LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

Exam in FRT110 Systems Engineering and FRTN25 Process Control

August 27, 2015, 08:00–13:00

Points and grades

All answers must include a clear motivation. Answers may be given in English or Swedish. The total number of points is 20 for Systems Engineering and 25 for Process Control. The maximum number of points is specified for each subproblem. Preliminary grading scales:

Systems Engineering:		Process Control:	
Grade 3:	10 points	Grade 3:	12 points
4:	14 points	4:	17 points
5:	17 points	5:	21 points

Acceptable Aid

Authorized *Formelsamling i reglerteknik* / *Collection of Formulae*. Standard mathematical tables like TEFYMA. Pocket calculator.

Results

The solutions are posted on the course home page, and the results will be transferred to LADOK as soon as the exams have been graded. Please contact Christian Grussler to view the exam.

1. The unit step response of a first-order process with a time delay is shown in Figure 1.

a. Give the transfer function $G_p(s)$ of the process. (2 p)

b. Design a PI-controller for the process using the lambda method with $\lambda = T$, where T is the time constant of the process. (1 p)

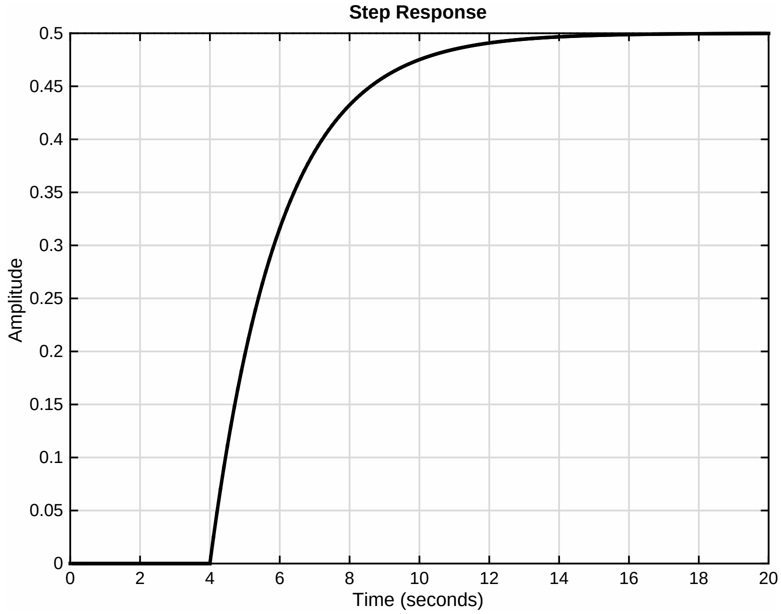

Figure 1 The step response in Problem 1.

2. Two stirred tanks are connected in series. The tanks have constant liquid volume $V > 0$ and constant flow $q > 0$. Mathematical modeling of the concentration dynamics gives the differential equations

$$V\dot{c}_1 = qu - qc_1$$
$$V\dot{c}_2 = qc_1 - qc_2$$

where c_1 and c_2 are the concentrations in the two tanks, and u is the concentration of the feed. The measurement signal is $y = c_2$.

Calculate the transfer function from u to y. Is the system asymptotically stable? (2 p)

3. Figure 2 shows the block diagram of a feedback control system. The process is given by

$$G_p(s) = \frac{1}{s(s+2)}$$

and the system is controlled by a P-controller with gain K.

a. Calculate the transfer functions from r to e and from d to e, respectively. (1 p)

b. For what values of K is the closed-loop system asymptotically stable? (1 p)

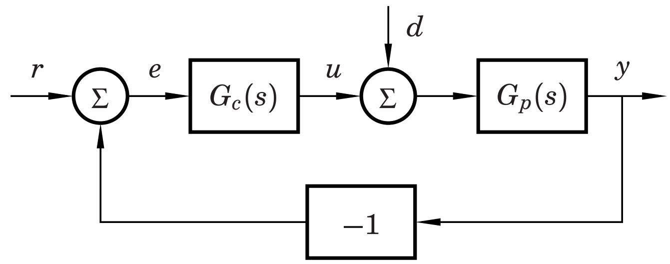

Figure 2 Block diagram in Problem 3

c. Calculate the stationary error $e(\infty)$ when d is a unit step load disturbance. The reference value r can be assumed to be zero. (1 p)

d. Calculate the stationary error $e(\infty)$ when r is a unit step reference change. The disturbance d can be assumed to be zero. (1 p)

e. The desired characteristic polynomial of a second-order closed-loop system is often specified as

$$s^2 + 2\zeta\omega s + \omega^2$$

Give a brief explanation of how the parameters ω and ζ affect the behavior of the closed-loop system.

What are ω and ζ if $K = 4$ in our feedback control system? (2 p)

4. A common method to control a drum boiler is illustrated in Figure 3. One valve controls the feed water to the boiler, and another valve controls the steam outlet. The goal is to keep the liquid level in the boiler (the dashed line) constant. The steam outlet is controlled by an external system and can hence be viewed as a disturbance.

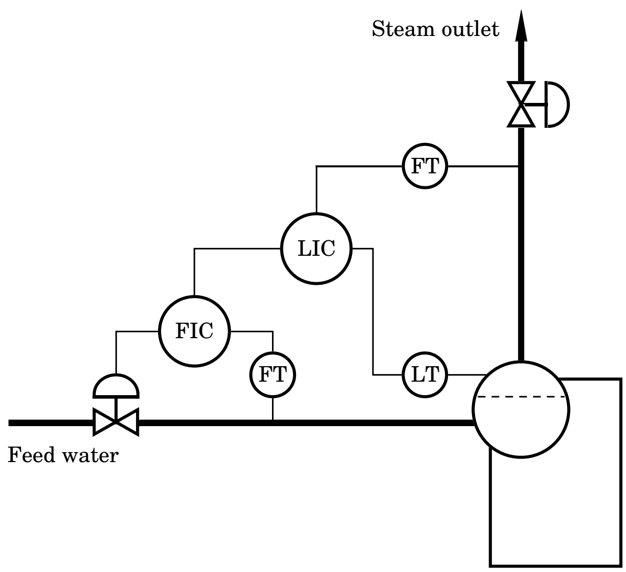

Figure 3 Drum boiler control in Problem 4

A block diagram of the open loop system is shown in Figure 4. Extend the diagram with the two controllers FIC and LIC in accordance with the P/I diagram in Figure 3. What control principles are used? (3 p)

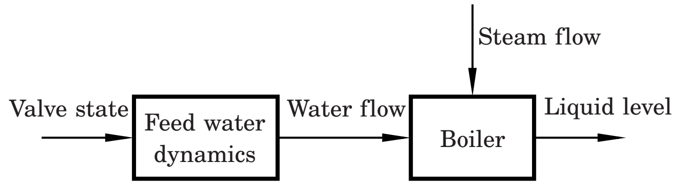

Figure 4 Block diagram of the open loop drum boiler system.

5. The Bode diagram of a stable linear system is shown in Figure 5. Determine the following using the diagram:

a. What is the static gain of the system? (1 p)

b. If the input signal is $u(t) = 10 \sin 20t$, what is the output signal $y(t)$? (1 p)

c. If the system is controlled using a P controller with gain $K = 10$, will the closed-loop system be stable? (1 p)

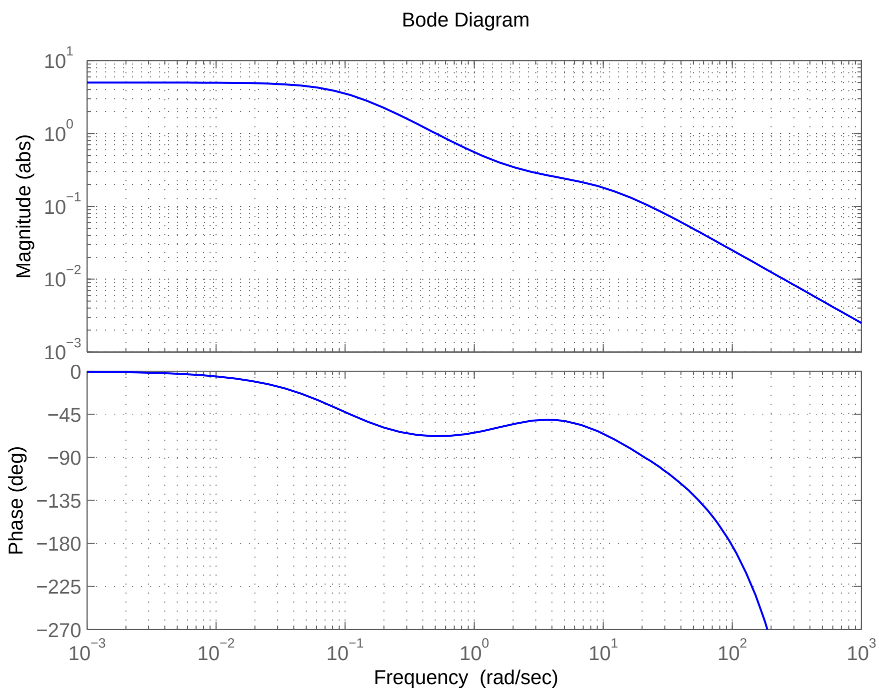

Figure 5 Bode diagram for the system in Problem 5

6. In 2009, four Canadian researchers published a dynamical model of a zombie epidemic.[1] The model contains three different stages that people can be in:

- Susceptibles, S
- Zombies, Z
- Removed, R

[1]Munz, P., Hudea, I., Imad, J., Smith, R.J. *When zombies attack!: Mathematical modelling of an outbreak of zombie infection*. Infectious Disease Modelling Research Progress. 2009.

Susceptibles die from natual causes with a rate δ; the dead humans are moved to the Removed category. Susceptibles can also become zombies after an encounter with a zombie; this happens with a rate ζ. Zombies are defeated with the rate α; the defeated zombies also end up in the Removed category. The model further assumes that the birth rate of humans is constant and equal to Π. The full model is given by

$$\begin{cases} \frac{dS}{dt} = \Pi - \beta SZ - \delta S \\ \frac{dZ}{dt} = \beta SZ + \zeta R - \alpha SZ \\ \frac{dR}{dt} = \delta S + \alpha SZ - \zeta R \end{cases}$$

Let us assume that a zombie outbreak happens under a short period of time, which means that the natural birth and death rates can be ignored. This gives the following simplified model, which will be used in our analysis:

$$\begin{cases} \frac{dS}{dt} = -\beta SZ \\ \frac{dZ}{dt} = \beta SZ + \zeta R - \alpha SZ \\ \frac{dR}{dt} = \alpha SZ - \zeta R \end{cases}$$

a. Calculate all stationary points (S^0, Z^0, R^0) of the simplified model. Can humans (susceptibles) and zombies co-exist in equilibrium? (1 p)

b. Linearize the simplified model around a stationary point where $S^0 > 0$. (2 p)

7. Only for FRTN25 Process Control.

The functionality of a washing machine can be described by the Grafcet program in Figure 6. When the operator hits the start button the prewashing cycle starts. When the prewashing has completed, the main washing cycle follows, and finally the laundry is rinsed and spun.

a. The laundry often gets clean enough without prewashing, so in order to save energy one should be able to skip this step. Modify the Grafcet program so that the Prewashing step is bypassed if the logic signal `short` is true. (1 p)

b. The Washing step can be divided into smaller parts. A simple description is given below.

1. Fill the machine with water by setting the signal `fill` to true.
2. When the machine is full this is indicated by the signal `full` becoming true. One should then *simultaneously* start the heating and the rotation by setting the signals `heat` and `rotate` to true.
3. When the washing is done, the signal `finished` becomes true. The heating and the rotation should then both stop. The water should also be emptied out by setting `empty` to true.

Translate the description above into Grafcet code. Only the signals mentioned in the description should be included. (2 p)

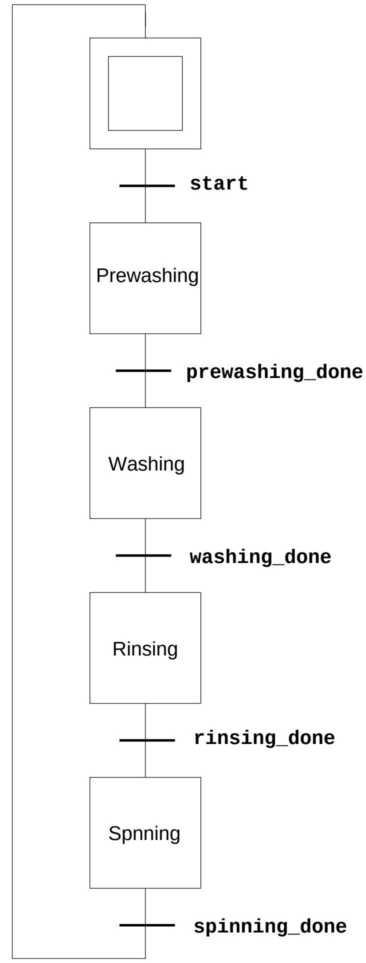

Figure 6 Grafcet program for Problem 7.

8. **Only for FRTN25 Process Control.** A mystical multivariable process in the basement of the Department of Automatic Control is described by the transfer matrix

$$G(s) = \begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} \\ \frac{3}{s+2} & \frac{4}{s+2} \end{pmatrix}$$

or, equivalently, by the state-space description

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} u \\ y &= x \end{aligned}$$

A recently graduated doctor in Automatic Control decides to test the waters and takes two P controllers out of a shelf. He neglects the cross-coupling and instead tunes the controllers as if there were two simple loops. The pairing of outputs and inputs is done as shown in the block diagram in Figure 7, and the gains of the controllers are chosen as $K_1 = 5$ and $K_2 = 3$.

a. Calculate the RGA of the process. Has the pairing of outputs and inputs been done wisely? (1 p)

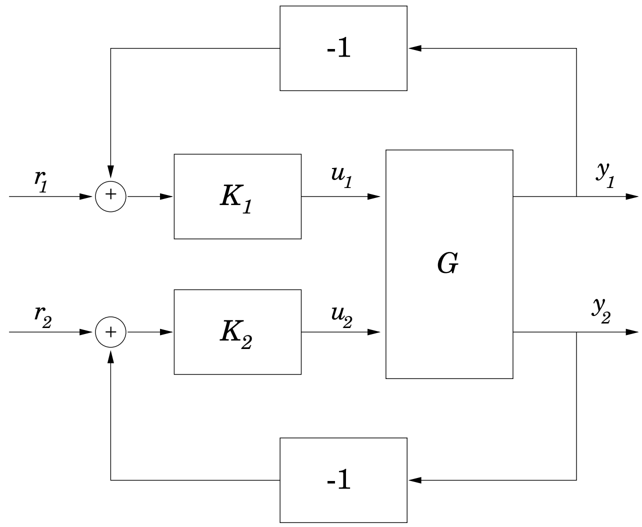

Figure 7 Multivariable control system in Problem 8.

b. Disregarding the reference values, the P controllers can be expressed in matrix form as

$$u = \begin{pmatrix} -5 & 0 \\ 0 & -3 \end{pmatrix} x$$

Combine this expression with the process state-space description above and show that the closed-loop system is indeed unstable under the current control configuration. (1 p)