Solutions to Exam in Systems Engineering/Process Control 2015-08-27

1 a. From the step response we identify the dead-time $L = 4$, the time constant $T = 2$, and the static gain $K_p = 0.5$. The process transfer function is hence

$$G(s) = \frac{0.5}{1+2s}e^{-4s}$$

b. Using the lambda tuning rules from the Collection of Formulae we obtain

$$K = \frac{1}{K_p}\frac{T}{L+T} = \frac{2}{3}$$
$$T_i = T = 2$$

2. Taking the Laplace transform of the equations we obtain

$$VsC_1(s) = qU(s) - qC_1(s)$$
$$VsC_2(s) = qC_1(s) - qC_2(s)$$

Solving for $C_2(s)$ we get

$$C_2(s) = \frac{1}{(\frac{V}{q}s+1)^2}U(s)$$

Since $y = c_2$, the transfer function from uU to y is hence $\frac{1}{(\frac{V}{q}s+1)^2}$. With $V, q > 0$, both poles are in the left half plane, and the system is hence asymptotically stable.

3 a. From the block diagram, we calculate the Laplace transform of the error:

$$E(s) = R(s) - G_P(s)\Big(D(s) + G_c(s)E(s)\Big)$$

Solving for $E(s)$ we obtain

$$\begin{aligned} E(s) &= \frac{1}{1+G_c(s)G_P(s)}R(s) - \frac{G_p(s)}{1+G_c(s)G_P(s)}D(s) \\ &= \frac{s(s+2)}{s(s+2)+K}R(s) - \frac{1}{s(s+2)+K}D(s) \end{aligned}$$

The transfer function from r to e is hence $\frac{s(s+2)}{s(s+2)+K}$, and the transfer function from d to e is $-\frac{1}{s(s+2)+K}$.

b. As seen in both transfer functions above, the closed-loop characteristic polynomial is $s(s+2)+K$. A second-order polynomial has both roots in the left half plane if and only if all coefficients are positive. Hence, the system is asymptotically stable for all $K > 0$.

c. Setting $R(s) = 0$ and $D(s) = \frac{1}{s}$, we obtain

$$E(s) = -\frac{1}{s(s+2)+K}\frac{1}{s}$$

Assuming $K > 0$, we can apply the final value theorem:

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} -\frac{1}{s(s+2)+K} = -\frac{1}{K}$$

d. Setting $D(s) = 0$ and $R(s) = \frac{1}{s}$, we obtain

$$E(s) = \frac{s(s+2)}{s(s+2)+K}\frac{1}{s}$$

Assuming $K > 0$, we can apply the final value theorem:

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s(s+2)}{s(s+2)+K} = 0$$

e. ζ determines the damping and ω the speed of the closed-loop system.

f. With $K = 4$ we have the characteristic polynomial $s^2 + 2s + 4$. We identify $\omega = 2$ and $\zeta = 0.5$.

4. The augmented block diagram is shown in Figure 1.

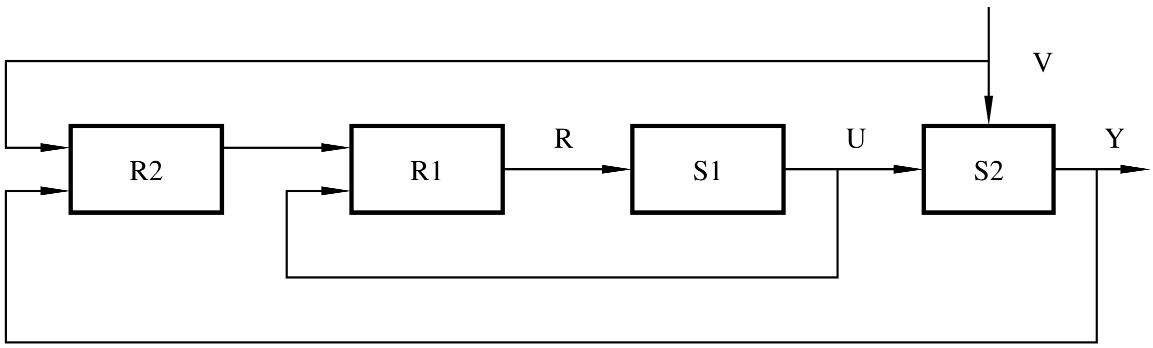

Figure 1

The present control principles are feedback (from dome level and water flow), feed-forward (from the steam flow) and cascade control (The control signal for the level controller is reference value for the flow controller).

5 a. We read out that the low-frequency gain goes to 5 and the phase approaches $0°$. Hence, the static gain is 5.

b. For $\omega = 20$ rad/s we read out approximately $|G(i\omega)| = 0.1$ and $\arg G(i\omega) = -90°$. The output signal is therefore $y(t) \approx \sin(20t - \pi/2)$.

c. At $\omega \approx 100$ we read out the gain $|G(i\omega)| \approx 0.02$. The gain margin is therefore approximately $A_m = 50$. Since $K < A_m$, we have a stable closed-loop system.

6 a. At stationary points, all time derivatives must be 0:

$$\begin{cases} 0 = -\beta SZ \\ 0 = \beta SZ + \zeta R - \alpha SZ \\ 0 = \alpha SZ - \zeta R \end{cases}$$

We see that in the first equation, either S or Z must be 0. Then the third equation gives that R is 0. In conclusion, there are 3 stationary points: $(S, Z, R) = (0, 0, 0)$, $(S, Z, R) = (S^0, 0, 0)$ och $(S, Z, R) = (0, Z^0, 0)$, where S^0 och Z^0 are arbitrary (positive) numbers. Hence, humans and zombies can not co-exist in an equilibrium.

b. The stationary point when $S = S^0 > 0$ is given in subtask a), by $(S, Z, R) = (S^0, 0, 0)$. Identify the system on standard form $\frac{dx}{dt} = f(x)$. This yields:

$$x = \begin{bmatrix} S \\ Z \\ R \end{bmatrix}$$

$$f(x) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} -\beta SZ \\ \beta SZ + \zeta R - \alpha SZ \\ \alpha SZ - \zeta R \end{bmatrix}$$

Define Δx according to

$$\Delta x = \begin{bmatrix} S - S^0 \\ Z - Z^0 \\ R - R^0 \end{bmatrix} = \begin{bmatrix} S - S^0 \\ Z \\ R \end{bmatrix}$$

The linearized model can then be written as

$$\frac{d\Delta x}{dt} = A\Delta x$$

where

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial Z} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial Z} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial Z} & \frac{\partial f_3}{\partial R} \end{bmatrix}\Bigg|_{(S,Z,R)=(S^0,0,0)}$$

Computation of the partial derivatives

$$\begin{array}{lll} \frac{\partial f_1}{\partial S} = -\beta Z & \frac{\partial f_1}{\partial Z} = -\beta S & \frac{\partial f_1}{\partial R} = 0 \\ \frac{\partial f_2}{\partial S} = \beta Z - \alpha Z & \frac{\partial f_2}{\partial Z} = \beta S - \alpha S & \frac{\partial f_2}{\partial R} = \zeta \\ \frac{\partial f_3}{\partial S} = \alpha Z & \frac{\partial f_3}{\partial Z} = \alpha S & \frac{\partial f_3}{\partial R} = -\zeta \end{array}$$

Inserting the stationary point in these derivatives gives the linearized system below.

$$\frac{d\Delta x}{dt} = \begin{bmatrix} 0 & -\beta S^0 & 0 \\ 0 & (\beta - \alpha)S^0 & \zeta \\ 0 & \alpha S^0 & -\zeta \end{bmatrix} \Delta x$$

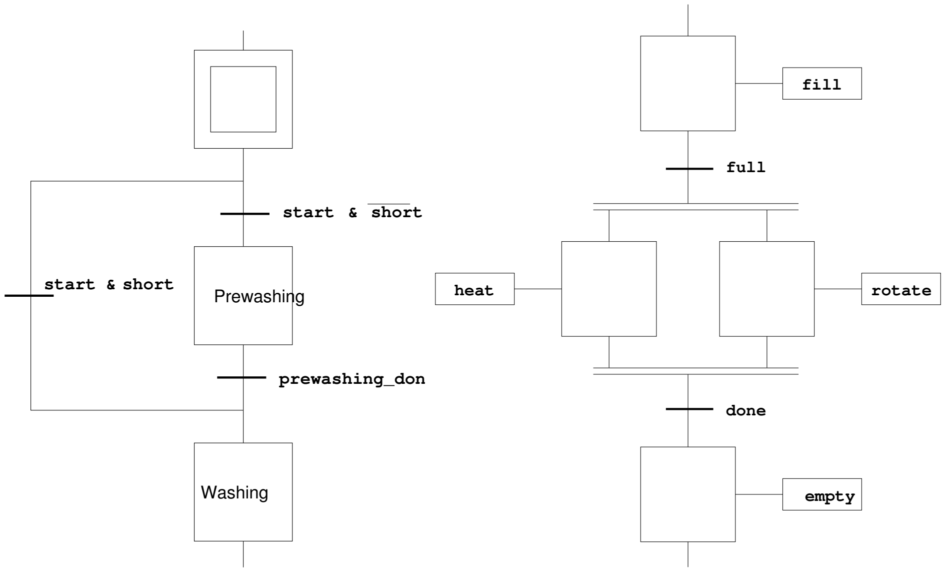

Figure 2 Suggested solution for Problem 7. Subproblem a) to the left and b) to the right.

7. A suggested solution is shown in Figure 2. Instead of using parallel branches in b), one could use that one step can have several outputs.

8 a. The RGA for the system is $\begin{bmatrix} -2 & 3 \\ 3 & -2 \end{bmatrix}$ and therefore we should pair the input and output signals according to u_1 — y_2 and u_2 — y_1, i.e., opposite of the given pairing.

b. Inserting the control law into the state-space description, we obtain the closed-loop system

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -5 & 0 \\ 0 & -3 \end{pmatrix} x = \begin{pmatrix} -6 & -6 \\ -15 & -14 \end{pmatrix} x$$

The characteristic polynomial is

$$\det \begin{pmatrix} s+6 & 6 \\ 15 & s+14 \end{pmatrix} = s^2 + 20s - 6$$

The system is unstable, since one coefficient of the characteristic polynomial is negative.