1.

a. Poles: -1, -2. No zero. The system is stable.

b.

$$y'' + 3y' + 2y = u$$

c. The output signal is $\frac{1}{(s+1)(s+2)}\frac{1}{s}$. Using inverse Laplace transform and table lookup, we have $y(t) = \frac{1}{2}(1 - 2e^{-t} + e^{-2t})$.

2.

a. Introduce

$$f_1(x_1, x_2, u) = x_1(2 - x_2) - u$$
$$f_2(x_1, x_2, u) = -x_2(100 - x_1).$$

Since $f_1(100, 2, 0) = f_2(100, 2, 0) = 0$ the given vector is a stationary point. We get

$$\frac{\partial f_1}{\partial x_1} = 2 - x_2 = 0,$$
$$\frac{\partial f_1}{\partial x_2} = -x_1 = -100$$
$$\frac{\partial f_2}{\partial x_1} = x_2 = 2,$$
$$\frac{\partial f_2}{\partial x_2} = -100 + x_1 = 0,$$
$$\frac{\partial f_1}{\partial u} = -1$$

$$\begin{pmatrix} \Delta x_1 \\ \Delta x_2 \end{pmatrix} = \begin{pmatrix} 0 & -100 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} u \tag{1}$$

b. The characteristic equation becomes $\det(sI - A) = s^2 + 200 = 0$ which has two roots on the imaginary axis. The system is hence not asymptotically stable (it is however stable).

c. Since the controllability matrix

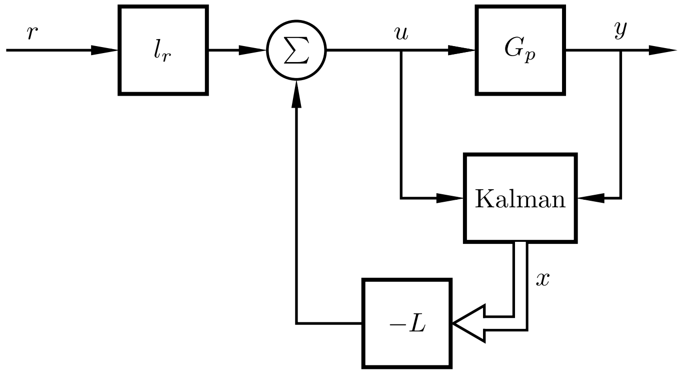
$$W_s = (\, B \quad AB \,) = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$$

is invertable, the system is controllable.

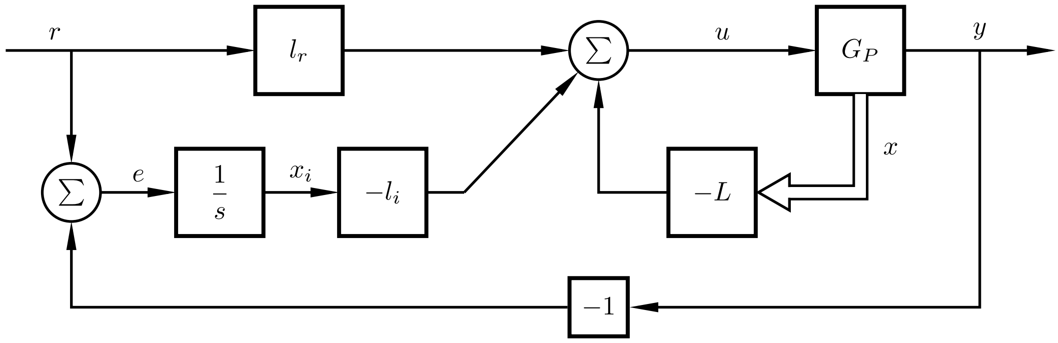
3.

a. See Figur 1.

b. See Figur 2.



Figur 1 Blockdiagram for state-feedback with Kalman filter.



Figur 2 Blockdiagram for state-feedback with Kalman filter with integral. action.

4 a. IFrom the figure of the periodic outsignal we can determine a period of approx. 1.6, i.e. the angular frequency is $\omega \approx \frac{2\pi}{1.6} \approx 4$, an the amplitude $A \approx 1.5$. The phase is given by

$$\begin{aligned} \sin(4 \cdot 14 + \phi) &= -1 \\ 4 \cdot 14 + \phi &= -\frac{\pi}{2} + n \cdot 2\pi, \quad n \in \mathbb{Z}. \end{aligned}$$

By choosing $n = 9$ we get

$$\phi = -\frac{\pi}{2} + 9 \cdot 2\pi - 4 \cdot 14 = -58^\circ.$$

Looking at the Bode-diagram at frequency $\omega = 4$ gives a phase of $\approx -58^\circ$ and a gain of ≈ 0.7. Therefore the signal is

$$u(t) = \frac{1.5}{0.7} \sin(4t) \approx 2 \sin(4t).$$

b. The systems static gain is 3, the diagram breaks down one (= slope 1) time at frequency 1, goes up one time at frequency 10 and in the end goes down two time at frequency 500. Therefore the transfer function is given by

$$G(s) = 3 \frac{\frac{s}{10} + 1}{(s+1)\left(\frac{s}{500} + 1\right)^2}.$$

c. The system is stable by assumption. The final value theorem gives

$$e(t) = \lim_{s \to 0} s \frac{1}{1 + GK} \frac{1}{s} = \frac{1}{1 + G(0)K}.$$

From the bode-diagram we get $G(0) = 3$, therefore the stationary error will be $e_\infty = \frac{1}{1+3K}$.

5.

a. In the Bode-diagram we can see that K_0, i.e. the process gain margin is

$$K_0 = \frac{1}{|G_P(i\omega_0)|} \approx 1/0.005 = 200,$$

at $\omega_0 \approx 14$. The period time is

$$T_0 = \frac{1}{\frac{\omega_0}{2\pi}} \approx 0.4.$$

(More precisely $K_0 \approx 204$ och $T_0 \approx 0.44$.) This gives the controller

$$G_\mathrm{P} = 0.5K_0 \approx 100, \qquad G_\mathrm{PI} = 0.45K_0 \left(1 + \frac{1.2}{sT_0}\right) \approx 90\left(1 + \frac{3}{s}\right).$$

Observe that the answer can be checked by the help of the Bode-diagram in problem b.

b. Because the process is asymptotically stable and the PI-controller has only one pol which lies in the origin, both open loop systems no poles in the right half plane och no multiple poles on the imaginary axis. Therefore, we can use the Nyquist theorem, which says that the closed loop system with a P-controller is stable (the gain margin is 2 by construction) and the closed loop system with the PI-controller is unstable (the gain margin is larger than 1).

c. We can see that the PI-regular gives a negative phase margin and an unstable closed loop system. Even the P-regulator as a much too small phase margin for a good control. A rule of thumb says that the phase margin should be at least 30 degrees. Possible enhancements are to decrease the gain and accept a lower cut-off frequency and a slower system, or to use a PID-controller which can increase the phase.

6 a. We start with determining the system's cut-off frequency

$$|G(i\omega_c)| = \frac{4}{\sqrt{\omega_c^2 + 4}} = 1$$
$$\omega_c = \sqrt{12}.$$

At this frequency the phase is

$$\arg G(i\omega_0) = -\arg(i\omega_0 + 2) = -\arctan\left(\frac{\omega_0}{2}\right)$$
$$= -\arctan\left(\frac{\sqrt{12}}{2}\right) = -\arctan\left(\sqrt{3}\right) = -\frac{\pi}{3}.$$

Therefore the phase margin becomes $\phi_m = \frac{2\pi}{3}$ and the delay margin

$$L_m = \frac{\phi_m}{\omega_0} = \frac{2\pi}{3\sqrt{12}} \approx 0.60 \text{ s}.$$

b. To get a delay margin of 0.75 seconds, we need to have

$$\phi_m = L_m \omega_c = 0.75\sqrt{12} \approx 2.60.$$

The gain margin has to increase by

$$\Delta\phi_m = 2.60 - \frac{2\pi}{3} \approx 29^\circ,$$

which gives $N = 3$. Moreover, the phase curve's top should lie at ω_c, which gives

$$b = \frac{\omega_c}{\sqrt{N}} = \frac{\sqrt{12}}{\sqrt{3}} = 2.$$

At this frequency the compensator's gain $K_K\sqrt{N}$ should be one, which gives

$$K_K = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{3}}.$$

The lead-compensator becomes

$$G_r^{ny}(s) = \sqrt{3}\frac{s+2}{s+2\cdot 3} \approx 1.73\frac{s+2}{s+6}.$$

7.

a. This controller is called Otto-Smith controller and the idea behind is to use it for systems with time delays of the form $e^{-sL}G(s)$. The hope is that one can design a controller just as for the process $G(s)$ which has no delay.

b.

$$\begin{aligned} U &= G_{R0}(R - Y + Y1 - Y2) = G_{R0}(E + \hat{G}_P(e^{-sL} - 1)U) \\ \Rightarrow U &= \frac{G_{R0}}{1 + (1 - e^{-sL})\hat{G}_P G_{R0}} E \end{aligned}$$

Det finns en liknande räkning i övningsuppgift 7.9b.

c.

$$G_R(s) = \frac{\frac{1}{s}}{1 + (1 - e^{-sL})\frac{1}{s(s+1)}\frac{1}{s}} = \frac{s+1}{s(s+1) + \frac{1-e^{-sL}}{s}} \to \frac{1}{L}, \quad \text{då } s \to 0.$$

For $L = 0$ there is integral action, which we can see from the figure because have $G_R(s) = G_{R0}(s)$ as controller. For $L > 0$, there is no integral action because $G_R(s)$ doesn't tend to ∞ as $s \to 0$. That is just the same as we have seen it in Lab 3 an its exercises 7.5 and 7.7.