

# Exercise Session 1

1. Describe your results on Handin 1a.
2. Describe your results on Handin 1b.
3. Show that level curves  $|T| = k$  are circles in the Nyquist diagram. Describe how they are mapped onto the Riemann sphere (north pole projection as in the lecture).
4. What is a Hall diagram? (mentioned in the song "Reglerteknik på bal")
5. Construct code to plot the GOF for a MIMO system (plot all singular values). Illustrate it on the Distillation Column design on the lecture with

$$P_{\delta}(s) = \frac{1}{50s + 1} \begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix} \begin{pmatrix} 1.2 & 0 \\ 0 & 0.8 \end{pmatrix}.$$

6. Verify the statement in Lecture 1 claiming that the distillation column process is not so sensitive to gain variations on the output channels: Determine the GOF (with two singular values) for nominal loop gain  $P(s)C(s)$  and perturbed loop gain  $P_{\delta}(s)C(s)$  with

$$P_{\delta}(s) = \frac{1}{50s + 1} \begin{pmatrix} 1.2 & 0 \\ 0 & 0.8 \end{pmatrix} \begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}, \quad C(s) = \frac{1}{s} P(s)^{-1}$$

7. ZD 2.11
8. ZD 4.3
9. Determine numerically the  $H_{\infty}$  distance between the two systems

$$G_0 = \frac{1}{s + 1}, \quad G_{\theta} = \frac{1}{s + 1} e^{-i\theta}.$$

for  $\theta = \{0.01, 0.1, 1\}$ . Can you find an explicit formula for the distance?