Session 5

LTV stability. Quadratic Lyapunov functions.

Reading Assignment

Rugh Ch 6,7,12 (skip proofs of 7.8, 12.6 and 12.7),14 (pp240-247), and (22,23,24,28), Pg 117-125 ZDG book.

Exercise 5.1 = Rugh 6.3 iii+iv

Exercise 5.2 = Rugh 6.13

Exercise 5.3 = Rugh 7.3

Exercise 5.4 = Rugh 7.20

Exercise 5.5 = Rugh 23.2

Exercise 5.6 = Rugh 8.12 with $F(t) = 0$

Exercise 5.7 Given two matrices $A \in R^{n\times m}$ and $B \in R^{m\times n}$ show that the following is equivalent: i) $I_n - AB$ is invertible, ii) $I_m - BA$ is invertible, iii) $\begin{bmatrix} I_n & A \\ B & I_m \end{bmatrix}$ is invertible.

Exercise 5.8 Fill in the details in the proof of Lemma 5.3 in [ZDG]:
a) Verify the formula for the transfer matrix from w to e given on p. 123.
b) Also show that if (A, B, C) and $(\hat{A}, \hat{B}, \hat{C})$ are stabilizable and detectable, then so is $\left(\tilde{A}, \begin{bmatrix} B & 0 \\ 0 & \hat{B} \end{bmatrix}, \begin{bmatrix} 0 & \hat{C} \\ C & 0 \end{bmatrix}\right)$, where $\tilde{A}$ is given on p.121.

Hand in problems

Exercise 5.9 For the linear state equation with

$$A(t) = \begin{cases} \begin{bmatrix} -1 & e^{-2t} \\ 0 & -3 \end{bmatrix}, & t \geq 0, \\ \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}, & t < 0. \end{cases} \tag{1}$$

Use a diagonal $Q(t)$ to prove uniform exponential stability.

Exercise 5.10 Use e.g. CVX to find a constant Lyapunov matrix Q verifying exponential stability for the system

$$\dot{x}(t) = A(t)x(t)$$

where for each t either $A(t) = A_1$ or $A(t) = A_2$ (i.e. $A(t)$ can jump between A_1 and A_2 at arbitrary times), where

$$A_1 = \begin{bmatrix} -1 & 3 \\ 0 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 3 \\ -1 & -6 \end{bmatrix}$$

What is the best exponential convergence rate $\lambda > 0$ you can guarantee?