LUNDS TEKNISKA HÖGSKOLA
Lunds universitet

Institutionen för
REGLERTEKNIK

Automatic Control, Basic Course FRT 010

Exam October 20, 2011

Points and grades

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem. Grades:

Betyg 3: least 12 points
4: least 17 points
5: least 22 points

Aids

Mathematical collections of formulae (e.g. TEFYMA), collections of formulae in automatic control, and calculators that are not programmed in advance.

Results

The results are presented on November 2, at the information board of the the department at the first floor in the M building and at the web page for the course. The exams are presented on the same day in the lab on the first floor at 12.00–12.30.

1. A system has the transfer function

$$G(s) = \frac{6}{s^2 + 4.5s + 2} - \frac{2}{2s+1}.$$

a. Determine the poles and the zeros of the system, and show them in a singularity plot. (1.5 p)

b. Is the system asymptotically stable, stable, or unstable Motivate! (0.5 p)

2. Pelle is to determine a state space representation of the differential equation

$$\ddot{y} + 3\dot{y} - 5\dot{u} = 7y + u$$

Pelle ses that it is a second-order differential equation, and realizes that he needs two states, and introduces

$$\begin{cases} x_1 = y \\ x_2 = \dot{y} \end{cases}$$

With this choice, he comes up with the solution

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 \\ -3 & 7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 5 \end{bmatrix} \dot{u} \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{cases}$$

a. Pelle has unfortunately made a mistake. Which? (1 p)

b. Help Pelle to make a correct state space representation. (1 p)

3. The step response of a system is given by

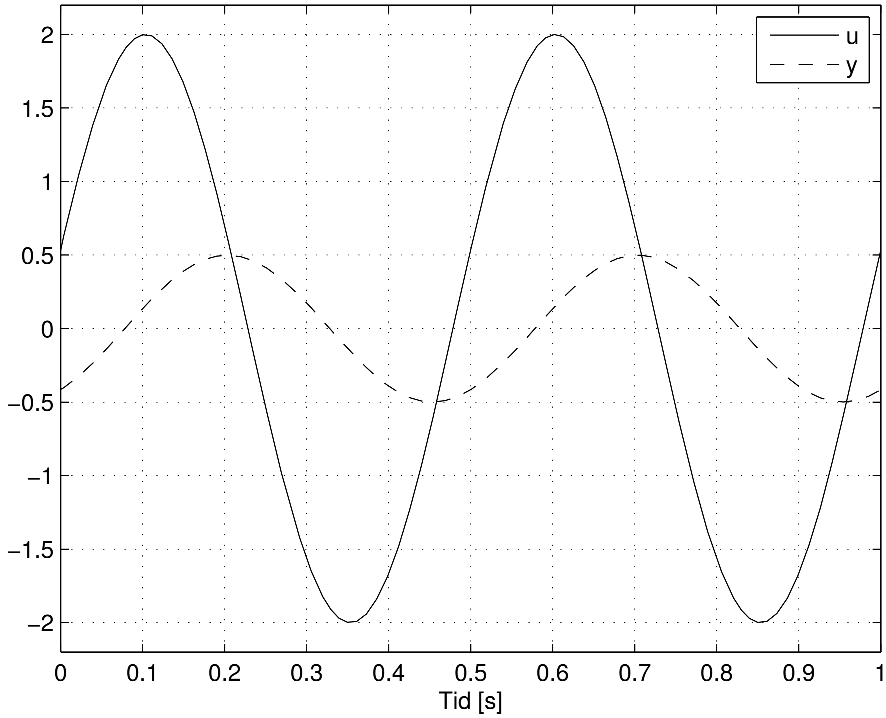
$$y(t) = 2(1 - e^{-3t})$$

Determine the transfer function of the system. (2 p)

4. The input and the output signals from a system with transfer function

$$G(s) = \frac{b}{s+a}$$

are shown in Figure 1 (see next page). Determine the model parameters a and b. (2 p)



Figur 1 Input signal $u(t)$ and output signal $y(t)$ for the system in Problem 4.

5. Which of the following statements are correct? Short motivations are required.

a. The poles of a linaear dynamic system are equal to the squares of the eigenvalues of the corresponding system matrix A. (0.5 p)

b. If you want to reduce the stationary error in a conrol system, it is suitable to introduce a lag compensator. (0.5 p)

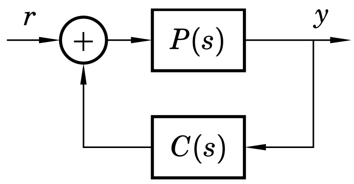
c. $G(s) = \frac{s^2+7s-5}{s^3+as^2+2s+9}$ is stable for $0 < a < 5$. (0.5 p)

d. The gain margin of a system is always equal to the sine of the phase margin. (0.5 p)

e. When the controller $G_R(s) = \frac{s^2+6s+5}{s^2}$ controls the process $G_P(s) = \frac{3}{s+2}$, the closed-loop system can track the reference $r(t) = t, t \geq 0$ without any stationary error. The closed-loop system is stable. (0.5 p)

f. The final-value theorem can be used on all system to determine the static gain. (0.5 p)

6. Consider the system with block diagram presented in Figure 2.



Figur 2 Block diagram in Problem 6.

a. Determine the transfer function between r and y. (1 p)

b. Suppose that the process transfer function $P(s)$ is

$$P(s) = \frac{1}{s+1}$$

Designe a controller $C(s)$ such that the closed-loop transfer function becomes 1. (1 p)

c. Why might it be difficult to use the controller derived in b) in practice? (1 p)

7. A system is given on state-space form as

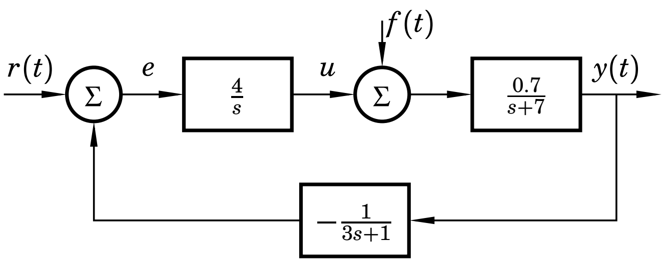
$$\begin{cases} \dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y = \begin{pmatrix} 1 & 0 \end{pmatrix} x \end{cases}$$

a. Is it possible to place the closed-loop poles arbitrarily using state feedback? (1 p)

b. Determine a state feedback such that the closed-loop poles are placed in $s = -3$. Assume that all states are measurable. (2 p)

c. Now assume that it's only possible to measure the output y. Is it possible to derive a Kalman filter such that the reconstruction error goes to zero arbitrarily fast? Give a suitable pole placement for the Kalman filter in this case. (Note! No Kalman filter has to be determined.) (1 p)

8. Consider the system presented in Figure 3. Determine the stationary error when both input signals $r(t)$ and $f(t)$ are steps. (3 p)



Figur 3 Block diagram for the system in Problem 8.

9. Nils has started his new job as a control engineer, and his first task is to improve the control of a rolling hoop. The present controller is too slow, i.e. the setpoint is not tracked fast enough. Attempts have been made to adjust the parameters in the controller, which is a PI controller, but it has resulted in a badly damped system.

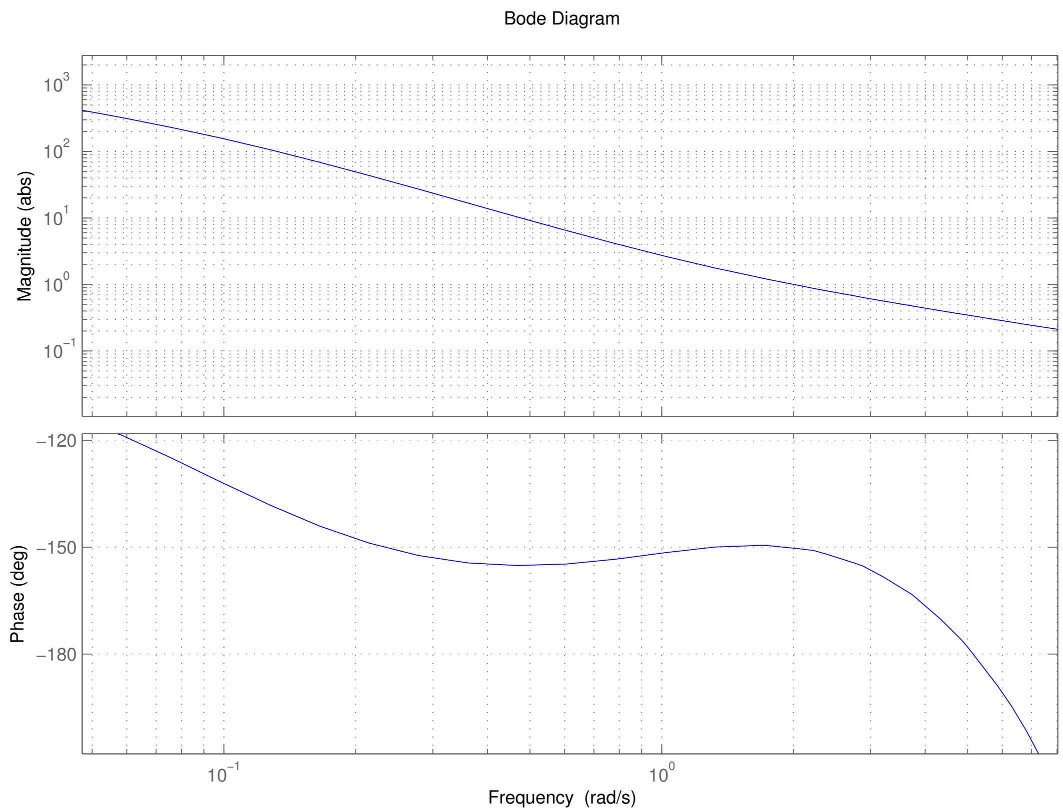
Nils is going to improve the control using a suitable compensator, but realizes that he needs a Bode plot of the open-loop system.

a. The rolling hoop is a stable process. Give a suggestion of an experiment that Nils can make to get the Bode plot. (1 p)

Nils succeeds in optaining the Bode plot, see Figure 4, but when he is going to design the compensator, he gets a blackout, and therefore needs your help.

Nils boss has decided that the speed of the system should be increased by a factor of three (i.e. the crossover frequency should be three times larger) whithout changing the phase margin.

b. Design a compensator that meets the specifications. (3 p)



Figur 4 Bode plot of the open-loop system in Problem 9.