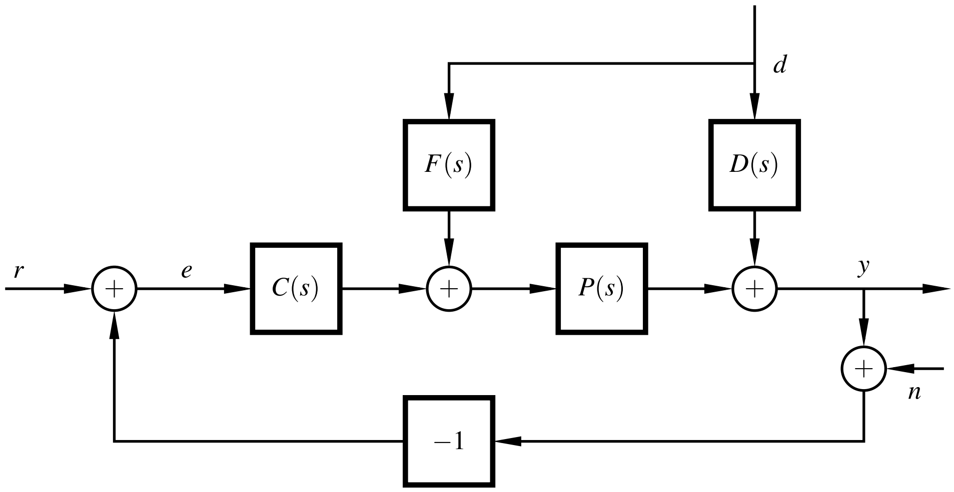

Figure 4 Block schedule for the system in Problem 5.

d. Is it possible to choose a $K > 0$ such that the closed loop system is stable and the absolute value of the stationary error $|e(\infty)|$ is less than 0.1? If so, what are the restrictions on K? (0.5 p)

4. The following nonlinear system on state-space form is given

$$\begin{aligned}\dot{x}_1 &= x_1(1-x_1)+x_2\\ \dot{x}_2 &= x_2(2-x_1)\end{aligned}$$

a. Find the three stationary points of the nonlinear system. (1 p)

b. Linearize the system around the only stationary point (x_1^0, x_2^0) where both x_1^0 and x_2^0 are nonzero. (1.5 p)

c. For the stationary point in **b.**, decide if the linearized system is stable, asymptotically stable, or unstable. (1 p)

5. The block schedule for a system is given in Figure 4.

a. Give the transfer functions from $r \to y$, $d \to y$, and $n \to y$. (2 p)

b. Is it possible to design a controller that follows reference signals r perfectly in steady state, that also removes the impact of noise n of all frequencies completely? Motivate your answer! (1 p)

c. Design the feedforward controller $F(s)$ so that all impact from the disturbance d on the output y is removed when

$$P(s) = \frac{1}{s+1}, \qquad C(s) = \frac{5s+3}{s}, \qquad D(s) = \frac{1}{s^2+3s+4}.$$

(1 p)

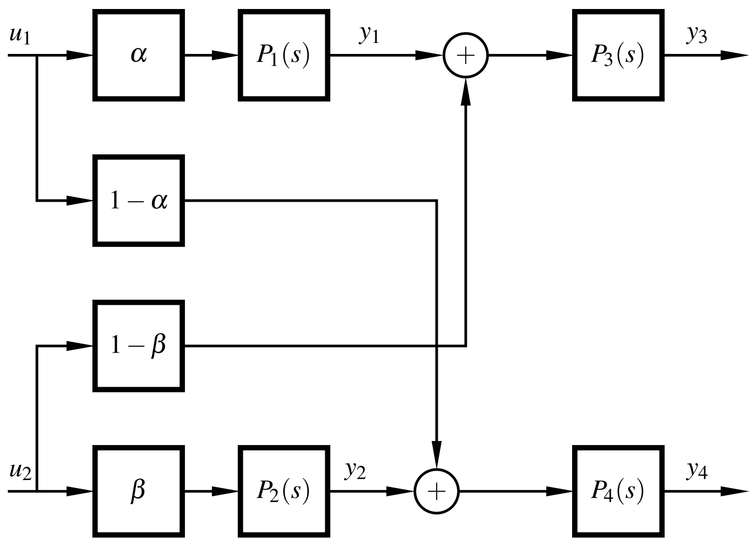

Figure 5 The block diagram for the system in problem 7.

6. Consider a process $G_P(s)$ with transfer function

$$G_P(s) = \frac{s+2}{s^2+2s+5}$$

a. Determine the poles and zeros of the process. Is the system stable? Asymptotically stable? (1 p)

b. Design a P-controller with gain K such that the poles of the closed loop system end up in -3 and -7. (2 p)

c. Is it possible to place the closed loop system poles arbitrarily using a P-controller? Motivate your answer. (0.5 p)

7. **Only for Process Control:**

You have a system described by the block diagram in Figure 5.

Remark: This system could for instance describe the linearized tank processes you used in laboration, but using both pumps simultaneously and letting part of the water from each pump go into the upper tank and the rest of the water directly into the lower tank on the opposite side. If you take our course on Multivariable control you will use this setup in one of the laborations.

a. Write the transfer function matrix from the two input signals u_1 and u_2 to the four different output signals y_1, y_2, y_3, y_4. (1.5 p)

b. Let's say you are only interested in controlling the outputs y_1 and y_2 (*corresponding to the upper tanks in the example*). How should you pair the inputs and outputs? Motivate your answer. (0.5 p)

c. Let's instead say you are only interested in controlling the outputs y_3 and y_4 (*corresponding to the lower tanks*). How should you pair the inputs and outputs for the case where all the subsystems have the same static gain $P_1(0) = P_2(0) = P_3(0) = P_4(0) = 1$ and the ratios are $\alpha = \beta = \frac{1}{4}$? Motivate your answer. (1 p)

8. **Only for Process Control:** A continuous time closed loop system is described by

$$Y(s) = \frac{25}{s+25}R(s).$$

a. Discretize the system using forward approximation with a sample time $h = 0.1$. Write the resulting system as a difference equation. (1 p)

b. When comparing step responses between the continuous time system and the discretized system, they do not agree at all. Why? (Hint: It has to do with stability.) Suggest a way to improve the discretization. (1 p)