Example - Control of Atomic Force Microscopes

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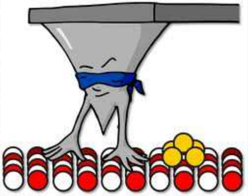
Contents and Purpose

- Accelerometer design
- Atomic Force Microscopes (AFM)
- AFM Model
- $G(s)$, Bode and Nyquist diagram
- Control design
  - I
  - PID - active resonance damping

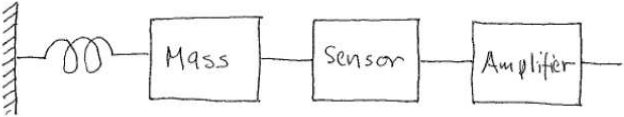


Some repetition of Laplace, Bode, Nyquist, PID-design using a nano example

Show that you have the tools to do a non-trivial control design

“Control can be used to overcome physical design restrictions”

Improved accelerometers using control



Want the accelerometer to be both sensitive and fast

Simple model of an accelerometer without control

$$m\ddot{x} + c\dot{x} + kx = mu, \qquad u = \text{ acceleration}$$

Laplace: $(ms^2 + cs + k)X(s) = mU(s)$

Accelerometer Analysis

$$X(s) = \frac{1}{s^2 + c/ms + k/m} U(s) = \frac{1}{s^2 + 2\zeta\omega_0 s + \omega_0^2} U(s)$$

Stationary solution $u = u_0$ gives $x = \frac{m}{k} u_0$

Sensitivity of the accelerometer: $S \sim m/k$

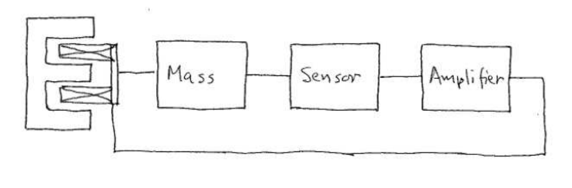
Bandwidth: $\omega_0 = \sqrt{k/m}$

Hence there is a fundamental design relation

$$\omega_0^2 S = \text{constant}$$

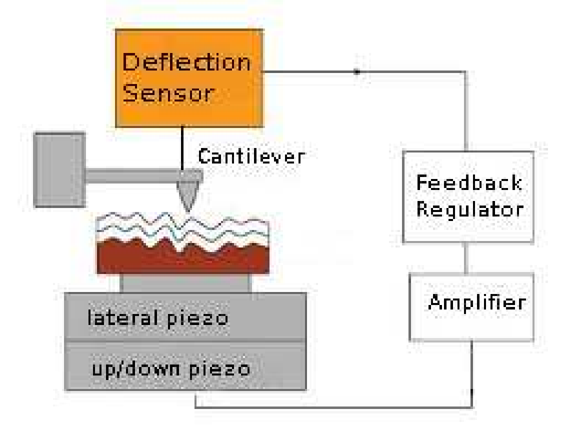
Compromise between sensitivity S and bandwidth ω_0

The advantage with Force Feedback



- The constraint $\omega_0^2 S =$ constant is eliminated if force feedback is used !
- The mass does not need to move, the sensor information is found in the control signal
- Bandwidth of a sensor with force feedback depends primarily on the tightness of the control loop
- Classic idea with tremendous impact
- Game changer in instrument design

AFM



Using an atomic force microscope (AFM) one can measure molecular forces between a fine tip and a surface

Force resolution: 0.1-1 nN, Distance resolution: 0.01 nm

Cantilever Model

The cantilever is an oscillative system, similar to the mass-spring system above

$$
\begin{aligned}
P(s) &= \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \\
&= \frac{\omega_0^2}{s^2 + \omega_0 s/Q + \omega_0^2}
\end{aligned}
$$

where $Q = 1/(2\zeta)$ is called the Q-factor of the resonance.

Can have $Q = 10 - 1000$ for cantilevers

Want zero stationary error, hence need integrator in the controller

Cantilever I-control

Lets start with an I-controller

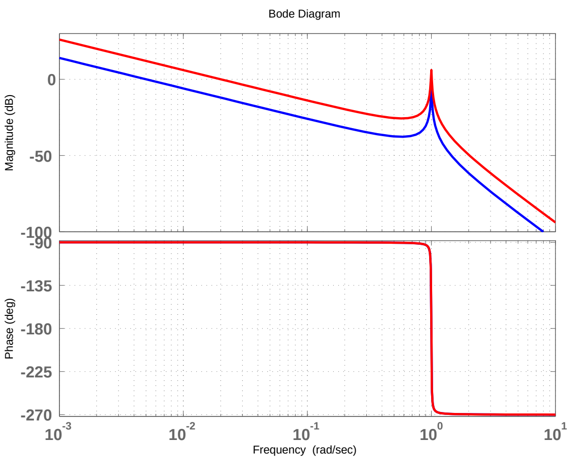
$$C(s) = \frac{k_i}{s}$$

What is the largest k_i that can be used?

Lets look on Bode and Nyquist diagram of

$$G_0(s) = C(s)P(s) = \frac{k_i}{s}\frac{\omega_0^2}{s^2 + \omega_0 s/Q + \omega_0^2}$$

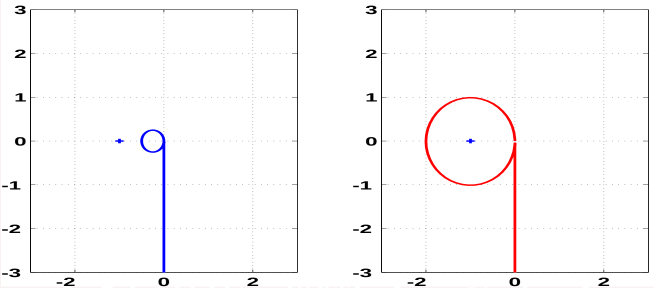
Bode diagram, I-control



$k_i = \omega_c = 0.005\omega_0$ (blue) , $\omega_0 = 1$

$k_i = \omega_c = 0.02\omega_0$ (red)

Nyquist diagram, I-control



$k_i = \omega_c = 0.005\omega_0$ (blue), will give stable closed loop

$k_i = \omega_c = 0.02\omega_0$ (red), will give unstable closed loop

$$G_0(iw_0) = -\frac{k_i Q}{w_0} > -1 \quad \Leftrightarrow \quad k_i < \frac{w_0}{Q}$$

Cantilever I-control is slow

Limitation: $k_i < \omega_0/Q$

Low frequencies: $G_0(s) \approx k_i/s$

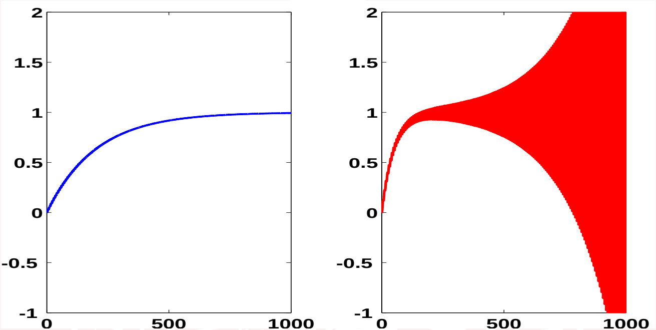
Cut-off frequency w_c: $|G_0(i\omega_c)| = 1 \Rightarrow w_c = k_i$, hence

$$\omega_c < \omega_0/Q$$

With $Q = 100$ the achievable bandwidth is only $\omega_c = 0.01\omega_0$

Not very good. It works, but it is slooow

Simulations, I-control



Simulations with $Q = 100, \omega_0 = 1$ and

- $k_i = \omega_c = 0.005\omega_0$ (blue, stable)
- $k_i = \omega_c = 0.02\omega_0$ (red, unstable)

The simulations support the theoretical analysis

Cantilevers, PID design

Let's try a PID design instead

$$C(s) = k_d s + k + k_i/s$$

We get

$$G_0(s) = P(s)C(s) = \frac{k_d s^2 + ks + k_i}{s(s^2 + \omega_0 s/Q + \omega_0^2)}$$

Idea: choose PID parameters k_d, k, k_i so characteristic polynomial becomes $(s + \omega_1)(s^2 + 2\zeta_1\omega_1 s + \omega_1^2)$

This gives

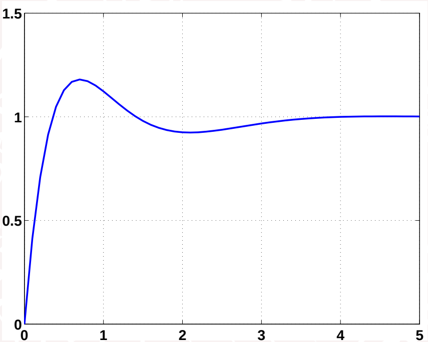
$$k_d = (2\zeta_1 + 1)\omega_1 - \omega_0/Q, \quad k = (2\zeta_1 + 1)\omega_1^2 - \omega_0^2, \quad k_i = \omega_1^3$$

Well-damped closed loop if $\zeta = 0.7$

ω_1 is related to the closed loop bandwidth

Cantilevers

Simulation with $\omega_1 = 2\omega_0$



More than 100 times faster!

Physical interpretation is that feedback control has virtually "changed the stiffness and mass" of the cantilever

PID design - Limitations

More simulations show that control signal magnitude is large if $\omega_1 >> \omega_0$, so there is an upper limit in practice, due to e.g.

- control signal saturation
- measurement noise amplification

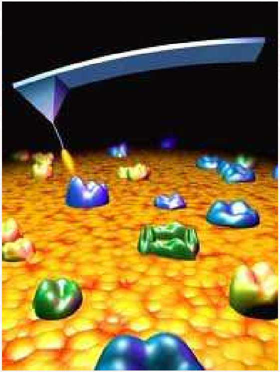
So there are limits to the magic

Limits are due to how good control loop one can design

Sub-nano accuracy achievable

Conclusions

- The control theory you have learned so far can be used to achieve acceptable control of an AFM
- Control can achieve “virtual change of physical parameters”



Presentation based on material from Karl Johan Åström, Lund