Robust Control 2018

Richard Pates and Bo Bernhardsson

Department of Automatic Control LTH,
Lund University

Design for Robust Performance

Next 3 lectures:

1. $\mathcal{H}_\infty$-norm specifications.
2. Robust stability and performance.
3. Controller synthesis via $\mathcal{H}_\infty$-norm optimisation.

Warning

“Optimisation can expose the weaknesses in thinking which are usually compensated for by soundness of intuition.”

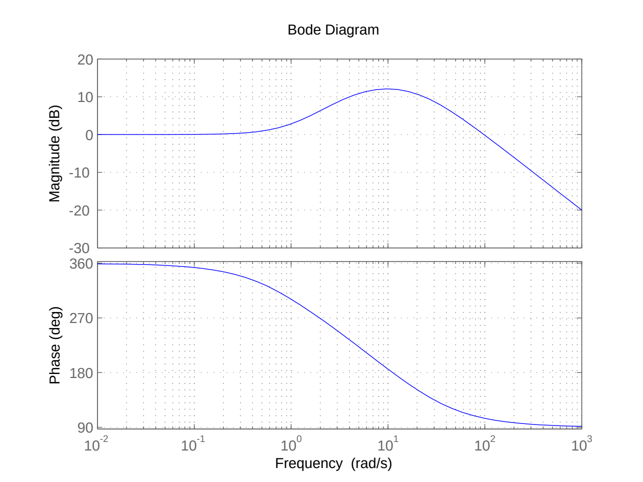
–F. Whittle

Plan of attack:

Today's topic: Formulate design specifications as $\mathcal{H}_\infty$-norm criteria.

- Check soundness of intuition.
- $\mathcal{H}_\infty$-norm specifications.
- Robust Performance:
  - from open loop to closed loop
  - from closed loop to open loop?
- The gang of four stability margin.

Understanding how systems behave



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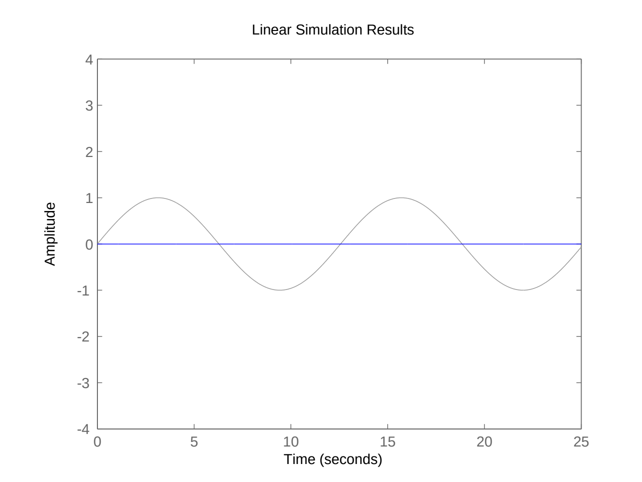

What will happen??

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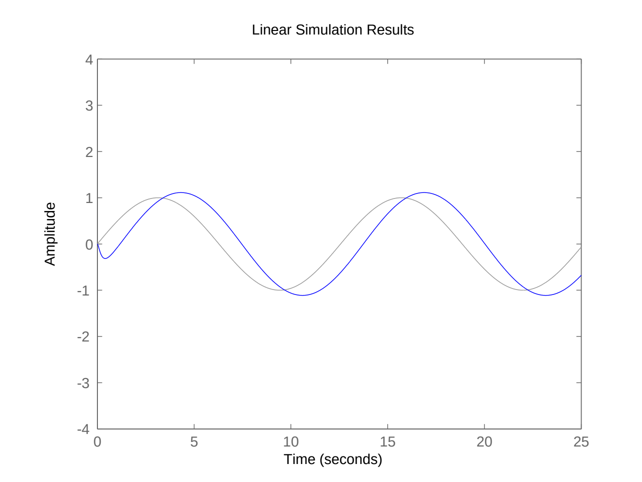

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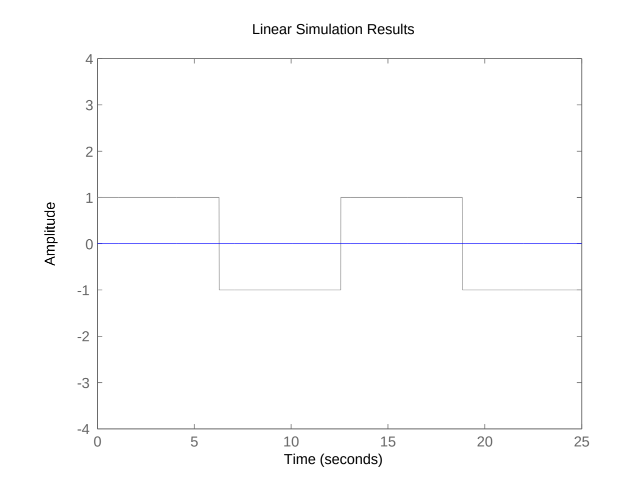

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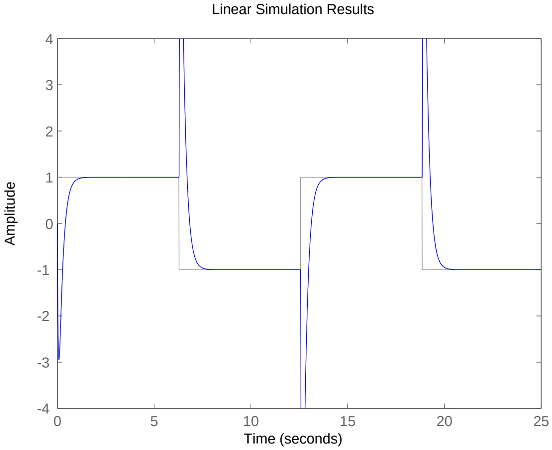

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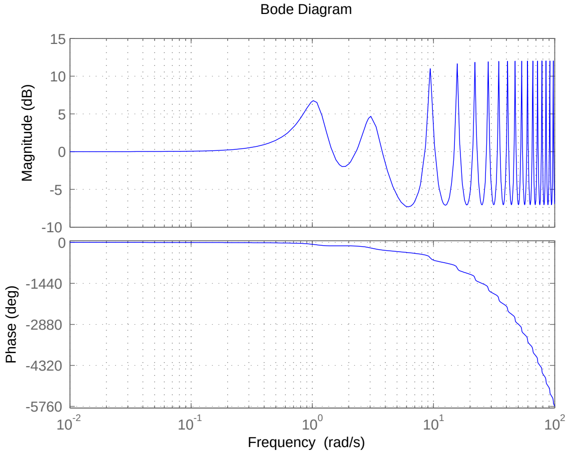

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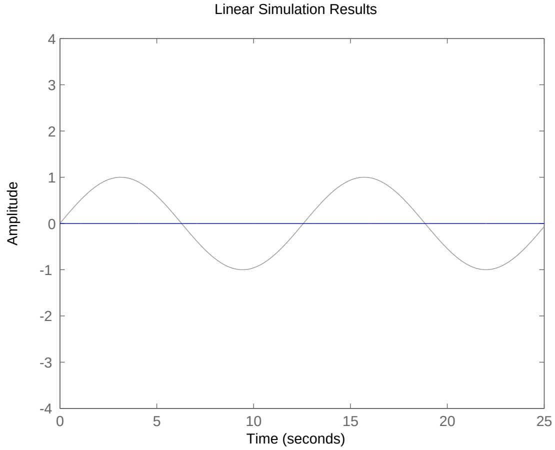

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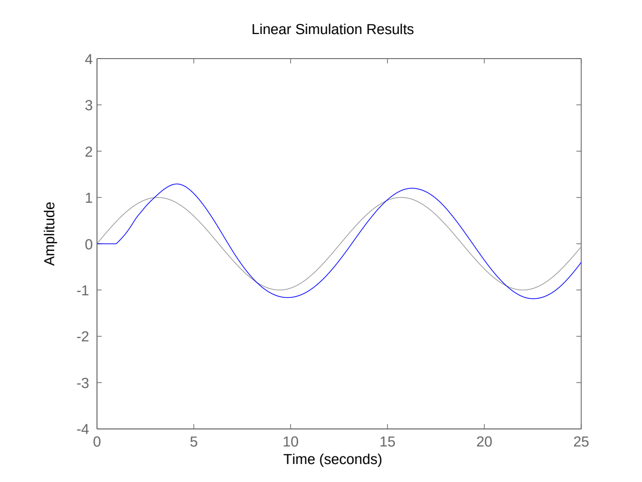

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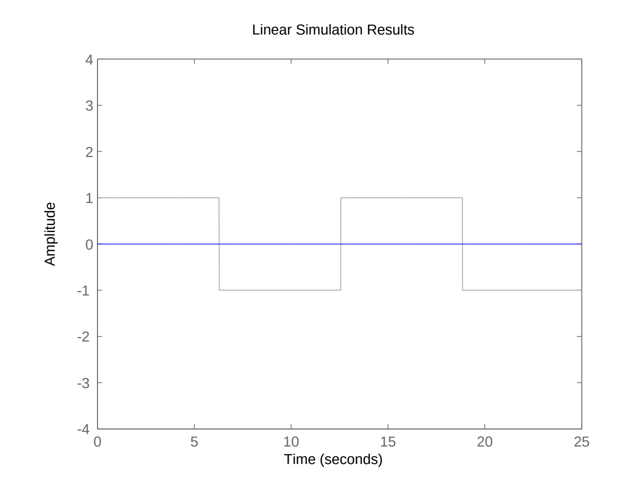

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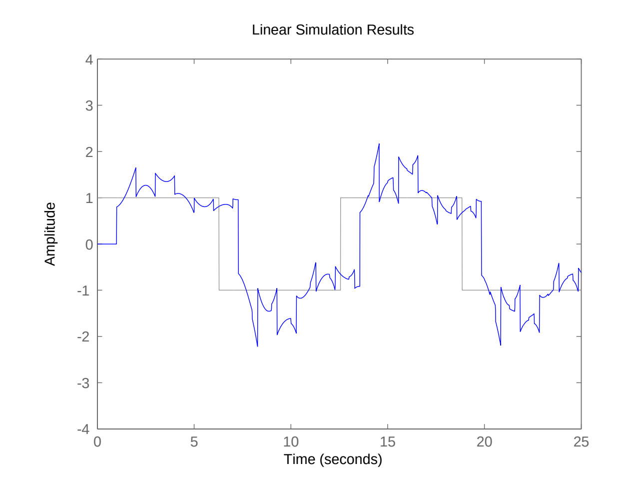

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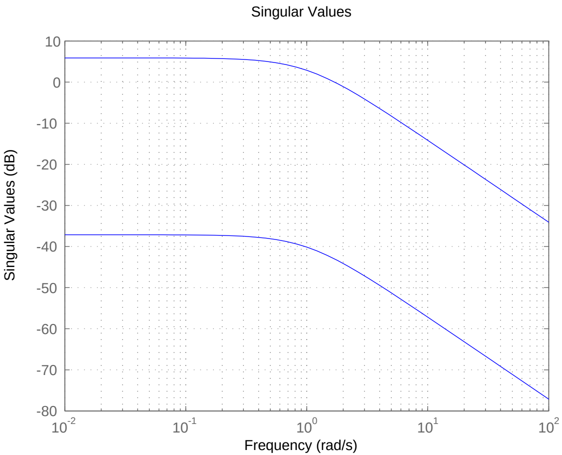

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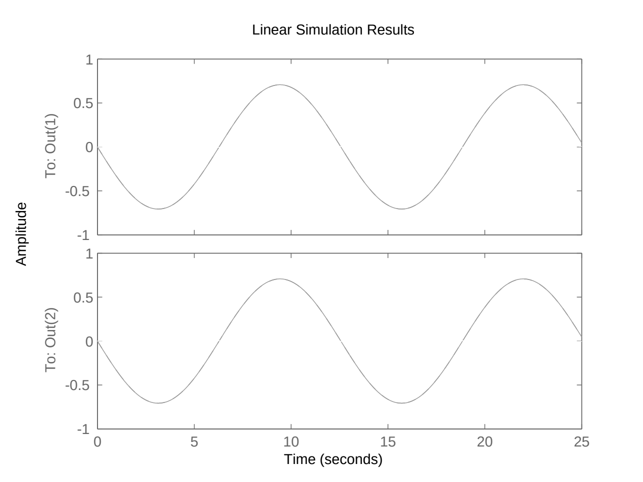

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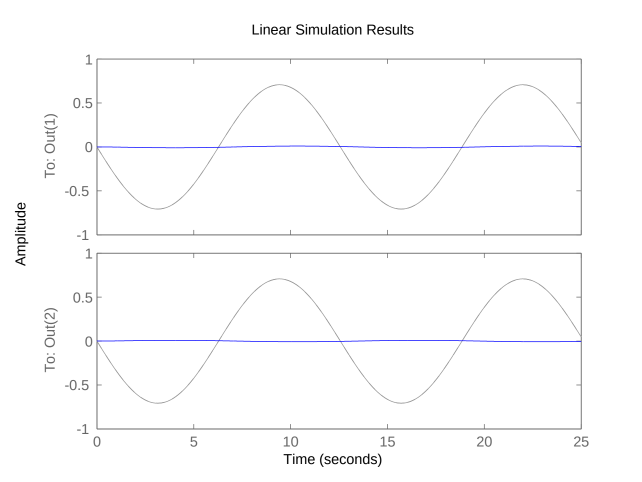

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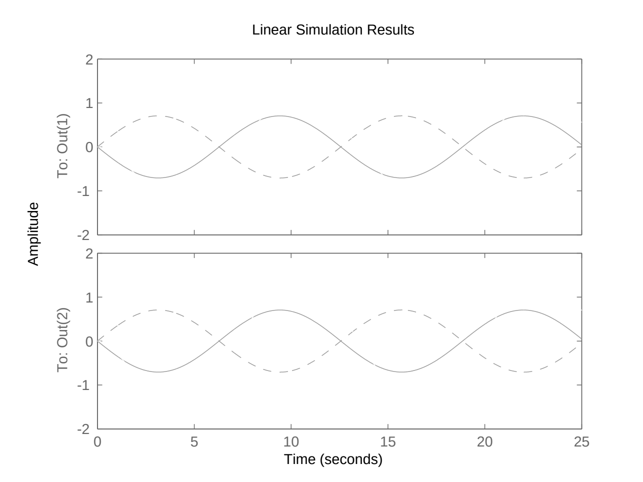

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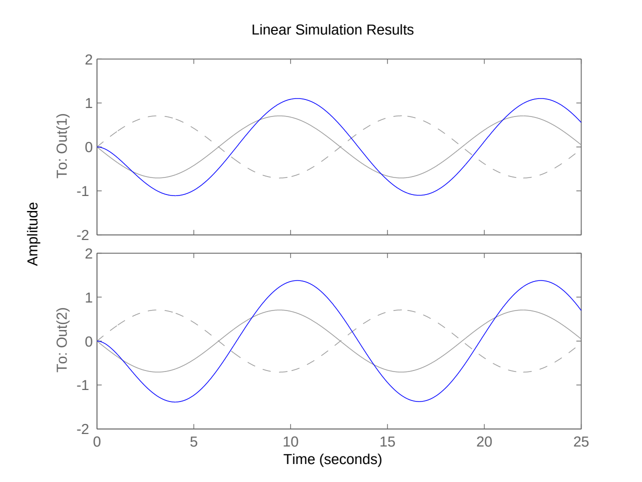

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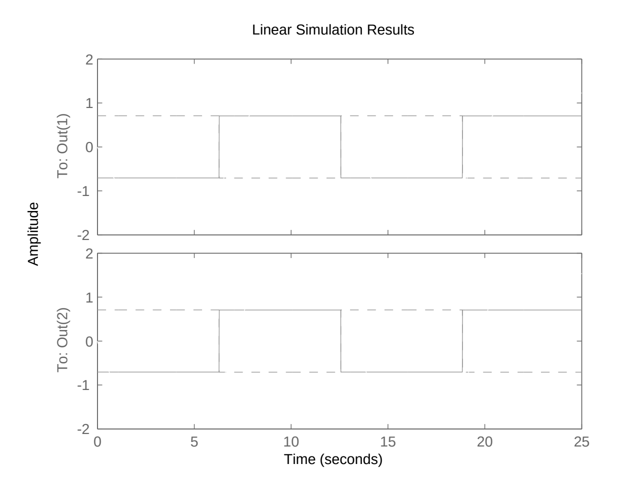

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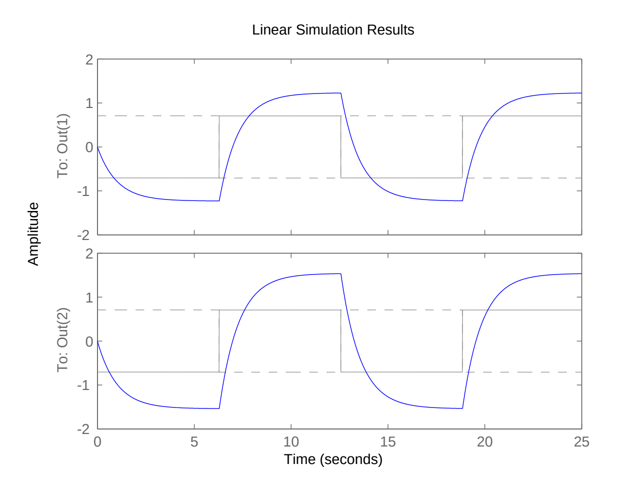

$\mathcal{H}_\infty$ norm specifications

$\mathcal{H}_\infty$ is the space of bounded holomorphic functions on the open right half plane, with norm

$$\|f(s)\|_\infty = \sup_{s \in \mathbb{C}_+} |f(s)|.$$

“I have never done anything ‘useful’. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world.”

$\mathcal{H}_\infty$ norm specifications

For stable real rational functions:

$$\|f(s)\|_\infty = \sup_{\omega \geq 0} |f(j\omega)|.$$

“I have never done anything ‘useful’. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world.”

$\mathcal{H}_\infty$ norm specifications

For matrices of stable real rational functions:

$$\|F(s)\|_\infty = \sup_{\omega \geq 0} \overline{\sigma}(F(j\omega)).$$

"I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."

$\mathcal{H}_\infty$ norm specifications

What does this tell us about how systems behave?

$\mathcal{H}_\infty$ norm specifications

Let $y(s) = T(s)r(s)$, and suppose we desire good tracking.

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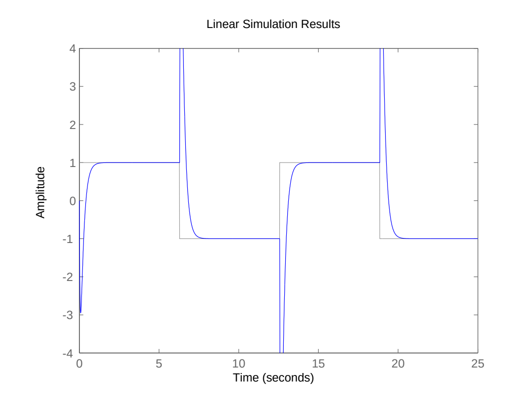

Impossible if $\|T(s)\|_\infty$ is small!

$\mathcal{H}_\infty$ norm specifications

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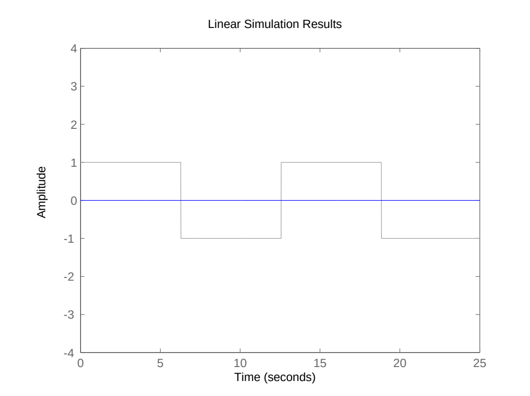

...possible if $\|T(s)\|_\infty$ is small!

$\mathcal{H}_\infty$ norm specifications

- Can rule out large bad behaviours.
- Doesn't mean behaviour is good...

$\mathcal{H}_\infty$ norm specifications

Revisit step tracking. Desire:

$$T(s)\frac{1}{s} \approx \frac{1}{s}$$

$\mathcal{H}_\infty$ norm specifications

Revisit step tracking. Desire:

$$(1 - S(s))\frac{1}{s} \approx \frac{1}{s}$$

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$\mathcal{H}_\infty$ norm specifications

Revisit step tracking. Desire:

$$(1 - S(s))\frac{1}{s} \approx \frac{1}{s}$$

$$\Rightarrow S(s)\frac{1}{s} \approx 0.$$

Check that

$$\|S(s)\frac{1}{s}\|_\infty \leq .05??$$

$\mathcal{H}_\infty$ norm specifications

Revisit step tracking. Desire:

$$(1 - S(s))\frac{1}{s} \approx \frac{1}{s}$$

$$\Rightarrow S(s)\frac{1}{s} \approx 0.$$

But

$$\|S(s)\frac{1}{s}\|_\infty = \infty$$

$\mathcal{H}_\infty$ norm specifications

Revisit step tracking. Desire:

$$(1 - S(s))\frac{1}{s} \approx \frac{1}{s}$$

$$\Rightarrow S(s)\frac{1}{s} \approx 0.$$

Check that

$$\|S(s)\frac{1}{s+T}\|_\infty \leq .05$$

Use weighting functions

$\mathcal{H}_\infty$ norm specifications

Use weighting functions and the $\mathcal{H}_\infty$ norm to capture desired performance.

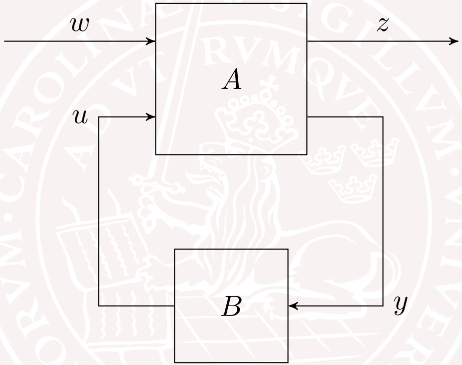
General advice:

- $\overline{\sigma}(S(j\omega))$ small over desired control bandwidth, and never too large.
- $\overline{\sigma}(T(j\omega))$ small at high frequencies (robustness).
- Check all the closed loop transfer functions, step responses... $\mathcal{H}_\infty$ norm is just a number!

$\mathcal{H}_\infty$ norm specifications

	$\|u\|_2$	$\|u\|_\infty$	$pow(u)$
$\|y\|_2$	$\|G\|_\infty$	∞	∞
$\|y\|_\infty$	$\|G\|_2$	$\|G\|_1$	∞
$pow(y)$	0	$\leq \|G\|_\infty$	$\|G\|_\infty$

Open Loop to Closed Loop



$$z(s) = A_{11}(s) + A_{12}(s)B(s)(I - A_{22}(s)B(s))^{-1}A_{21}(s)$$

Open Loop to Closed Loop

$$\mathcal{F}_l(A,B) = A_{11}(s) + A_{12}(s)B(s)(I - A_{22}(s)B(s))^{-1}A_{21}(s)$$

Find $A(s)$ for the following:

- $\mathcal{F}_l(A,B) = P(s) + \Delta(s)$, where $B(s) = \Delta(s)$.
- $\mathcal{F}_l(A,B) = W_1(s)(I + P(s)C(s))^{-1}$, where $B(s) = C(s)$.
- $$\mathcal{F}_l(A,B) = \begin{bmatrix} P(s) \\ I \end{bmatrix} (I - C(s)P(s))^{-1} \begin{bmatrix} -C(s) & I \end{bmatrix},$$

  where $B(s) = C(s)$.

Open Loop to Closed Loop

Read Chapter 10 of Robust and Optimal Control:

- Summing LFTs gives an LFT.
- Multiplying LFTs gives an LFT.
- Taking LFTs of LFTs gives an LFT.

Performance and robustness requirements can be specified with the LFT.

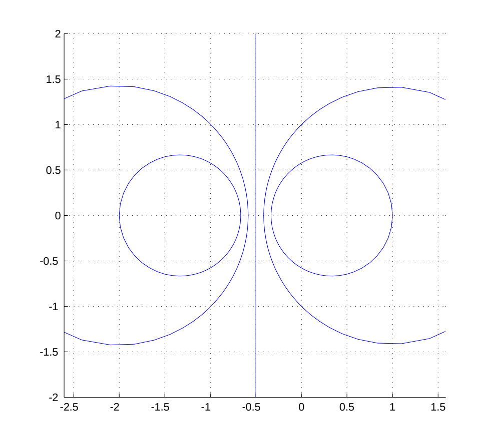
Open Loop to Closed Loop

Why use the LFT??

Open Loop to Closed Loop

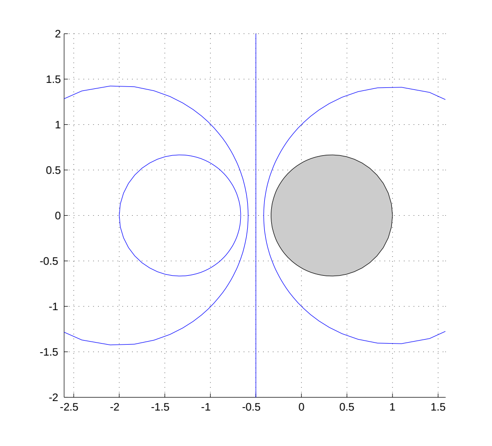
Why use the LFT??
Formalises the mapping from controller and uncertainty to design specifications.

Closed Loop to Open Loop?



$$\|T(j\omega)\|_\infty \leq \gamma$$

Closed Loop to Open Loop?



$$\|T(j\omega)\|_\infty \leq \frac{1}{2}$$

Closed Loop to Open Loop

Use the projective line!

$$\begin{bmatrix} A \\ B \end{bmatrix} \mapsto AB^{-1}.$$

Closed Loop to Open Loop

For example $T = G(I+G)^{-1}$.

$$T_P = \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \begin{bmatrix} G \\ I \end{bmatrix}$$

Closed Loop to Open Loop

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$$\Rightarrow G_P = \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} \begin{bmatrix} T \\ I \end{bmatrix}$$

Closed Loop to Open Loop

For example $T = G(I + G)^{-1}$.

$$T_P = \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \begin{bmatrix} G \\ I \end{bmatrix}$$

$$\Rightarrow G_P = \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} \begin{bmatrix} T \\ I \end{bmatrix}$$

$\Rightarrow G = T(I - T)^{-1}$.

Normalised Coprime Factors

$N(s), M(s) \in \mathcal{H}_\infty$ are right-coprime if there exist $X(s), Y(s) \in \mathcal{H}_\infty$ such that

$$X(s)M(s) + Y(s)N(s) = I.$$

Normalised Coprime Factors

$N(s), M(s) \in \mathcal{H}_\infty$ is a normalised right-coprime factorisation of P if:

- $P(s) = N(s)M(s)^{-1}$.
- $N(s), M(s)$ are coprime.
- $N(j\omega)^* N(j\omega) + M(j\omega)^* M(j\omega) = I$.

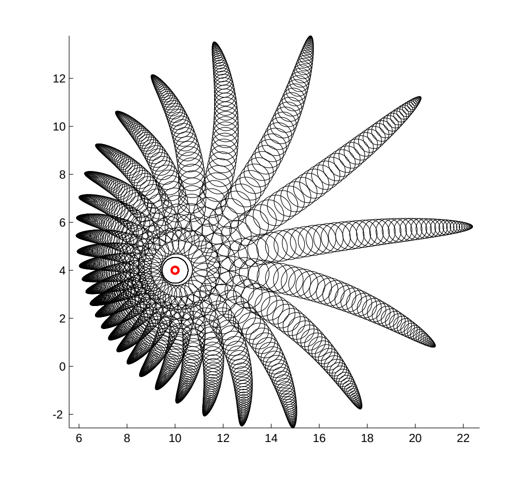
Normalised Coprime Factors

Normalised coprime factor uncertainty:

$$G_P = \begin{bmatrix} N(s) \\ M(s) \end{bmatrix} + \begin{bmatrix} \Delta_N(s) \\ \Delta_M(s) \end{bmatrix}, \left\| \begin{bmatrix} \Delta_N(s) \\ \Delta_M(s) \end{bmatrix} \right\|_\infty \leq \gamma.$$

Normalised Coprime Factors

Frequency by frequency:



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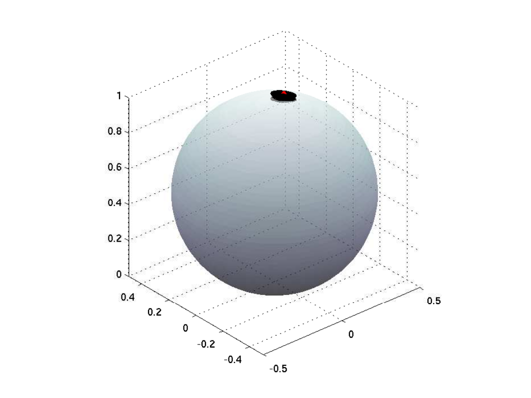

Gang of Four Stability Margin

Assuming stability,

$$b_{P,C} = \left\| \begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} \begin{bmatrix} -C & I \end{bmatrix} \right\|_{\infty}^{-1}.$$

Note $b_{P,C} = b_{C,P}$.

Gang of Four Stability Margin

Robustness properties:

- $b_{P,C} \leq \gamma$ if and only robust to coprime factor uncertainty of size γ.
- Robust to additive, multiplicative, inverse additive and inverse multiplicative uncertainty.

Gang of Four Stability Margin

Performance properties

- All 8 closed loop transfer functions are bounded.
- If $\underline{\sigma}(P(j\omega))$ is big, then $S_i(j\omega), S_o(j\omega)$ are small.
- If $\overline{\sigma}(P(j\omega))$ is small, then $T_i(j\omega), T_o(j\omega)$ are small.

Put $\bar{P}(s) = W_1(s)P(s)W_2(s)$, and check $b_{\bar{P},C}$!