Bicycle Dynamics and Control

thanks to K. J. Åström

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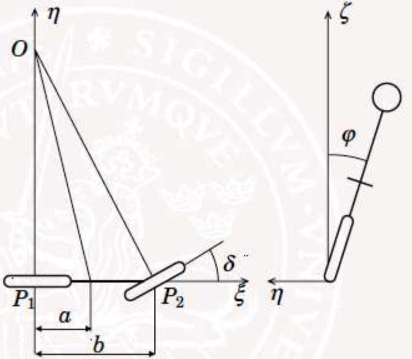
Thanks to Richard Klein and Anders Lennartsson

Some Interesting Questions

- How do you stabilize a bicycle?
  - By steering or by leaning?
- Do you normally stabilize a bicycle when you ride it?
- Why is it possible to ride with no hands?
- How is stabilization influenced by the design of the bike?
- Why does the front fork look the way it does?
- The main message:
  - A bicycle is a feedback system!
  - The front fork is the key!

Tilt Dynamics

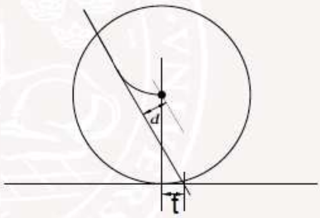
Like an inverted pendulum model



Transfer function: $\varphi(s) = k\frac{s + V_0/a}{s^2 - mg\ell/J}\delta(s), \qquad k = \frac{am\ell V_0}{bJ}$

The Front Fork

With a positive trail the front wheel lines up with the velocity (caster effect). The trail also creates a torque that turns the front fork into the lean. A static torque balance gives



$$\delta = -k_1\varphi + k_2 T$$

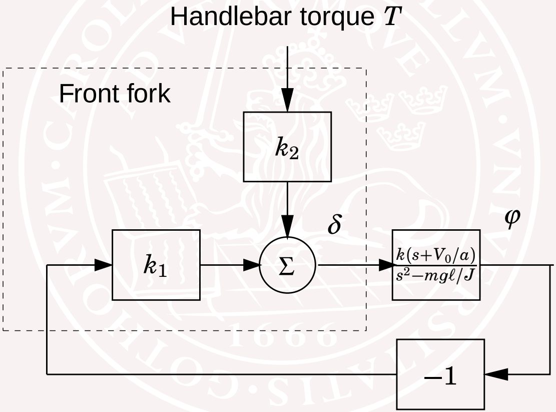
Control variable: Handlebar torque T

Process variables: Steering angle δ, tilt angle φ

Block Diagram of a Bicycle

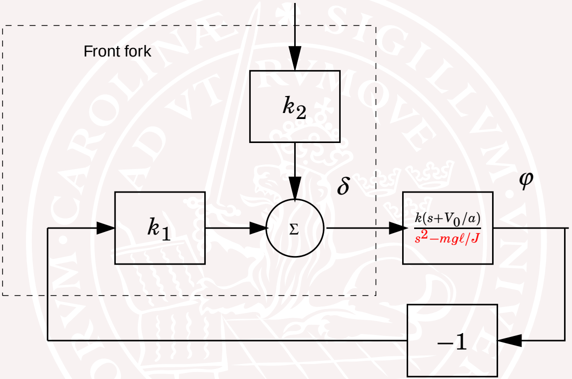
Control variable: Handlebar torque T

Process variables: Steering angle δ, tilt angle φ



A feedback system

How Steer Torque Influences Steer Angle



Transfer function from T to δ is

$$\frac{k_2}{1+k_1\frac{k(s+V_0/a)}{s^2-mg\ell/J}} = k_2\frac{s^2-mg\ell/J}{s^2+k_1ks+k_1kV_0/a-mg\ell/J}$$

Analysis

This equation is stable if $k_1 k V_0 / a > mg\ell / J$

Since $k = \dfrac{am\ell V_0}{bJ}$ this becomes

$$V_0 > V_c = \sqrt{bg/k_1}$$

where V_c is the critical velocity.

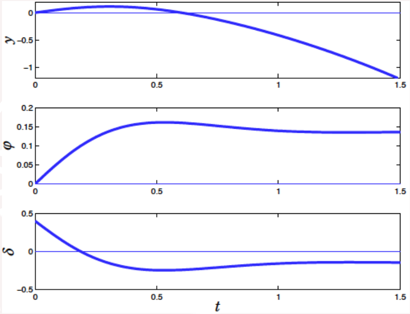
By the design of the front-fork the system is self-stabilizing for high enough speeds!

For low speeds, an extra controller (you) uses T to stabilize φ

Leaning was not modeled above. Mainly used when turning.

Think about this next time you bike!

Warning: Response for T = impulse (without lean)



The position on the road, y, moves initially in the intended direction, but due to fork dynamics then in the wrong direction

Some motorcycle accidents are caused by this effect, which is due to the right half plane zero.

Understanding control dynamics can save your life!

Rear Wheel Steering

Many people have seen theoretical advantages in the fact that front-drive, rear-steered recumbent bicycles would have simpler transmissions than rear-driven recumbents and could have the center of mass nearer the front wheel than the rear. The U.S. Department of Transportation commissioned the construction of a safe motorcycle with this configuration. It turned out to be safe in an unexpected way: No one could ride it.

F. R. Whitt and D. G. Wilson (1974) Bicycling Science - Ergonomics and Mechanics. MIT Press Cambridge, MA

Same equations as above, but with $V_0 < 0$

Similar analysis as above shows that this leads to a transfer function which can be uncontrollable !

Klein's Un-ridable Bike

Klein’s Ridable Bike

Klein's Adapted Bikes for Children with Disabilities

Over a dozen clinics for children and adults with a wide range of disabilities, including Down syndrome, autism, mild cerebral palsy and Asperger's syndrome. More than 2000 children aged 6-20 have been treated, see

http://www.losethetrainingwheels.org