Exercise Session 2

1. Zhou 6.8.

2. Show that for any compatibly dimensioned A, B, $\overline{\sigma}(AB) \geq \underline{\sigma}(A)\overline{\sigma}(B)$. Use this to show that

$$b_{P,C} \geq \frac{1}{\epsilon} \Longrightarrow \overline{\sigma}\left((I - P(j\omega)C(j\omega))^{-1}\right) \leq \frac{\epsilon}{\underline{\sigma}(P(j\omega))}.$$

Why is this useful?

3. Find a transfer matrix $A(s)$ such that $A(s)$ does not depend on $C(s)$, and

$$\begin{bmatrix} P(s) \\ I \end{bmatrix} (I - C(s)P(s))^{-1} \begin{bmatrix} -C(s) & I \end{bmatrix} = \mathcal{F}_l(A(s), C(s)).$$

4. Show that if C is invertible,

$$(A + BQ)(C + DQ)^{-1} = \mathcal{F}_l(M, Q),$$

where

$$M = \begin{bmatrix} AC^{-1} & B - AC^{-1}D \\ C^{-1} & -C^{-1}D \end{bmatrix}.$$

Can we also write A, B, C, D in terms of $M_{11}, M_{12}, M_{21}, M_{22}$?

5. Show that for $0 \leq \gamma < 1$ the following subset of the complex plane

$$\left\{z : z = \frac{\Delta_a}{1 + \Delta_b}, |\Delta_a|^2 + |\Delta_b|^2 \leq \gamma^2\right\}$$

is a circle centered on the origin. What is the radius of the circle as a function of γ? What happens as $\gamma \to 1$? What does this tell us about normalised coprime factor uncertainty?

6. Define

$$\rho(X, Y) = 1/\overline{\sigma}\left(\begin{bmatrix} X \\ I \end{bmatrix} (I - YX)^{-1} \begin{bmatrix} -Y & I \end{bmatrix}\right).$$

Show that $\rho(X, Y) = \rho(Y, X)$. Hint 1: if U is a tall matrix that satisfies $U^*U = I$, then $\overline{\sigma}(UA) = \overline{\sigma}(A)$. Try using right and left coprime factorisations of X, Y in conjunction with this fact. Hint 2: Hint 1 will let you solve the problem but I'm sure there is a better way!