Solutions to Exam in Systems Engineering/Process Control 2015-06-05

1 a. y_1 and y_2 are used for feedback control in a cascaded structure. The master level controller (LIC) calculates the flow setpoint for the slave flow controller (FIC). y_3 measures the outflow, which can be seen as a disturbance. The level controller uses feedforward from this signal to compensate for the disturbance.

b. If y_2 is removed, there will be no feedback from the tank level, which is the quantity we are trying to control. Hence, y_2 should not be removed. Removing y_1 (and FIC) would disable the inner loop of the cascade controller, and removing y_3 would disable the feedforward. Both of these actions would decrease the control performance but still leave a working system.

2 a. In stationarity we have $\dot{x} = 0$, which yields $0 = -8(x_2^0)^3 + \sqrt{1} \leftrightarrow x_2^0 = 0.5$. Further, $x_1^0 = 0$. Finally, we have $y^0 = (x_2^0)^2 = 0.25$. Let

$$\begin{aligned} f_1(x_1, x_2, u) &= -8x_2^3 + \sqrt{u} \\ f_2(x_1, x_2, u) &= x_1 \\ g(x_1, x_2, u) &= x_2^2 \end{aligned}$$

The partial derivatives are

$$\begin{aligned} &\frac{\partial f_1}{\partial x_1} = 0 && \frac{\partial f_1}{\partial x_2} = -24x_2^2 && \frac{\partial f_1}{\partial u} = \frac{1}{2\sqrt{u}} \\ &\frac{\partial f_2}{\partial x_1} = 1 && \frac{\partial f_2}{\partial x_2} = 0 && \frac{\partial f_2}{\partial u} = 0 \\ &\frac{\partial g}{\partial x_1} = 0 && \frac{\partial g}{\partial x_2} = 2x_2 && \frac{\partial g}{\partial u} = 0 \end{aligned}$$

Inserting the stationary values and introducing the variables $\Delta x = x - x^0, \Delta u = u - u^0$ and $\Delta y = y - y^0$, we get the linearized system

$$\begin{aligned} \dot{\Delta x} &= \begin{pmatrix} 0 & -6 \\ 1 & 0 \end{pmatrix} \Delta x + \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} \Delta u \\ \Delta y &= \begin{pmatrix} 0 & 1 \end{pmatrix} \Delta x \end{aligned}$$

b. The transfer function is calculated as

$$G(s) = C(sI - A)^{-1}B = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s & 6 \\ -1 & s \end{pmatrix}^{-1} \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} = \frac{0.5}{s^2 + 6}$$

The two poles are located in $\pm\sqrt{6}\,i$. Hence, the system is marginally stable (i.e., stable but not asymptotically stable).

3. Larger ω_0 gives faster step response, and poles further away from the origin. Larger ζ gives a more damped step response, and smaller angle between the poles and the negative real axis (see page 45 in the course book). Hence, the correct matching is 1–D, 2–C, 3–B, 4–A.

4 a. Laplace transformation of the equation yields

$$3sY(s) + Y(s) - e^{-0.1s}U(s) = 0$$

Solve for $Y(s)$ to obtain

$$Y(s) = \frac{e^{-0.1s}}{3s+1}U(s)$$

Now, the transfer function can be identified as

$$G(s) = \frac{e^{-0.1s}}{3s+1}$$

- The system is asymptotically stable, so the static gain is given by $G(0) = 1$.
- The time constant is identified as $T = 3$.
- The deadtime is identified as $L = 0.1$.

b. We have $U(s) = 1$, and hence $Y(s) = \frac{e^{-0.1s}}{3s+1}$. Inverse Laplace transformation gives

$$y(t) = \begin{cases} 0, & t < 0.1 \\ \frac{1}{3}e^{-(t-0.1)/3}, & t \geq 0.1 \end{cases}$$

5 a. The gain is calculated as

$$|G(i\omega)| = |K(1 + i\omega T_d)| = K\sqrt{1 + (\omega T_d)^2}$$

b. Very large input frequency gives a very large gain, and the controller is very sensitive to high-frequency noise. This can be coped with by using the following approximation of the derivative term, where the derivative gain is limited by N:

$$D(s) = \frac{sKT_d}{1 + sKT_d/N}$$

6 a. This is a PI-controller, since it is in the form

$$G_1(s) = K\left(1 + \frac{1}{sT_i}\right)$$

The s in the denominator corresponds to time integration, and forms the integral part together with the control parameters.

b. Setting $l_1 = l_2 = n = 0$, we obtain

$$\begin{aligned} Y(s) &= G_2(s)G_1(s)(R(s) - Y(s)) \\ Y(s) &= \underbrace{\frac{G_2(s)G_1(s)}{1 + G_2(s)G_1(s)}}_{=G_{yr}(s)} R(s) \end{aligned}$$

Inserting the given transfer functions, we calculate

$$G_{yr}(s) = \frac{-1.5(s+0.2)(s-2)}{(s+0.2)(s+3)} = \frac{-1.5(s-2)}{s+3}$$

The zero is hence located in $+2$, and the pole is located in -3.

(The process pole in -0.2 is cancelled by the controller zero in the same location—this is typical for Lambda tuning.)

c. Since the system is asymptotically stable (pole strictly in left half-plane), the static gain can be found as $G(0) = 1$. This implies that there will be no static error, since we will have $y(t) = r(t)$ in stationarity (again assuming $l_1 = l_2 = n = 0$).

7 a. The phase-margin is 90 deg as seen by the intersection of the unit-circle and the Nyquist curve. The Nyquist curve intersects the negative real axis at about -0.2 (the actual value is $-\frac{1}{6}$), which gives an amplitude margin of 5 (respectively 6).

The static gain is given by $G(0)$, which is about 1.6 (the actual value is $\frac{5}{3}$).

b. In the Laplace domain, we have

$$E(s) = G(s)(D(s) - KE(s)) \Leftrightarrow E(s) = \frac{G(s)}{1+KG(s)}D(s)$$

With $D(s) = \frac{1}{s}$, we have

$$e(\infty) = \lim_{s\to 0} sE(s) = \frac{G(0)}{1+KG(0)} = \frac{5/3}{1+5K/3}$$

The stationary error decreases with larger K, and for stability we need $K < 6$. Hence, the smallest stationary error that is possible is 0.15.

8. One solution is shown in Figure 1. More elaborate solutions are also possible.

9. For each row in Λ (each corresponding to an output), one should find a column (each corresponding to an input) with a non-negative element close to one. In row 1, the first element is 0.931. In row 2, the fourth element is 1.154. In row 3, the only positive element is the second one, 3.314. In row 4, the third element is 0.90. A suitable pairing is hence y_1–u_1, y_2–u_4, y_3–u_2, y_4–u_3.

10. The control signal vector is given by

$$U(s) = G_1(s)(R(s) - G_2(s)U(s))$$

Solving for $U(s)$ we obtain

$$(I + G_1(s)G_2(s))U(s) = G_1(s)R(s)$$
$$U(s) = (I + G_1(s)G_2(s))^{-1}G_1(s)R(s)$$

The multivariable transfer function from r to u is hence given by

$$G_{ur}(s) = (I + G_1(s)G_2(s))^{-1}G_1(s)$$

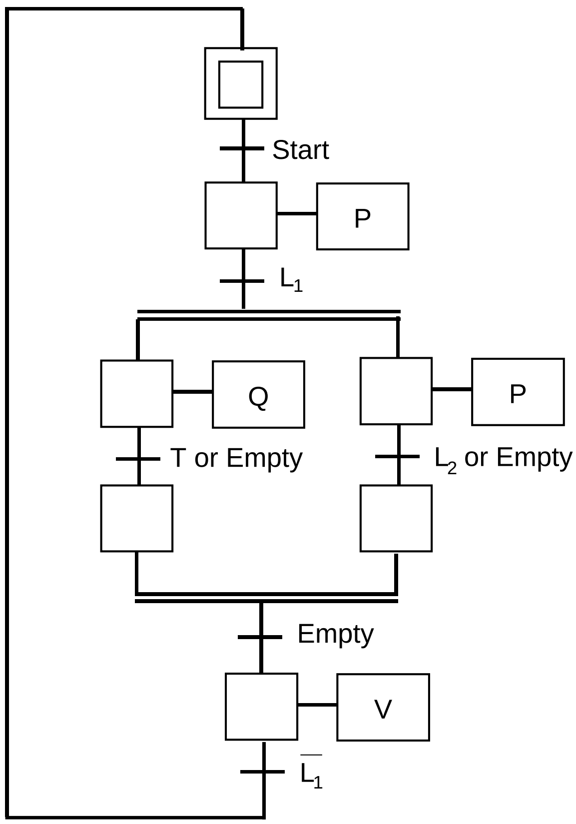

Figure 1 GRAFCET in Problem 8