Example - Control of Atomic Force Microscopes

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Contents and Purpose

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- Atomic Force Microscopes (AFM)
- AFM Model
- $G(s)$, Bode and Nyquist diagram
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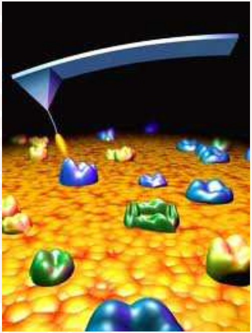
  - I
  - PID - active resonance damping

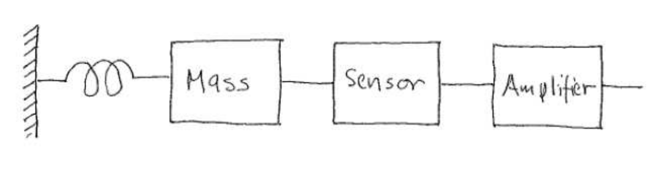


Some repetition of Laplace, Bode, Nyquist, PID-design using a nano example

Show that you already now have the tools to do a non-trivial control design

“Control can be used to overcome physical design restrictions”

Improved accelerometers using control



Want the accelerometer to be both sensitive and fast

Simple model of an accelerometer without control

$$m\ddot{x} + c\dot{x} + kx = mu, \qquad u = \text{ acceleration}$$

Laplace: $(ms^2 + cs + k)X(s) = mU(s)$

Accelerometer Analysis

$$X(s) = \frac{1}{s^2 + c/ms + k/m} U(s) = \frac{1}{s^2 + 2\zeta\omega_0 s + \omega_0^2} U(s)$$

Stationary solution $u = u_0$ gives $x = \frac{m}{k} u_0$

Sensitivity of the accelerometer: $S \sim m/k$

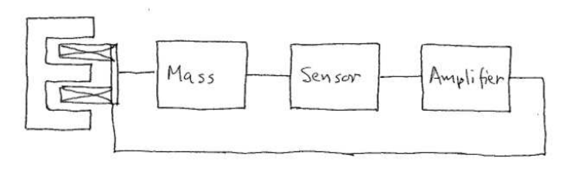
Bandwidth: $\omega_0 = \sqrt{k/m}$

Hence there is a fundamental design relation

$$\omega_0^2 S = \text{constant}$$

Compromise between sensitivity S and bandwidth ω_0

The advantage with Force Feedback



The constraint $\omega_0^2 S =$ constant is eliminated if force feedback is used !

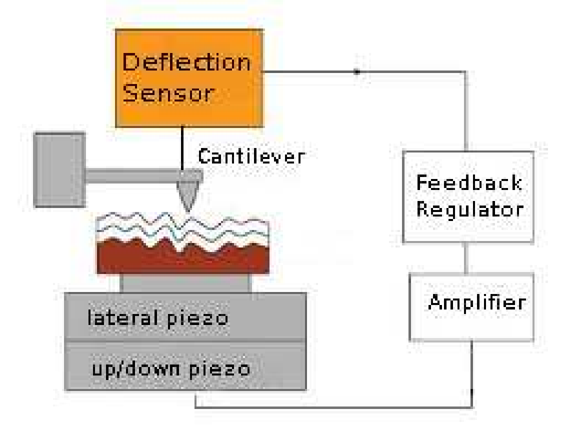
The mass does not need to move, the sensor information is found in the control signal

Bandwidth of a sensor with force feedback depends primarily on the tightness of the control loop

Relieves the designer of a difficult compromise

Higher sensitivity/bandwidth achievable

AFM



Using an atomic force microscope (AFM) one can measure molecular forces between a fine tip and a surface

Force resolution: 0.1-1 nN, Distance resolution: 0.01 nm

Cantilever Model

The cantilever is an oscillative system, similar to the mass-spring system above

$$
\begin{aligned}
P(s) &= \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \\
&= \frac{\omega_0^2}{s^2 + \omega_0 s/Q + \omega_0^2}
\end{aligned}
$$

where $Q = 1/(2\zeta)$ is called the Q-factor of the resonance.

Can have $Q = 10 - 1000$ for cantilevers

Want zero stationary error, hence need integrator in the controller

Cantilever I-control

Lets start with an I-controller

$$C(s) = \frac{k_i}{s}$$

With an I-controller the step-response error e satisfies

$$\int_0^\infty e(t) = 1/k_i \qquad \text{(nice exercise)}$$

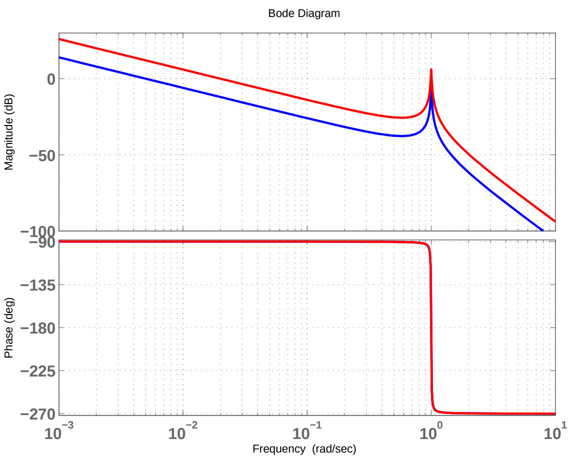
What is the largest k_i that can be used?

Draw Bode and Nyquist diagram of

$$G_0(s) = C(s)P(s) = \frac{k_i}{s}\frac{\omega_0^2}{s^2 + \omega_0 s/Q + \omega_0^2}$$

(Blackboard)

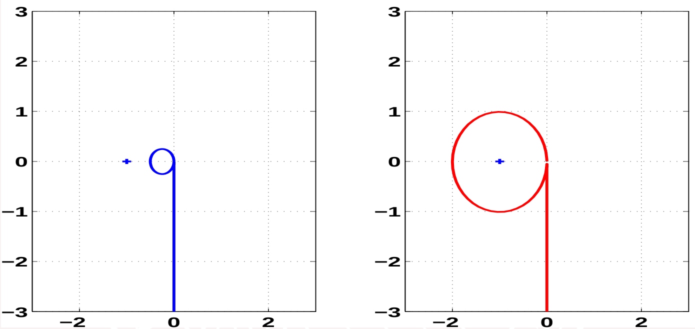
Bode diagram, I-control



$k_i = \omega_c = 0.005\omega_0$ (blue) , $\omega_0 = 1$

$k_i = \omega_c = 0.02\omega_0$ (red)

Nyquist diagram, I-control



$k_i = \omega_c = 0.005\omega_0$ (blue), will give stable closed loop

$k_i = \omega_c = 0.02\omega_0$ (red), will give unstable closed loop

Cantilever I-control

Nyquist diagram of $G_0(s) = C(s)P(s)$ intersects negative real axis for $s = i\omega_0$, and we have

$$G(i\omega_0) = -k_i Q/\omega_0$$

Stability condition: $k_i < \omega_0/Q$

Cantilever I-control

Alternative derivation: Closed loop characteristic polynomial is

$$s^3 + \omega_0 s^2/Q + \omega_0^2 s + k_i \omega_0^2$$

Third order polynomial $s^3 + as^2 + bs + c$ stable if $a, b, c > 0$ and $ab > c$ so we have stability when

$$\omega_0/Q \cdot \omega_0^2 > k_i \omega_0^2$$

Stability condition: $k_i < \omega_0/Q$ (same as above)

Cantilever I-control

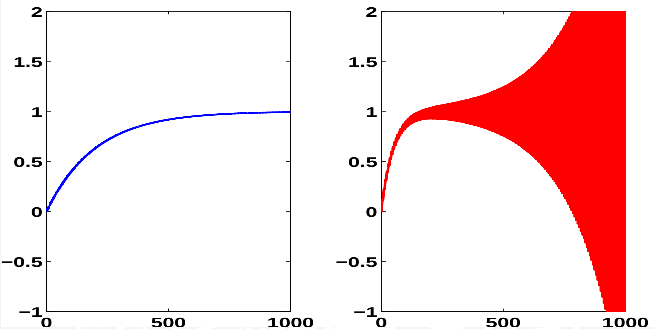
For low frequencies we have $G_0(s) \approx k_i/s$. If we define closed loop bandwidth as the frequency ω_c where $1 = |G_0(i\omega_c)| = k_i/\omega_c$ we get

$$\omega_c < \omega_0/Q$$

With $Q = 100$ the achievable bandwidth is only $\omega_c = 0.01\omega_0$

Not very good. It works, but it is slooow

Simulations, I-control



Simulations with $Q = 100, \omega_0 = 1$ and

- $k_i = \omega_c = 0.005\omega_0$ (blue, stable)
- $k_i = \omega_c = 0.02\omega_0$ (red, unstable)

The simulations support the theoretical analysis

Cantilevers, PID design

Let’s try a PID design instead

$$C(s) = k_d s + k + k_i/s$$

We get

$$P(s)C(s) = \frac{k_d s^2 + ks + k_i}{s(s^2 + \omega_0 s/Q + \omega_0^2)}$$

Idea: choose PID parameters k_d, k, k_i so characteristic polynomial becomes $(s + \omega_1)(s^2 + 2\zeta_1\omega_1 s + \omega_1^2)$

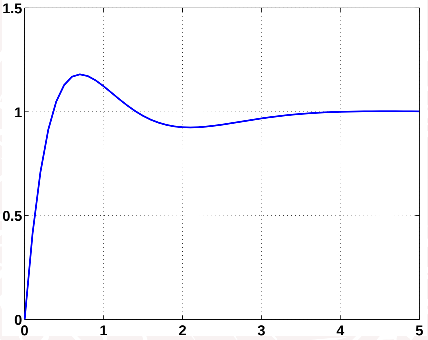
This gives

$$k_d = (2\zeta_1 + 1)\omega_1 - \omega_0/Q, \quad k = (2\zeta_1 + 1)\omega_1^2 - \omega_0^2, \quad k_i = \omega_1^3$$

This is a well-damped closed loop, where the parameter ω_1 is related to the closed loop bandwidth

Cantilevers

Simulation with $\omega_1 = 2\omega_0$



More than 100 times faster!

Physical interpretation is that feedback control has virtually "changed the stiffness and mass" of the cantilever

PID design - Limitations

More simulations show that control signal magnitude is large if $\omega_1 >> \omega_0$, so there is an upper limit in practice, due to e.g.

- control signal saturation
- measurement noise amplification

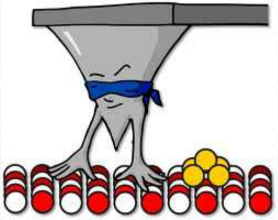
So there are limits to the magic

Limits are due to how good control loop one can design

Sub-nano accuracy achievable

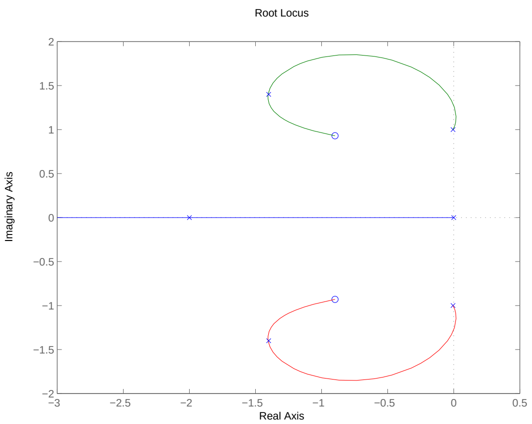
Conclusions

- The control theory you have learned so far can be used to achieve acceptable control of an AFM
- Control can achieve “virtual change of physical parameters”



Presentation based on material from Karl Johan Åström, Lund and University of Santa Barbara cooperation on Atomic Force Microscope control design

Extra: Root Locus



PID design has closed loop poles in $-2, \ -1.4 \pm 1.4i$