Handin 3

1. Draw the block diagram of the negative feedback interconnection of

$$\begin{bmatrix} P(s) \\ I \end{bmatrix} (I + C(s)P(s))^{-1} \begin{bmatrix} C(s) & I \end{bmatrix}, \begin{bmatrix} \Delta_1(s) & \Delta_2(s) \\ \Delta_3(s) & \Delta_4(s) \end{bmatrix} =: \Delta,$$

using only six blocks (one for each of $P(s), C(s), \Delta_1(s), \Delta_2(s), \Delta_3(s), \Delta_4(s)$). How is the $\mathcal{H}_\infty$ norm requirement $b_{P,C} \geq \gamma$ related to robustness with respect to Δ? Now consider the additive uncertainty set

$$\Delta_{\text{add}} = \{P_\Delta(s) : P_\Delta(s) = P(s) + \Delta(s), \|\Delta(s)\|_\infty < \gamma\}.$$

If $b_{P,C} \geq \gamma$, is the negative feedback interconnection of $P_\Delta(s)$ and $C(s)$ stable for all $P_\Delta(s) \in \Delta_{\text{add}}$? What does this tell us about coprime factor uncertainty when compared to additive uncertainty? Can you make similar claims about the multiplicative uncertainty set

$$\Delta_{\text{mult}} = \{P_\Delta(s) : P_\Delta(s) = P(s)(I + \Delta(s)), \|\Delta(s)\|_\infty < \gamma\}?$$

Hint: Turn the performance requirement $\|(I + C(s)P(s))^{-1}C(s)\|_\infty \leq \frac{1}{\gamma}$ into a statement about robust stability.

2. Zhou 8.12.