Control System Design - QFT

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Lecture - QFT Design

QFT Design

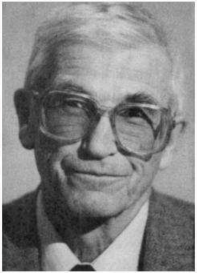
“Quantitative Feedback Theory” (QFT) is a good choice when the main problem is model uncertainty.

One often develops a good intuitive understanding of the design problem while using QFT.

Carpentry

Its main strength is for SISO design, but MIMO and nonlinear extensions exist. Will only discuss SISO design here

Background - History



Was proposed and developed by Horowitz in the 1960s.
Based on Bode's work in the 1950s.

– *Why use feedback instead of open loop control ?*

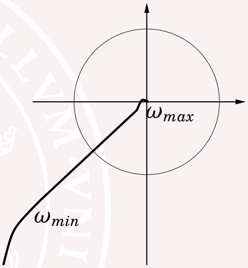
– *Feedback reduces influence of disturbances and system uncertainty, feedforward doesn't.*

– *Cost of control*

Bode's Ideal Loop Transfer

The loop transfer function

$$L(s) = \left(\frac{s}{\omega_{gc}}\right)^n$$



gives a phase margin that is invariant to gain variations.

The slope $n = -1.5$ gives the phase margin $\varphi_m = 45°$.

Horowitz extended Bode's ideas to deal with arbitrary plant variations not just gain variations in the QFT method.

Background - Feedback against uncertainty

Consider a static gain process and static controller

$$y = Pu, \qquad u = C(r - y)$$

Want $y = PC/(1+PC)r \approx r$, say $PC/(1+PC) \in [T_{min}, 1]$

For $P \in [0.1, 1]$

- $T_{min} = 0.75$ need $C \geq 30$
- $T_{min} = 0.90$ need $C \geq 90$
- $T_{min} = 0.99$ need $C \geq 990$

For $P \in [0.01, 1]$

- $T_{min} = 0.75$ need $C \geq 300$
- $T_{min} = 0.90$ need $C \geq 900$
- $T_{min} = 0.99$ need $C \geq 9900$

Tighter spec or more uncertain system $\Rightarrow$ need higher controller gain.

Cost of High Gain Feedback

- Control effort
- Measurement noise
- Dynamic instability

The aim of QFT design is to use precisely the right amount of feedback

High gain exactly where needed

The Cost of Feedback

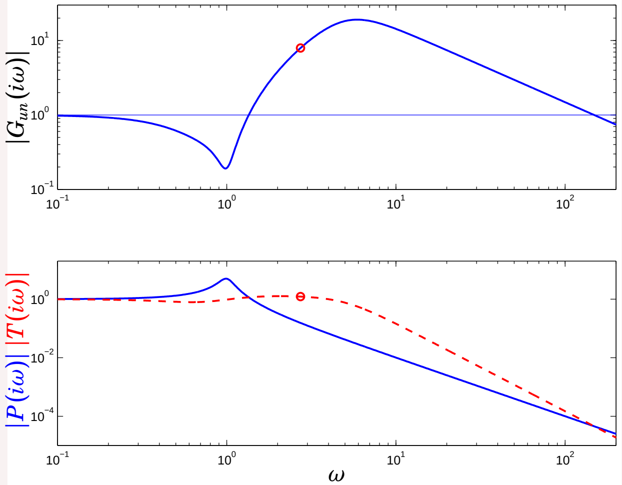
The cost of feedback is that a controller which has high gain over a wide frequency range is required. The high controller gain means that small measurement errors may generate large control signals that could cause saturations in the system. If there is measurement noise the high gain may also cause control actions that may wear out the actuators.

The transfer function from measurement noise n to the control signal for a closed loop system is

$$-G_{un} = \frac{C}{1 + PC} = \frac{T}{P}$$

An estimate of the cost of control can be obtained by plotting the gain curves of the transfer functions G_{un} or T and P. Reasonable measures are $\max |G_{un}(i\omega)|$ eller $||G_{un}||_2$.

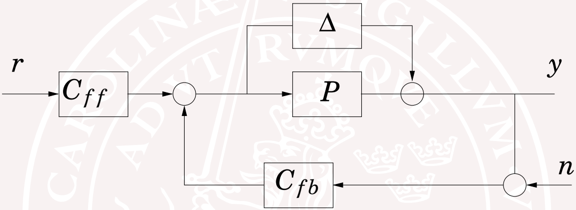
The Cost of Feedback



Notice that controller gain is high way beyond
the gain crossover frequency marked with the red dots

Feedback vs Feedforward - Risk for Instability

Influence of model uncertainty



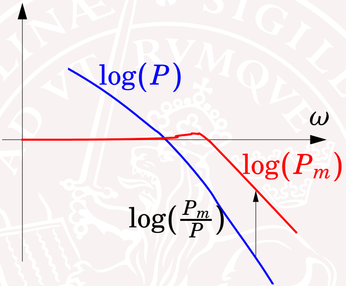
Model $y = P_m r = \frac{PC_{ff}}{1+PC_{fb}} r$ (note $P_m = TF$). Real system is stable if

$$|\Delta| \left| \frac{C_{fb}}{1 + PC_{fb}} \right| < 1, \quad \forall \omega$$

$$|\Delta| < \left| \frac{P}{P_m} \right| \left| \frac{C_{ff}}{C_{fb}} \right|, \quad \forall \omega$$

Nice interpretation

Cost of Feedback - Robust stability for 2DOF



$$|\Delta| < \left|\frac{P}{P_m}\right| \left|\frac{C_{ff}}{C_{fb}}\right|, \quad \forall\omega$$

Regions where **gain is increased** and where **feedback is used** require low model uncertainty

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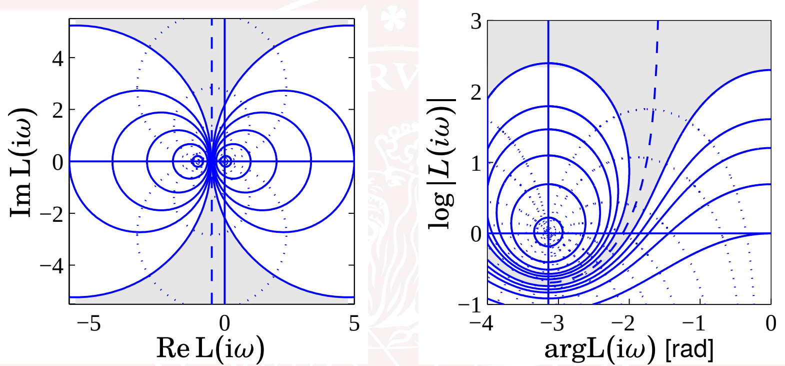
Hall and Nichols Chart

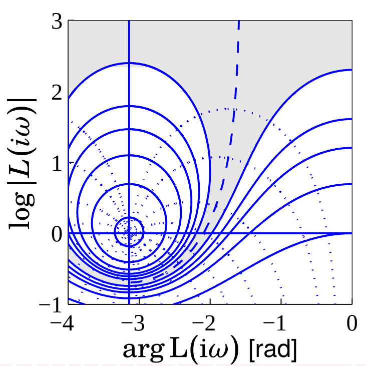


Hall is a Nyquist plot with level curves of gain and phase for the complementary sensitivity function T. Nichols=$\log$ Hall.

Both make it possible to judge T from a plot of PC

The Robustness Valley $\mathrm{Re}\,\mathrm{L}(\mathrm{i}\omega) = -1/2$ dashed

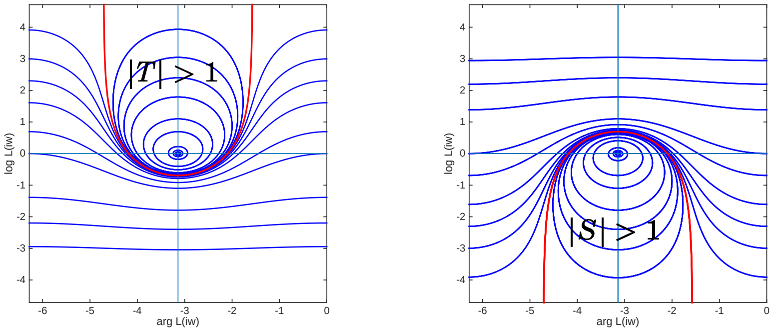
The Nichols Chart



Advantages with nichols chart

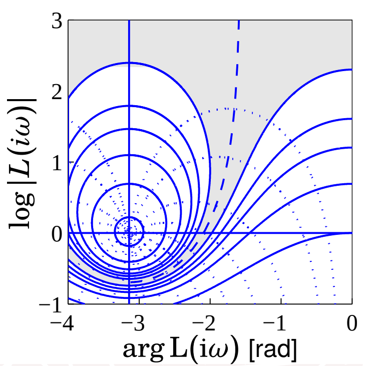
- logarithmic gain scale shows wide range
- $\log(PC) = \log(P) + \log(C)$ and $\arg(\mathrm{PC}) = \arg(\mathrm{P}) + \arg(\mathrm{C})$

Nichols Chart - Level curves of T and S



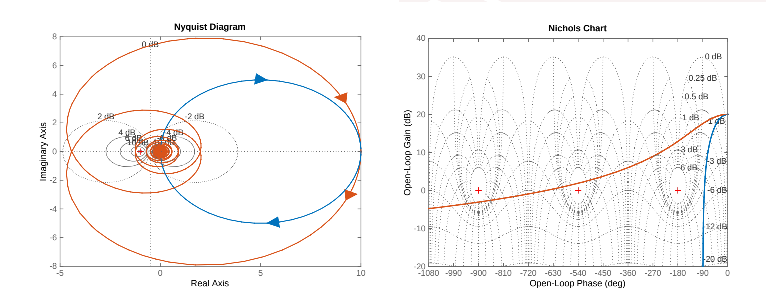
TAT: Explain the symmetry between the level curves of $|T| = |\frac{L}{1+L}|$ and $|S| = |\frac{1}{1+L}|$

The Nichols Chart



TAT: Interpret the Nyquist theorem in the Nichols diagram. How do you count encirclements of -1?

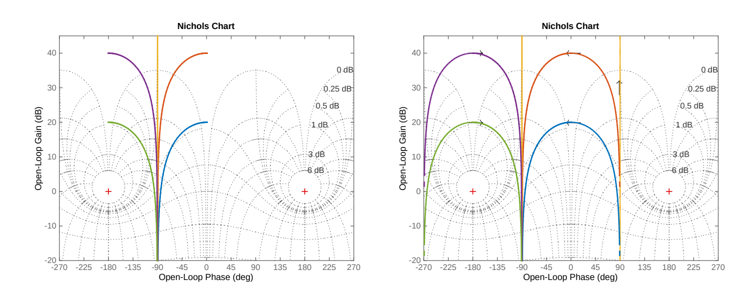
Example



$$\frac{10}{s+1}, \qquad \frac{10e^{-s}}{s+1}$$

TAT: Will the closed loop systems be stable? How many unstable poles will there be?

Example

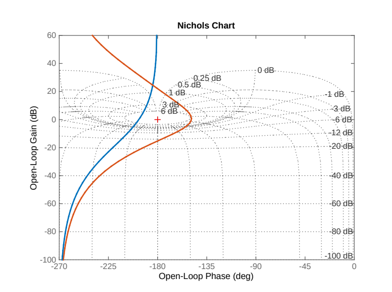


Only positive frequencies (left), both negative and positive (right)

$$G(s) = \frac{10}{s-1}, \quad \frac{10}{s-0.1}, \quad \frac{10}{s}, \quad \frac{10}{s+0.1}, \quad \frac{10}{s+1}$$

All five systems are stable when closed with -1.

Example - Ball and Beam

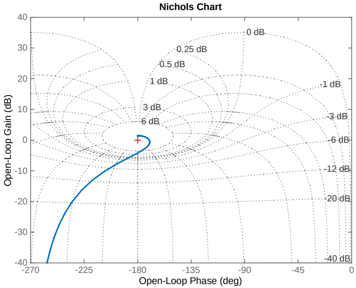


This is our lead-lag design for the (outer) control loop of the ball and beam process used in the basic course

$$P(s) = \frac{10}{s^2(1+0.1s)}, \qquad P(s)C(s) = \frac{10}{s^2(1+0.1s)} \frac{s+a}{s} \frac{K(1+sb)}{1+sb/N}$$

TAT: What is the gain and phase margins?

Example - Inverted pendulum



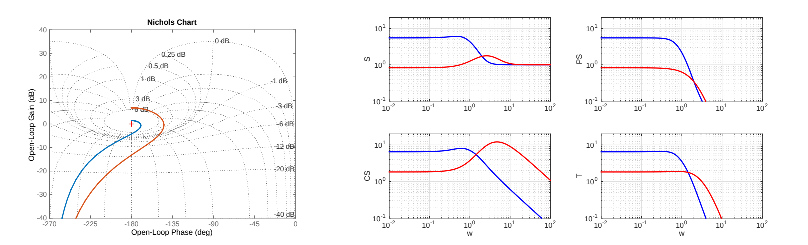
This is the state-feedback with Kalman filter design for the inverted pendulum $G(s) = \frac{1}{s^2-1}$ in the basic course.

Controller poles: $-0.5 \pm 0.5i$, observer poles: $-1 \pm i$.

Phase margin=9 degrees, $\omega_{gc} = 0.6$ rad/s

Too slow design (remember rule of thumb $\omega_{gc} > 2p$)

Example - Inverted pendulum, improved design



Better design: controller poles $-2, -2$, observer poles=$-4, -4$

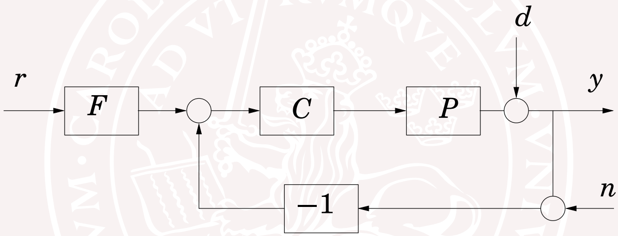
$M_s = 1.8(6.1)$, phase margin $\phi_m = 36°(9°)$, $\omega_{gc} = 1.8(0.6)$ rad/s

Note that higher controller gain is needed

QFT Methodology

Unknown process $P(s) \in \mathcal{P}$

2DOF controller structure



$$u = C(i\omega)(-y + F(i\omega)r)$$

First design feedback $C(i\omega)$ to reduce effects of uncertainty and disturbances, considering structure of process variations

Then find prefilter $F(i\omega)$ to shape reference response

QFT Methodology

Unknown process $P(s) \in \mathcal{P}$

Would be nice if we only needed to design for one process case $P_{nom}(s)$

The QFT design methodology achieves this !

QFT Design Steps

SPECs Translate requirements to approximate frequency domain specifications, e.g. on elements of GOF

Nominal Choose a nominal process $P_{nom}(s)$ and a suitable frequency set ω_k ("representative" frequencies)

Bounds For each frequency ω_k determine the region of $C(i\omega_k)$ for which the specs can be satisfied for all processes $\mathcal{P}$ for some $F(i\omega_k)$. Plot the regions $C(i\omega_k)P_{nom}(i\omega_k)$ in the Nichols diagram (the Horowitz bounds)

C Find a controller C so that $L = CP_{nom}$ satisfies all bounds, i.e. for which a F might exist.

F Find, if possible, a F so that all freq domain specs are satisfied

Check Simulate and verify that time domain specs are satisified.

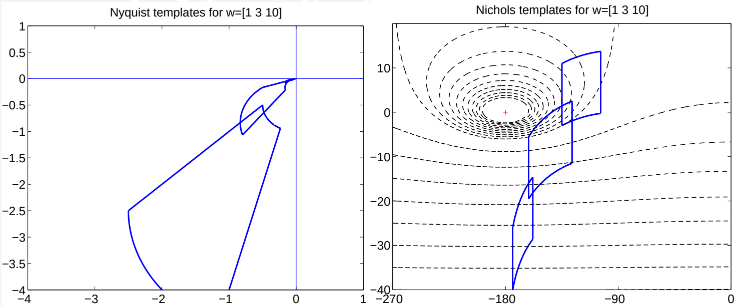
If you fail, then either the specs where too tight, or you didnt work hard enough.

Lecture - QFT Design

Templates

For each frequency, plot all possible complex process gains

$$P(s) = \frac{ka}{s(s+a)}, \quad k \in [1, 5], \quad a \in [1, 4]$$



Easier to work with the Nichols diagram

The regions $\{P(i\omega_k)\}_{P \in \mathcal{P}}$ are called "templates"

A Nice Observation

Note: The set $\{P(i\omega)C(i\omega)\}_{P\in\mathcal{P}}$ has the same shape as the template $\{P(i\omega)\}_{P\in\mathcal{P}}$! Not true in Nyquist.

Template is moved by (180/pi*angle(C),20*log10(abs(C)))

For each frequency ω_k:

- Move the template around in the Nichols chart
- Mark all positions of the nominal point for which the entire template satifies the requirement

Gives a region of allowed controller gains $C(i\omega)$.

"Horowitz Bounds"

Template Calculations

How to compute template $P(i\omega_k, \theta)$, with N uncertain parameters $\theta = (\theta_1, \ldots, \theta_N)$?

Possible methods

1. Compute $P(s, \theta)$ for a large set of random parameters θ.
2. Grid the parameter space
3. Compute for edges of parameter space, hope this gives template

1,2 are inefficient and 3 is not guaranteed to work.

Worst cases give useful insight

Template Calculations

Choice of frequency set.

- Low frequencies to guarantee disturbance rejection performance
- cross-over and process resonances
- high frequencies for e.g. measurement noise and unmodelled dynamics

Use common sense.

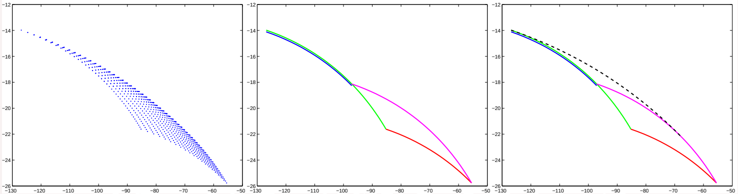
Template calculation - Method 3

```
template=[];
a=amin:da:amax;
for k=[kmin kmax]
    g=k*a/s./(s+a);
    template=[template g];
end
k=kmin:dk:kmax;
for a=[amin amax]
    g=k*a/s/(s+a);
    template=[template g];
end
```

Template Calculations

Example where method 3 fails

$$P(s,a,b) = \frac{1}{(s+a)(s+b)}, \quad a \in [1,5], \;\; b \in [1,3]$$



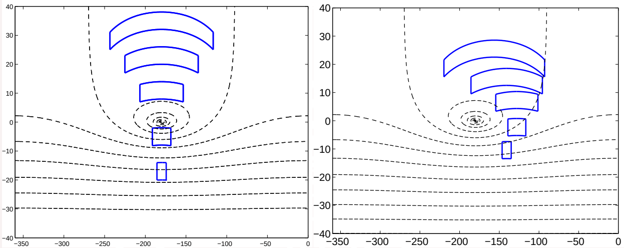
- $P(i\omega, 1, [1,3])$ blue
- $P(i\omega, 5, [1,3])$ red
- $P(i\omega, [1,5], 1)$ green
- $P(i\omega, [1,5], 3)$ magenta
- Missing boundary: $P(i\omega, a, b)$ where $a = b \in [1,3]$ (black)

Varying number of unstable poles

What if number of unstable poles varies?

$$P(s) = \frac{k}{s(s-a)}, \quad k \in [10, 20], \quad a \in [-1, 1]$$

$$C(s) = \frac{3(s+1)}{s+10}$$



Closed loop system stable for all parameters.

Left part of template (unstable open systems) crosses 1 time, right part (stable open systems) crosses 0 time. Fine.

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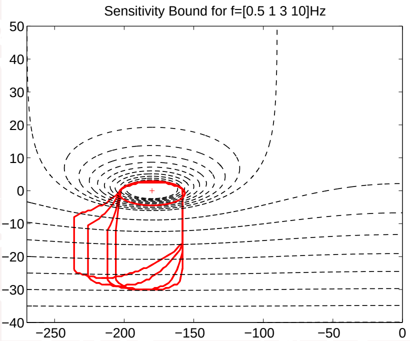
Specifications

Requirement on sensitivity function

$$S(i\omega) = |\frac{1}{1 + P(i\omega)C(i\omega)}| < 2.5$$



$$P(i\omega) = \frac{ka}{i\omega(i\omega + a)}, \quad k \in [1, 5], \quad a \in [1, 4]$$

Sensitivity Bound Computation

```
fasv=-270:2:0;
for fas = fasv
 for ampdb=-40:0.5:10
  c=10^((ampdb-ampnom)/20)*exp(i*(fas-fasnom)*pi/180);
  l=template*c;  s=1./(1+l);
  if max(abs(s)) > spec,
    bound=[bound ampdb]; fasbound = [fasbound fas]; break
  end
 end
end
for fas = fasv(end:-1:1)
 for ampdb=10:-0.5:-20
  c=10^((ampdb-ampnom)/20)*exp(i*(fas-fasnom)*pi/180);
  l=template*c;  s=1./(1+l);
  if max(abs(s)) > spec,
    bound=[bound ampdb]; fasbound = [fasbound fas]; break
  end
 end
end
bound=[bound bound(1)];fasbound=[fasbound fasbound(1)];
plot(fasbound,bound,'r','Linewidth',2);hold on;
```

Specifications - GOF

All frequency domain specifications on e.g. elements in the GangOfFour can be treated similarly.

For each frequency ω_k determine the set of $C(i\omega_k)$ for which the spec is satisfied

Plot the set $P_{nom}(i\omega_k)C(i\omega_k)$ in the Nichols chart

Choose the frequency set wisely ! Might need to iterate as design proceeds

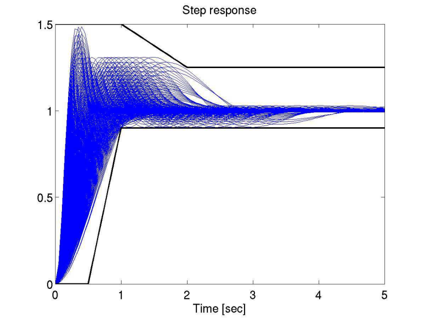
What about time domain requirements?

Usually hard to translate to frequency domain

Some approximation and creativity is usually needed.

Specifications - time domain

Time domain requirement on step response $y = \frac{PCF}{1+PC} r$



How do we

- translate it into a frequency domain specification?
- split it into separate requirements on C and F?

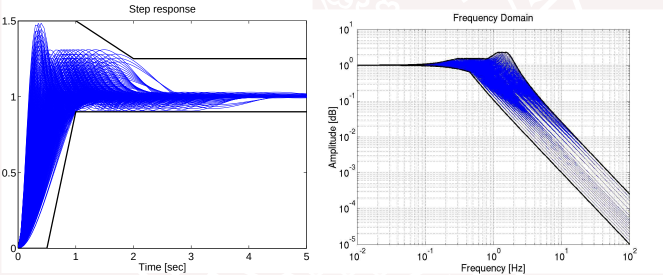
A typical way of approximating the requirement is the following

Time Domain Spec

Study the set of all 2nd order systems

$$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

satisfying the time domain bound.



If the closed loop system $\frac{PCF}{1+PC}$ lies between the black curves, and behaves similarly to a 2nd order system, then the time domain spec should hopefully be satisfied. With some luck.

Time domain spec to frequency bound

```
for w=[2:0.25:4.5 5:1:10]
  for zeta = [0:0.025:1 1.2:0.2:2];
    g=w^2/(s^2+2*w*zeta*s+w^2);
    [y,t]=step(g,timev);
    if max(y-upperbound)<=0 & min(y-lowerbound)>=0
      figure(1)
      plot(t,y);hold on;
      amp = abs(squeeze(freqresp(g,2*pi*frv))');
      ampmax = max([ampmax;amp]);
      ampmin = min([ampmin;amp]);
      figure(2)
      loglog(frv,abs(amp)); hold on;
    end
  end
end
```

Specifications - example

To be able to later find F satisfying

$$a(\omega) \leq \left| \frac{PCF}{1+PC} \right| \leq b(\omega)$$

a necessary and sufficient condition is that

$$\frac{\max_{P \in \mathcal{P}} \left| \frac{P(\omega)C(\omega)}{1+P(\omega)C(\omega)} \right|}{\min_{P \in \mathcal{P}} \left| \frac{P(\omega)C(\omega)}{1+P(\omega)C(\omega)} \right|} \leq \frac{b(\omega)}{a(\omega)}, \qquad \forall \omega$$

Let's use this to calculate bounds !

Bounds

We get the following requirements

Freq [Hz]	b/a
0.01	1.0011
0.02	1.0043
0.1	1.107
0.2	1.47
0.5	3.1

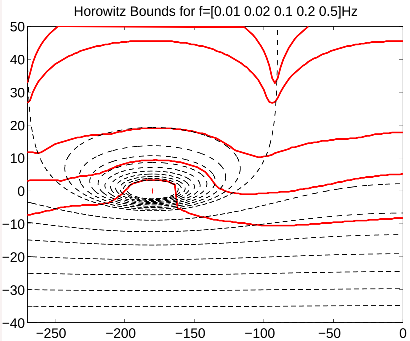
For each ω_k determine set $\{C(i\omega_k)\}$ that satisfies

$$\frac{\max_{P\in\mathcal{P}}\left|\frac{P(\omega_k)C(\omega_k)}{1+P(\omega_k)C(\omega_k)}\right|}{\min_{P\in\mathcal{P}}\left|\frac{P(\omega_k)C(\omega_k)}{1+P(\omega_k)C(\omega_k)}\right|} \leq \frac{b(\omega_k)}{a(\omega_k)}$$

Then plot region $\{P_{nom}(i\omega_k)C(i\omega_k)\}$ in Nichols diagram

Horowitz Bounds

$$P(s) = \frac{ka}{s(s+a)}, \quad k \in [1,5], \quad a \in [1,4], \quad k_{nom} = a_{nom} = 1.$$



Robustness valley !

Freq [Hz]	b/a
0.01	1.0011
0.02	1.0043
0.1	1.107
0.2	1.47
0.5	3.1

Bound computation

```
bound=[];
fasv=-270:2:0;
for fas = fasv
 for ampdb=-0:0.1:30
  c=10^((ampdb-ampnom)/20)*exp(i*(fas-fasnom)*pi/180);
  l=template*c;
  t=l./(1+l);
  delta = 20*log10(max(abs(t))) - 20*log10(min(abs(t)));
  if delta < specdB,
    bound=[bound ampdb];
    break
  end
 end
end
plot(fasv,bound,'r','Linewidth',2)
```

Merging the bounds

When all requirements have been translated to frequency domain bounds on $C(i\omega)$, the final bounds are calculated as intersections, for each frequency.

Need not be connected sets, can be empty

If empty, then we have proved the requirements are too tough (disregarding possible approximations we have done) !

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Example

Let's design a controller for the process

$$P(s) = \frac{ka}{s(s+a)}, \quad k \in [1, 5], \quad a \in [1, 4]$$

satisfying

- Step response satisfying time domain spec above
- $|S(i\omega)| < 2.5$ for all ω

Design frequencies chosen as

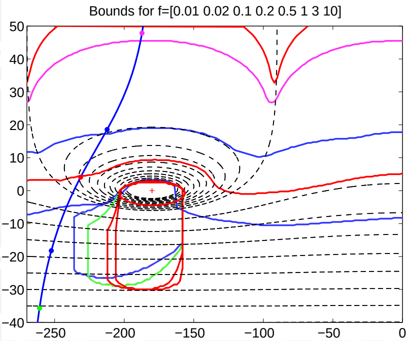
$$\omega = [0.01, \ 0.02, \ 0.1, \ 0.2, \ 0.5, \ 1, \ 3, \ 10]$$

Nominal process chosen as $k_{nom} = 1, \ a_{nom} = 1.$

Bounds have been computed above.

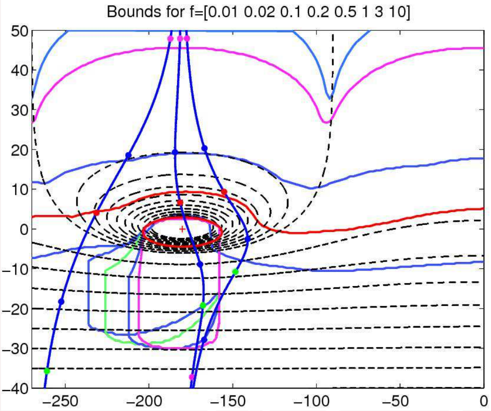
Design - Lead lag design

The controller is designed by hand using lead-lag design.
Lets start with $C(s) = \frac{4}{s}$



Low frequency gains is roughly right, but about 100 degrees more phase advance is needed around 0.5Hz

Design - Lead lag design

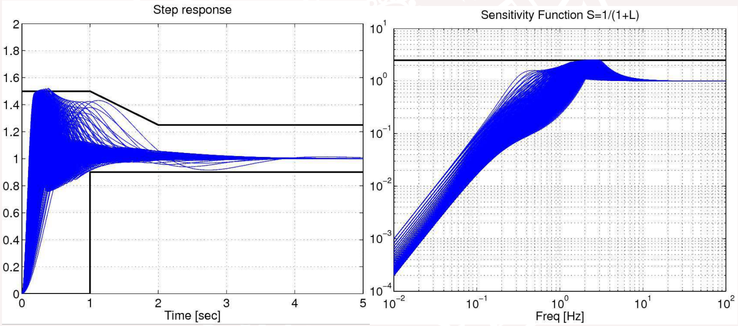


$$C(s) = \frac{4}{s}$$
$$C(s) = \frac{5(s+2)^2}{s(s+5)}$$
$$C(s) = \frac{15(s+1)(s^2+3.6s+4)}{s(s+3)(s+5)}$$

Bound for $\omega = 3$ barely satisfied.

Verification

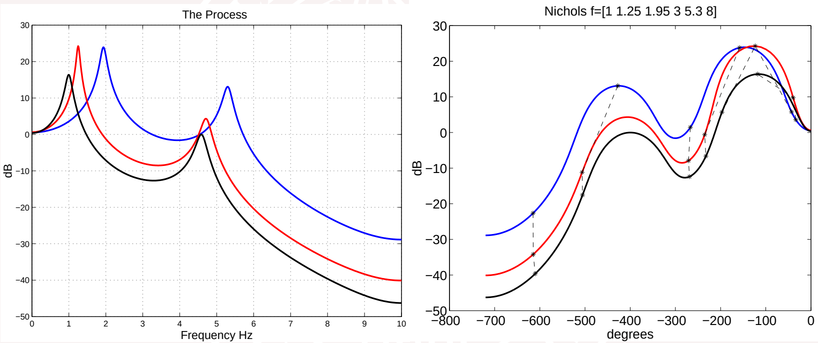
$$C(s) = \frac{15(s+1)(s^2+3.6s+4)}{s(s+3)(s+5)}$$



Close to ok. More fine tuning could be done.

Example - Handin 2

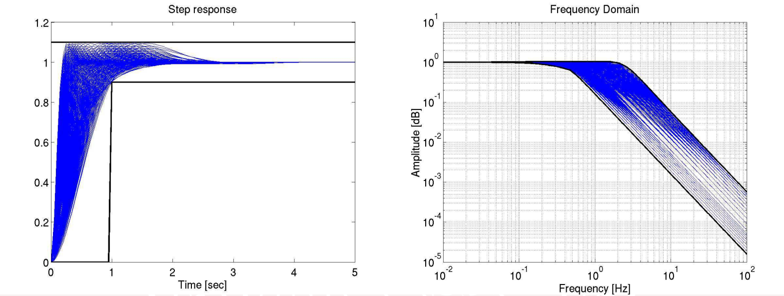
The process set $\mathcal{P} = \{P_0, P_{0.5}, P_1\}$



Choose P_1 (black) as the nominal plant

Handin 2 - Spec A,B

Rise time < 1 sec, overshoot < 10%



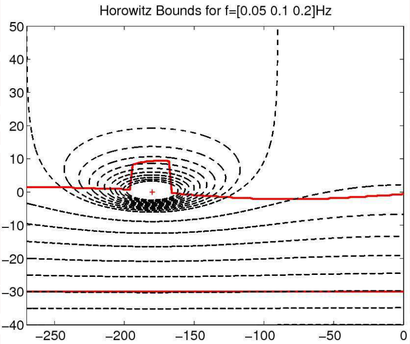
$f = 0.05$ Hz gives $b/a = 1.013$

$f = 0.1$ Hz gives $b/a = 1.05$

$f = 0.2$ Hz gives $b/a = 1.18$

Spec A,B Bounds

Translating this to bounds



Very little process uncertainty for low frequencies, hence modest requirements on gain.

Indicates that step response requirement should not be that difficult to meet. Can be solved by feedforward tuning.

Spec C

Settling time 1.2 sec for output disturbance filtered by $1/A(z)$.

Integral action and closed loop bandwidth of approx 2 rad/s needed, so lets require

$$\left|\frac{S}{A}(i\omega_k)\right| < K\left|\frac{i\omega_k}{i\omega_k + 2}\right|$$

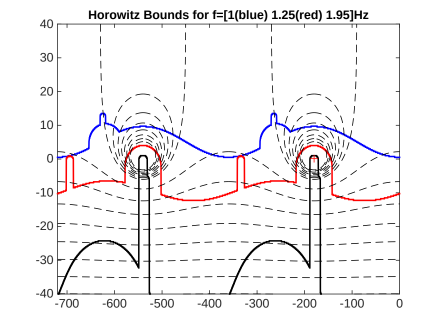
Mainly a requirement on attenuation near resonances, where $1/A$ is large.

Choose frequencies $\omega_k \in 2\pi[1, 1.25, 1.95]$.

Suitable constant K=20 found by trial and error.

Spec C Bounds

Translating this to bounds at $[1,\ 1.25,\ 1.95]$Hz



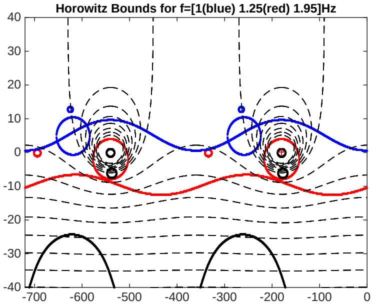
Main trouble seems to be achieving sufficiently high gain for P1 for f around 1 Hz.

Bounds become somewhat messy.

Lets wait with bounds for the 2nd resonance around 5 Hz to later. Perhaps it isnt needed.

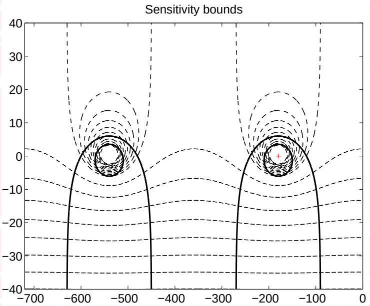
Spec C Bounds - a closer look

The bounds are really disconnected regions. A better plot is



Spec E+F

Sensitivity less than 1 for f<0.2Hz, and less than 6dB for all frequencies



SpecE bound shown for $f = 0.2$ and specF bound for $f \approx 0$

Spec G,H

Delay margin and input sensitivity less than 10dB in [8,10]Hz.

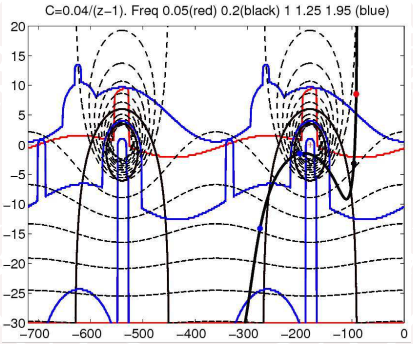
Sensitivity constraint seemed to give sufficient guarantees on phase margin, and delay margin was automatically satisfied

Spec H limits the controller gain at high frequencies.

Decided to check this afterwards. The Nicols chart was becoming messy enough.

Design Step 1

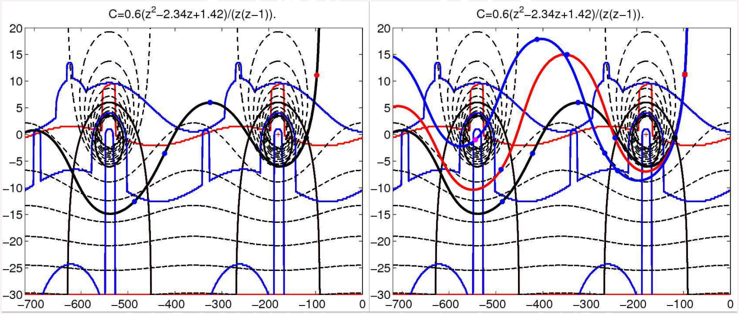
We need an integrator so lets introduce that. Also choose gain so low frequency requirements are fullfilled, $C = 0.04z/(z-1)$



To satisfy requirement at f=1 (blue) it seems easiest to move this point to somewhere around -300 to -360 degrees and gain = 2 to 5dB

Design Step 2

$C = 0.7(z^2 - 2.34z + 1.42)/(z(z-1))$



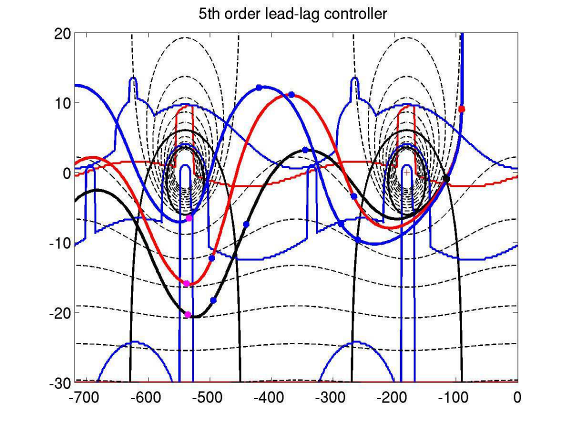
Plotting all three cases shows that there are problems for P0 around 3 Hz

Need to compute sensitivity bound for 3Hz

Also considered to introduce bound for attenuation near 2nd resonance, but turned out not to be needed

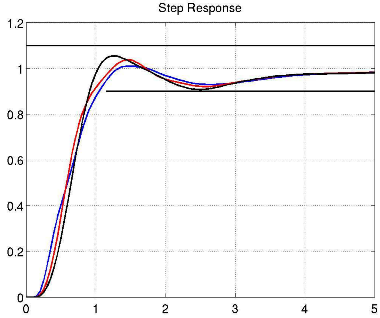
Design Step 3

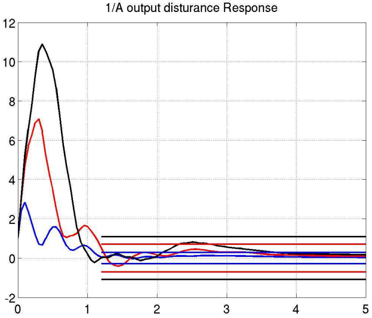
Final 5th order lead-lag controller



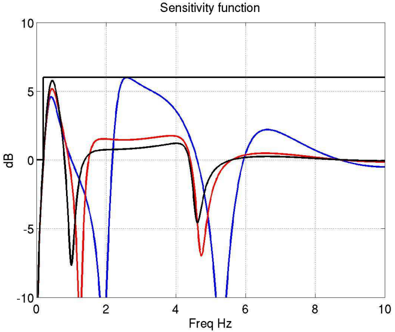
Feedforward was easily found by hand tuning the step response.

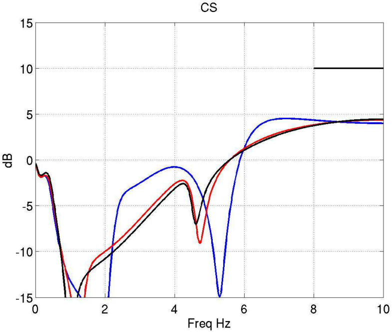
Handin 2 - Result





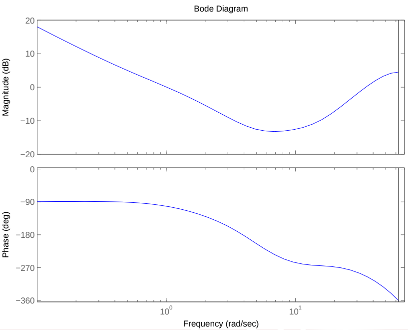
Handin 2 - Result

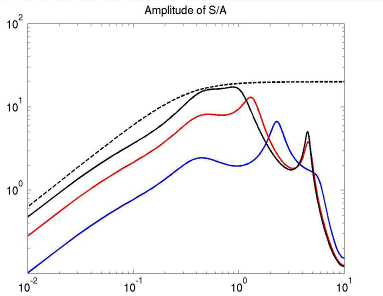




Handin 2 - Result

Controller and Spec C bound (disturbance rejection)





S/A largest for process P1 (black). Main problem around 0.5-1Hz. Note that 2nd resonance is much smaller, worst case is P0 (blue) near 3Hz

Summary - Pros and Cons

The robust design problem is transformed into a design problem for only one process, the nominal case P_{nom}. This simplifies design considerably.

Specifications initially in time domain might need some creativity to transform to frequency domain

The splitting into separate designs of feedback C and feedforward F simplifies things

The designer still needs to handtune the controller

Good tools help

Hard to know what to do if design fails. Keep trying, or?

Lecture - QFT Design

Tools

QFTIT - free software based on Sysquake, new version for Mac available (for us!)

QFTCT - matlab toolbox

Interactive bounds computation and controller design possible with these tools

There are also some commercial toolboxes available

Summary

- Probably the best approach for structured uncertainty
- Cost of feedback
- Nichols
  - Large range
  - Templates just translate

References

Isaac Horowitz, Synthesis of Feedback Systems (1963), Academic Press.

J. D'Azzo, C. Houpis, Feedback Control System, Analysis and Synthesis, (1966) McGraw Hill

Isaac Horowitz, Survey of QFT, Int. J. Control, 1991, pp 255-291

QFTIT and QFTCT tools (google it)

Go and try one of these QFT tools now !