

Systems Engineering/Process Control

Hand-In Assignment 2

Dynamics for Biodegradation of Wastewater

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Figure 1 Aerated lagoons are commonly used for wastewater treatment in Swedish pulp industry. The picture shows an aerated lagoon in Korsnäs.

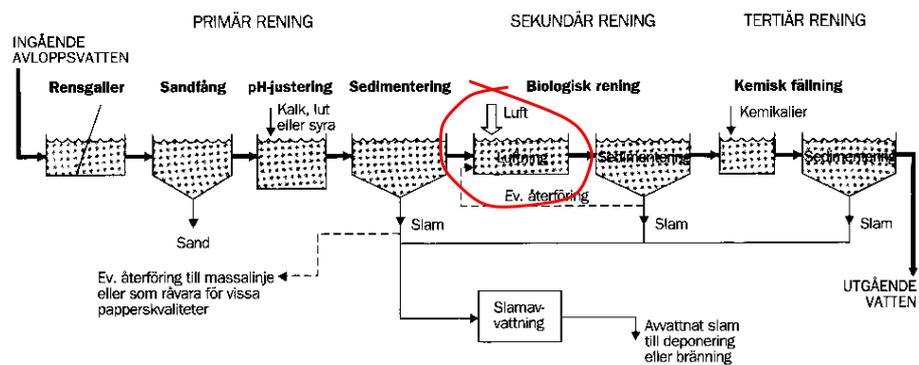


Figure 2 Flow diagram of the wastewater treatment process. Only a few wastewater plants encompass all these steps. In this assignment, the encircled subsystem is studied.

1. Introduction

The purpose of the hand-in assignment is to study the dynamics for biodegradation of for instance process water. We will find the equilibrium point of the system and linearize the system around this point. Using simulations we will study the accuracy of the resulting linear approximation. Finally we will examine how a PI controller can be used to control the water quality by varying the water flow.

You will use MATLAB with Control System Toolbox and Simulink in the assignment.

The assignment is performed in groups of three students each and should take approximately one day to complete.

Background

In addition to the concepts studied in Assignment 1, you should also be familiar with

- linearization
- simulation using Simulink

Two Simulink models are available on the course homepage: `damm.mdl` for simulation and `damm_lin.mdl` for linearization. (“Damm” means pond.)

2. Problem Formulation¹

In the pulp and paper industry, large amounts of wastewater containing dissolved organic carbon is produced. Before the wastewater can be released into a body of water, it is important to reduce the amount of organic carbon, since carbon compounds are consumed by heterotrophs that in turn consume large amounts of oxygen in the recipient.

The organic quality of the water is commonly expressed in terms of its BOD, *biochemical oxygen demand*. BOD is an experimentally determined quantity that describes how much oxygen is needed for the microbial metabolism of organic compounds in the water. If the water contains 1 gm^{-3} BOD, then 1 g dissolved O_2 is required to convert the organic material in 1 m^3 wastewater into CO_2 and water. (Sometimes, BOD_5 or BOD_7 are specified, which correspond to the amount of oxygen consumed in 5 or 7 days.)

The conversion rate can be limited by the supply of oxygen or by the supply of carbon. High substrate levels may also lead to saturation. For O_2 and BOD, it has been shown experimentally that the decomposition rate can be described by a Michaelis–Menten expression for the total rate,

$$r([\text{O}_2], [\text{BOD}]) = \frac{[\text{O}_2] \cdot \alpha}{\beta + [\text{O}_2]} \cdot \frac{[\text{BOD}] \cdot \gamma}{\delta + [\text{BOD}]}$$

¹Parts of the problem formulation have been borrowed from the the exercise compendium on Material Balances and Reactor Calculations by Per Warfvinge.

where we in this assignment assume the parameters

$$\begin{aligned}\alpha &= 2 \text{ (g m}^{-3} \text{ h}^{-1}\text{)}^{0.5} \\ \beta &= 2 \text{ g m}^{-3} \\ \gamma &= 5 \text{ (g m}^{-3} \text{ h}^{-1}\text{)}^{0.5} \\ \delta &= 10 \text{ g m}^{-3}\end{aligned}$$

Oxygen for the decomposition is also supplied from the atmosphere via diffusion. This mass transport is described by the flow rate

$$F_{\text{O}_2} = kA([\text{O}_2]_{\text{sat}} - [\text{O}_2])$$

In the assignment we will study the dynamics of biodegradation of BOD in an aerated lagoon with the following nominal parameters:

Mass transport coefficient	k	0.1 m h ⁻¹
Area of the lagoon	A	3000 m ²
Volume of the lagoon	V	1600 m ³
Oxygen saturation level at 20°C	$[\text{O}_2]_{\text{sat}}$	10 g m ⁻³
Oxygen level in the inflow	$[\text{O}_2]_{\text{in}}$	5 g m ⁻³
Nominal BOD level in the inflow	$[\text{BOD}]_{\text{in}}$	50 g m ⁻³
Nominal water flow	Q^0	100 m ³ h ⁻¹

The lagoon has been dimensioned to keep the BOD level in the outflow under the limit 20 g m⁻³.

We assume that the inflow and outflow are both equal to Q , implying that the volume V will be constant. Introducing $x_1 = [\text{O}_2]$ and $x_2 = [\text{BOD}]$, a mass balance gives the following equations, which will be the starting point for our analysis:

$$\begin{aligned}\frac{dx_1}{dt} &= -r(x_1, x_2) + \frac{Q}{V}([\text{O}_2]_{\text{in}} - x_1) + \frac{kA}{V}([\text{O}_2]_{\text{sat}} - x_1) \\ \frac{dx_2}{dt} &= -r(x_1, x_2) + \frac{Q}{V}([\text{BOD}]_{\text{in}} - x_2)\end{aligned}$$

We will later on assume that the flow Q can be manipulated. A block diagram of the process is shown in Figure 3.

Problem 1. Use simulation to determine what the oxygen and BOD levels converge to in stationarity, assuming the nominal values for the BOD level in the inflow and the water flow. (Hint: Read off the stationary values using four significant digits to have high enough precision for the following problems.) Does the process manage to keep the BOD level under the limit?

Use the Simulink model `damm.mdl`. □

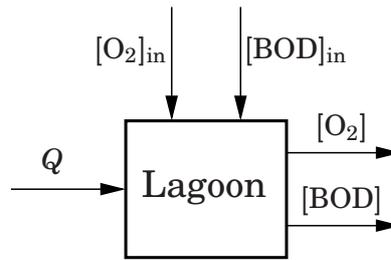


Figure 3 Block diagram of the aerated lagoon, where the flow Q is the input signal and where the oxygen level $[O_2]$ and the carbon level $[BOD]$ are output signals. The oxygen and BOD levels in the inflow, $[O_2]_{in}$ and $[BOD]_{in}$ are viewed as process disturbances.

Problem 2. Linearize the process around the equilibrium

$$x^0 = \begin{pmatrix} x_1^0 \\ x_2^0 \end{pmatrix}$$

that you found in Problem 1. Note that the stationary input is given by $u^0 = Q^0$. Write the answer in the form

$$\begin{aligned} \frac{d\Delta x}{dt} &= A\Delta x + B\Delta u \\ \Delta y &= C\Delta x \end{aligned}$$

Use the command `linmod` and the Simulink model `damm_lin.mdl`. □

Problem 3. Calculate the transfer functions from Δu to Δy_1 and Δy_2 respectively. Where are the poles of the linear system located? What time constants (measured in hours) does this correspond to? □

Problem 4. Calculate the static gains of the linear system from Δu to Δy_1 and Δy_2 respectively. Using this, estimate how much the levels of BOD and oxygen will change after a long time if the flow increases by $1 \text{ m}^3/\text{h}$. Is the behavior reasonable? (Try to do a physical interpretation.) □

Problem 5. Plot the step responses for the nonlinear system and the linearized system in the same figure. How well do the responses agree?

- Simulate the nonlinear step response using the model `damm.mdl`. Set the initial values of $[O_2]$ and $[BOD]$ to the values you found in Problem 1. (Double click on the “Damm” block to change the parameters of the lagoon.) Set the input signal to $Q^0 + 1$ to simulate a step in the input.
- Simulate the linear step response using the command `step`. Do not forget to add the stationary BOD level to the resulting output before you compare it to the nonlinear step response.

□

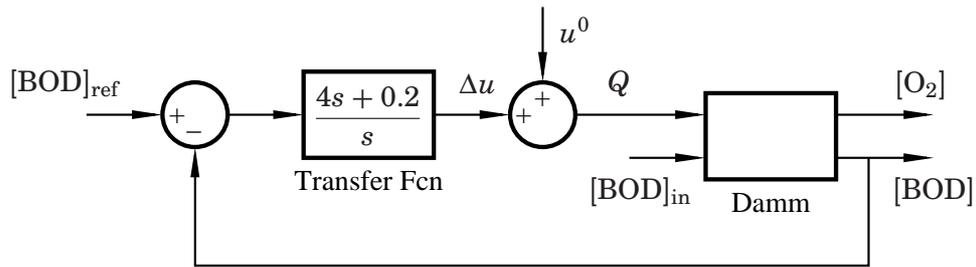


Figure 4 Block diagram for PI control of the BOD level in the lagoon.

Problem 6. We will now examine how the process responds to a change in the BOD level in the inflow. Use `damm.mdl` and set $[BOD]_{in} = 55 \text{ g m}^{-3}$. Set the initial values of $[O_2]$ and $[BOD]$ in the lagoon to the values you found in Problem 1. Does the process still manage to keep the BOD level under the limit? □

Problem 7. Finally we want to control the BOD level in the outflow so that it is kept close to a setpoint even if the BOD level in the inflow varies. For this purpose, implement a PI controller,

$$\Delta U = \frac{4s + 0.2}{s} ([BOD]_{ref} - [BOD])$$

in Simulink. Start with `damm.mdl` and extend the model according to the block diagram in Figure 4. Use the block *Continuous / Transfer Fcn* for the controller and the block *Sources / Constant* for $[BOD]_{ref}$, u^0 , and $[BOD]_{in}$. Simulate the closed-loop system using the setpoint $[BOD]_{ref} = 18.5 \text{ g m}^{-3}$ and incoming BOD level $[BOD]_{in} = 55 \text{ g m}^{-3}$. Set the initial values of $[O_2]$ and $[BOD]$ to the values you found in Problem 1.

Is the closed-loop response satisfactory? If not, adjust the controller parameters so that the limit value for the $[BOD]$ is not exceeded.

Plot the BOD level, the oxygen level and the flow using your final controller. How long does it take for the oxygen level to recover after the disturbance? What happens to the oxygen level in the lagoon? □

A. List of Useful MATLAB Commands

```
figure
plot
hold on
legend
linmod
ss
tf
pole
dcgain
step
```