LUNDS UNIVERSITET

Institutionen för
REGLERTEKNIK

Automatic Control for FiPI

Exam March 16, 2015 14–19

Points and Grades

Solutions and answers to all questions should be clearly motivated. The exam consists of in total 25 points.

Betyg 3: at least 12p
4: at least 17p
5: at least 22p

Allowed help

Mathematical tables (TEFYMA or similar), collection of formulae, calculators without preprogramming.

Results

Will be presented in LADOK before March 31. More info on course home page.

1. Consider the system $Y(s) = G(s)U(s)$ where

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

a. Calculate the poles and zeros of the system. Is the system stable? (1.5 p)

b. Find a differential equation relating $u(t)$ and $y(t)$. (0.5 p)

c. What will $y(t)$ be if $u(t)$ is a step function (the system is at rest at $t = 0$)? (1 p)

2. The following equations

$$\dot{x_1} = x_1(2 - x_2) - u$$
$$\dot{x_2} = -x_2(100 - x_1),$$

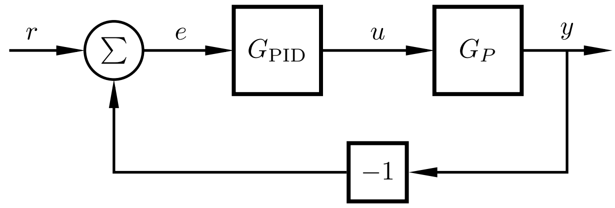
describe a dynamical model for the populations of preys and predators (e.g. fish x_1, and shark x_2). We assume it is possible to influence the system by the signal $u \geq 0$ (e.g. by fishing).

a. Verify that $(x_1, x_2, u_0) = (100, 2, 0)$ is a stationarly point and linearise the system around this point. (2 p)

b. Is the linearised system asymptotically stable? (1 p)

c. Is the linearised system controllable? (1 p)

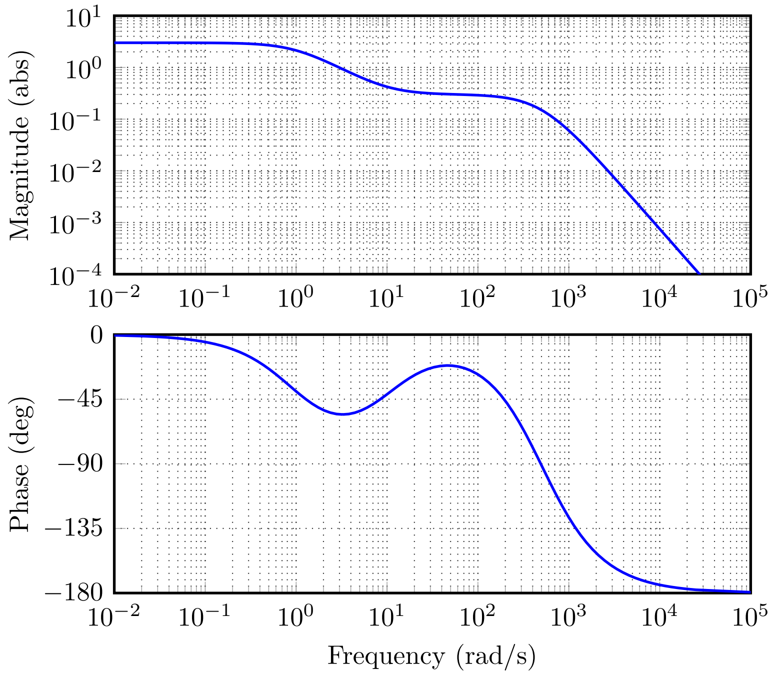
3. A process with transfer function G_P and inpout u and output y should be controlled so that y follows a reference signal r. This can be done with a PID-controller with transfer function G_{PID} according to the block diagram in Figure 1.



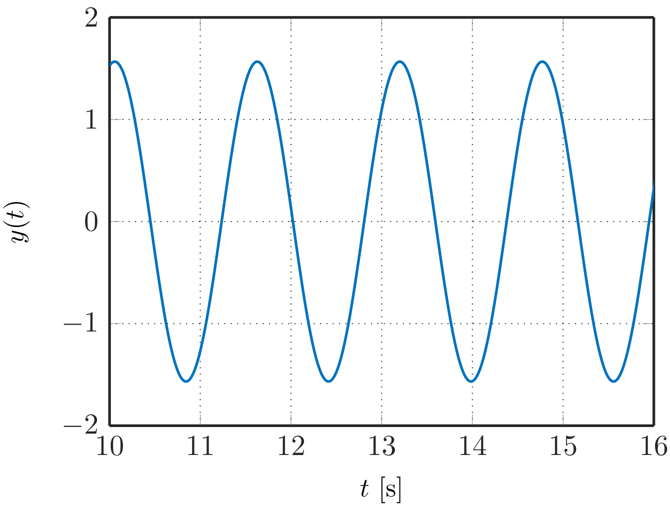
Figur 1 Block diagram for PID-controller in Problem 3

a. Suppose we instead want to use state feedback with a Kalman filter that estimates the states. Draw a block diagram for this controller structure. You may use a "Kalman-filter"-block. (1 p)

b. Suppose instead that we can measure all states x and that we want to control the process with state feedback without a Kalman filter. We however want to get integral action in our state feedback controller to eliminate stationary errors. Draw a block diagram for this controller structure. (1 p)



Figur 2 Bode diagram for Problem 4.



Figur 3 Output signal in Problem 4a.

4. The Bode diagram for an open loop asymptotically stable system without complex poles are shown in Figure 2.

a. Figure 3 shows the output from the system when possible transients have decayed. The output signal is sinusoidal, determine the input signal. (2 p)

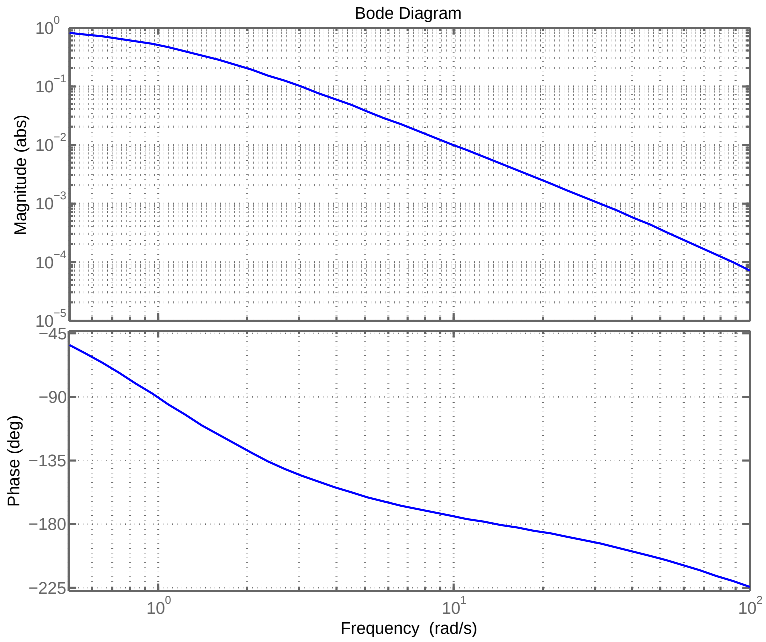
b. Using the Bode diagram, determine the system transfer function $G(s)$. (2 p)

c. What stationary error do we get when the system is controlled with a P-controller with static gain K, assuming the closed loop system is stable? Your answer should be expressed in K and not include any other paramters. (1 p)

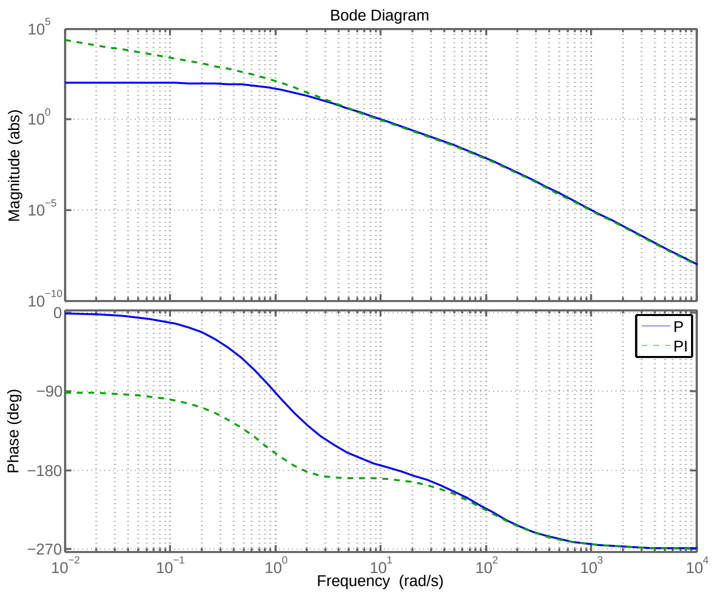
5. The Bode diagram for an asymptotically stable process G_P is shown in Figure 4.

a. Use the Ziegler-Nichols frequency method to find a P-controller and PI-controller G_R for the process. (2 p)

b. A correct solution of ProblemK 5 a gives loop transfer functions $G_0 = G_P G_R$ with Bode diagrams as in Figure 5. None of the controllers give satisfactory control. Explain why. (1 p)



Figur 4 Bode diagram for the process G_P in Problem 5



Figur 5 Bode diagram for $G_0 = G_p G_R$ in Problem 5 b. The solid lines correspond to the P-controller and the dashed to the PI.

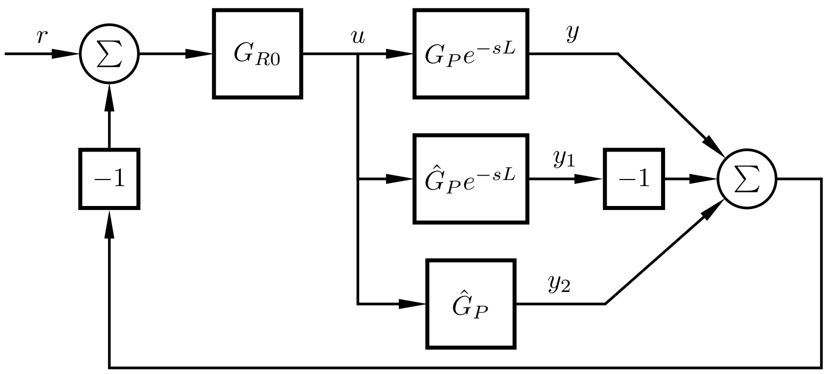
6. A system consists of process and controller given by

$$G_P(s) = \frac{4}{s+2}, \quad G_R(s) = 1$$

a. Calculate the system cut-off frequency ω_c. (1 p)

b. How large is the delay margin L_m? (1 p)

c. We want the closed loop system to be stable for time delays up to $L_m = 0.75$ seconds. Design a suitable compensation link G_R^{ny} that achieves this without changing the cut-off frequency. (2 p)



Figur 6 Block diagram for the controller structure in Problem 7

7.

a. Explain briefly the idea with the controller structure in Figure 6. For what kind of processes is it normally used? (1 p)

b. Show that the transfer function $G_R(s)$ for the complete controller, i.e. the transfer function from $e = r - y$ to u, is given by

$$G_R(s) = \frac{G_{R0}(s)}{1 + (1 - e^{-sL})\hat{G}_P(s)G_{R0}(s)}.$$

(1.5 p)

c. Suppose now that $G_{R0}(s) = \dfrac{1}{s}$ and $\hat{G}_P(s) = \dfrac{1}{s(s+1)}$. For what $L \geq 0$ does the complete controller have $G_R(s)$ integral action? Motivate the answer with a calculation of $G_R(s)$ for $s \to 0$. (1.5 p)