



**LUNDS**  
UNIVERSITET

Institutionen för  
**REGLERTEKNIK**

## Tentamen i Systemteknik/Processreglering

2014-08-28

### Poängberäkning och betygssättning

Lösningar och svar till alla uppgifter skall vara klart motiverade. Tentamen omfattar 7 uppgifter om totalt 20 poäng (Systemteknik) eller 8 uppgifter om totalt 25 poäng (Processreglering). Poängberäkningen finns markerad vid varje uppgift. Preliminära betygsgränser:

Systemteknik:	Processreglering:
Betyg 3: 10 poäng	Betyg 3: 12 poäng
4: 14 poäng	4: 17 poäng
5: 17 poäng	5: 21 poäng

### Tillåtna hjälpmedel

Matematiska tabeller (TEFYMA eller motsvarande), formelsamling i reglerteknik samt icke förprogrammerade räknare.

### Tentamensresultat

1. The population dynamics of wolves ( $V$ ) and reindeer ( $R$ ) in a small region in eastern Siberia can be expressed as

$$\begin{aligned}\dot{V} &= 0.5V + 0.1R - 0.1V^2 \\ \dot{R} &= 0.5R - 4V - \frac{2}{300}RV\end{aligned}$$

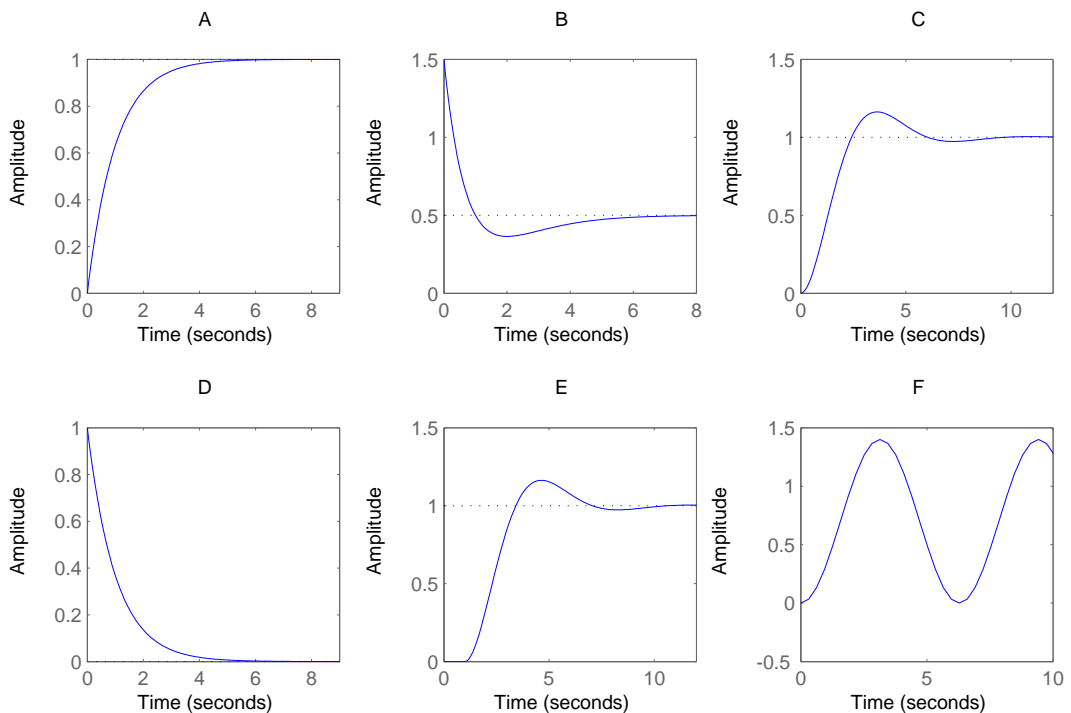
- a. Find all stationary points for the system. (1.5 p)
- b. Linearize the system around the stationary point which gives the highest value for  $R$ . Comment on the stability properties of the linearized system. (3 p)
2. Does the transfer function

$$G(s) = \frac{s+2}{s^3+3s^2+4s+10}$$

have all its poles strictly in the left half-plane? (1 p)

3. Match the transfer functions  $G_1$ – $G_4$  with four of the step responses A–F in Figure 1. The magnitude of the step is 1 and the step is applied at time  $t = 0$ . Don not forget to motivate your answer. (2 p)

$$\begin{aligned}G_1(s) &= \frac{0.7}{s^2+1} & G_2(s) &= \frac{1}{s+1} \\ G_3(s) &= \frac{s}{s+1} & G_4(s) &= \frac{e^{-s}}{s^2+s+1}\end{aligned}$$



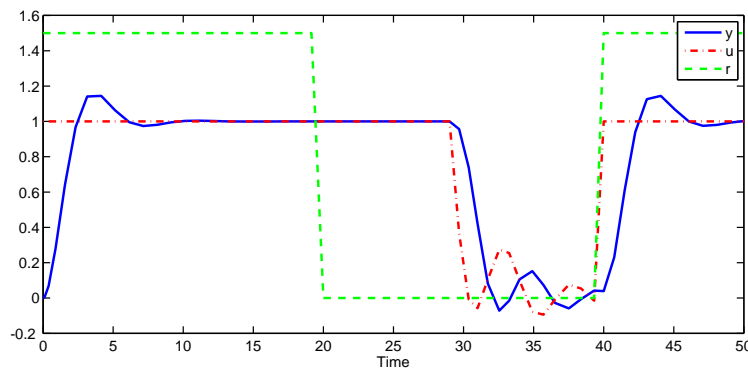
Figur 1 The step responses A–F for Problem 3.

4. Consider the system with transfer function

$$G(s) = \frac{1}{(s-2)(s+4)}$$

The system is to be controlled using a P regulator with gain  $K$ . Find the closed-loop transfer function and the lowest value for  $K$  for which the closed-loop system is stable. (1.5 p)

5. Your colleague is complaining about poor performance in a control system she designed some time ago. She is showing you the output of the process and is kindly asking for your assistance in the matter.



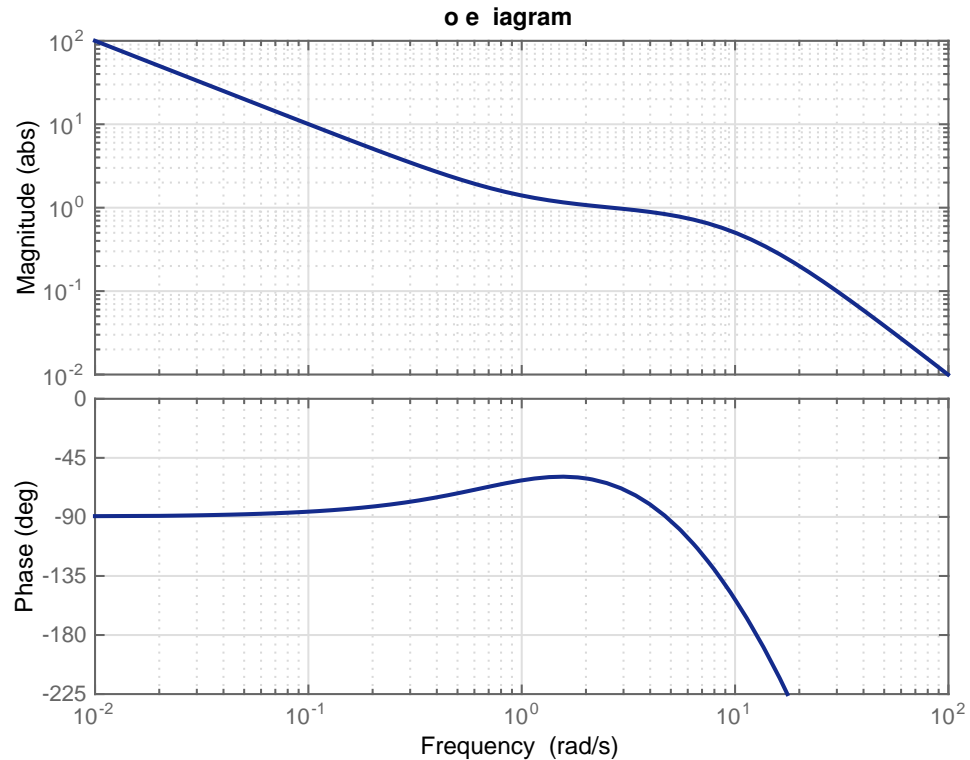
**Figure 2** The process output of a poorly controlled system.

- a. As you can see from the process output in Figure 5, the output  $y$  does not follow the reference  $r$  very well, especially at time  $t \in [20, 30]$ . What kind of controller do you think your colleague has implemented and what is making this controller perform poorly? Motivate your answer. (2 p)
- b. Suggest a strategy to mitigate the severe control issues while keeping the general structure of the controller. (2 p)
6. A process is described by the differential equation

$$\dot{y} + y - u = 0 \quad (1)$$

where  $u$  is the control signal and  $y$  is the measured output.

- a. Find the transfer function  $G_P(s)$  from  $U(s)$  to  $Y(s)$ . Is the process stable? (2 p)
- b. Design a feedback PI-controller  $G_R(s)$  such that the closed-loop poles are placed in  $-2 \pm i$ . (2 p)
- c. If a reference signal  $r(t) = 2 \sin t$  is fed to the controller, what will the output signal be (after initial transients have died out)? (2 p)
7. Consider the problem whose Bode diagram is shown in Figure 3. Find the system's gain and phase margins. (1 p)



**Figur 3** Bode diagram for the system in Problem 7.

- 8. Only for Process Control:** Consider the multivariable system

$$G(s) = \begin{pmatrix} \frac{2a}{(s+2)(50s+1)} & \frac{e^{-s}}{(10s+1)(2s+1)} \\ \frac{3}{(10s+1)(2s+1)} & \frac{2}{(2s+1)^2} \end{pmatrix}. \quad (2)$$

The system should be controlled using two PID controllers

- a. Calculate the relative gain array, RGA, for the system. (1 p)
- b. How should the inputs and outputs be paired if  $a = 0$ ? Comment upon the interaction. (0.5 p)
- c. How should the inputs and outputs be paired if  $a = 3$ ? Comment upon the interaction. (0.5 p)
- d. Let  $a = 3$ . Determine a decoupling matrix that decouples the system dynamics in stationarity and gives the corresponding decoupled system static gains of one. Is the decoupler realizable? (1 p)

- 9. Only for Process Control:** Consider the system

$$\dot{y} + 2y = 3u,$$

where  $y(t)$  is the output and  $u(t)$  is the input.

- a. Discretize the system using the forward (Euler) approximation and the sampling interval  $h$ . (1 p)
- b. Discretize the system using the backward approximation and the sampling interval  $h$ . (1 p)