Lecture 1 - Self Study Material

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Zhou/Doyle Book

Read ZD 2.5-2.6

Skim through ZD 3.1-3.5

Read ZD 4

SVD

Find out what the singular value decomposition of a matrix is.

$$A = U\Sigma V^T$$

How can we use SVD to find a low-rank approximation of a matrix?

What is the condition number of a matrix and how does it relate to the singular values?

What numerical problems can you get with a high condition number?

Norms

What is the definition of a norm? What is an induced norm?

By default $\|A\|$ stands for the induced 2-norm, i.e.

$$\|A\| = \sigma_1(A)$$

More on Doyle's counter example

Watch the control bootcamp video ”Intro to Robust control”

`https://www.youtube.com/watch?v=Y6MRgg_TGy0&t=7s`

Also get the paper and plot the GOF for the example.

H_∞ norm computation

H_∞-norm computation requires a search

Theorem $\|C(sI-A)^{-1}B+D\|_\infty < \gamma$ if and only if

- $C(sI-A)^{-1}B$ is asymptotically stable
- $\sigma_{\max}(D) < \gamma$, hence $R = \gamma^2 I - D^*D > 0$
- H has no eigenvalues on the imaginary axis, where

$$H = \begin{pmatrix} A+BR^{-1}D^*C & BR^{-1}B^* \\ -C^*(I+DR^{-1}D^*)C & -(A+BR^{-1}D^*C)^* \end{pmatrix}$$

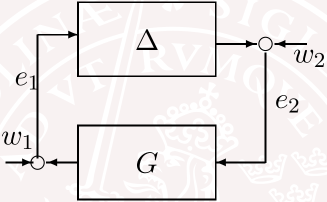
H_∞ norm computation - LMI alternative

$\|G\|_\infty < \gamma$ is equivalent to that there exists $P = P^* > 0$ such that

$$\begin{bmatrix} PA + A^*P & PB & C^* \\ B^*P & -\gamma I & D^* \\ C & D & -\gamma I \end{bmatrix} < 0$$

Linear Matrix Inequality. Convex optimization to find such P

The Small Gain Theorem



Suppose $G \in RH_\infty^{p\times m}$. Then the closed loop system (G, Δ) is internally stable for all

$$\Delta \in \mathcal{B}RH_\infty := \{\Delta \in RH_\infty^{m\times p} \ : \ \ \|\Delta\|_\infty \leq 1\}$$

if and only if $\|G\|_\infty < 1$.