Extremum-seeking Control

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Short introduction

- Non-model based real-time optimization

[1] M.Krstić H.Wang, Stability of extremum seeking feedback for general nonlinear dynamic systems, Automatica 36, 2000

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  - E.g. a nonlinear equilibrium map with a local minimum
- Popular around the middle of the 1950s
- Revival with proof of stability [1]
- Very attractive with the increasing complexity of engineering systems

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Examples of application

- active flow control
- aeropropulsion
- colling systems
- wind energy
- human exercise machines
- optimizing the control of non-isothermal valve actuator
- timing control of HCCI engine combustion
- formation flight optimization
- beam matching adaptive control
- optimizing bioreactors
- control of beam envelope in particle accelerators

Problem statement

Consider a SISO nonlinear model

$$\dot{x} = f(x, u), \quad (1)$$
$$y = h(x) \quad (2)$$

- $x \in \mathbb{R}^n$ is the state
- $u \in \mathbb{R}$ is the input
- $y \in \mathbb{R}$ is the output (or the performance function
- $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}$ are smooth

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Suppose that we know a control-law

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parametrized by a scalar parameter θ.

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The closed-loop system

$$\dot{x} = f(x, \alpha(x, \theta))$$

has equilibria parametrized by θ.

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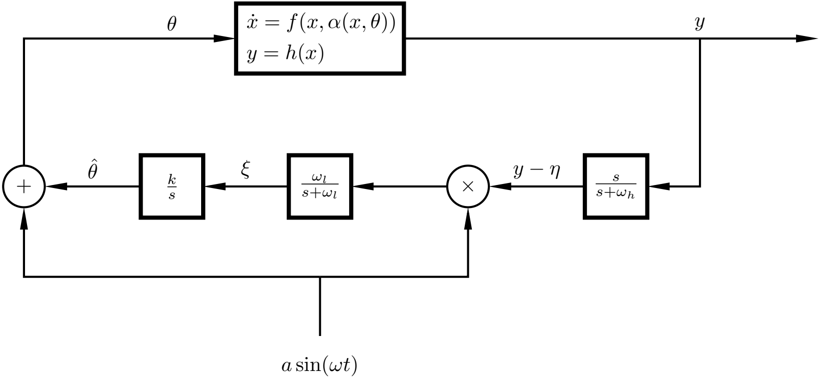
- *There exists a $\theta^* \in \mathbb{R}$ such that*

$$(h \circ l)'(\theta^*) = 0, \tag{4}$$
$$(h \circ l)''(\theta^*) > 0 \tag{5}$$

The feedback scheme

- Perturb the plant with a *slow* periodic signal $a\sin(\omega t)$



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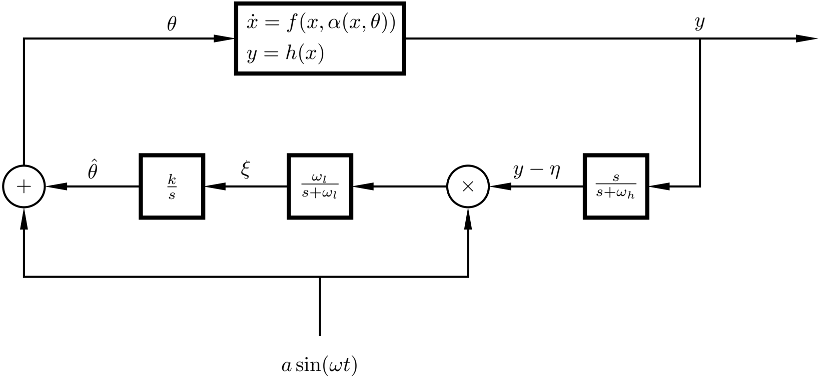

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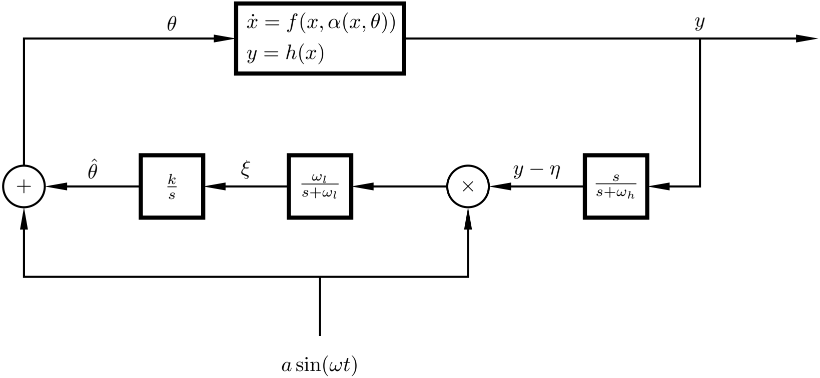

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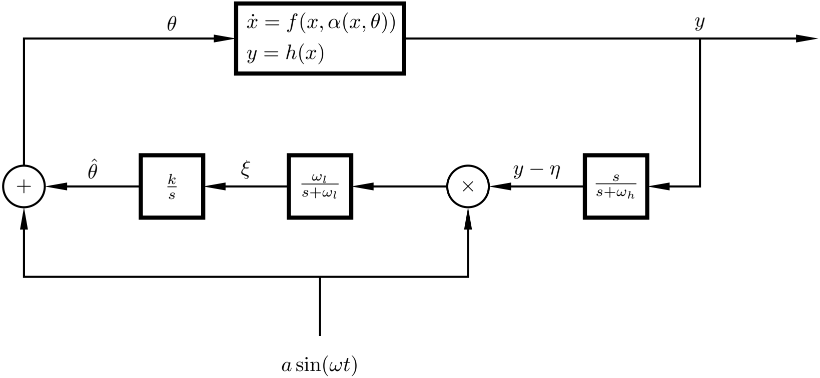

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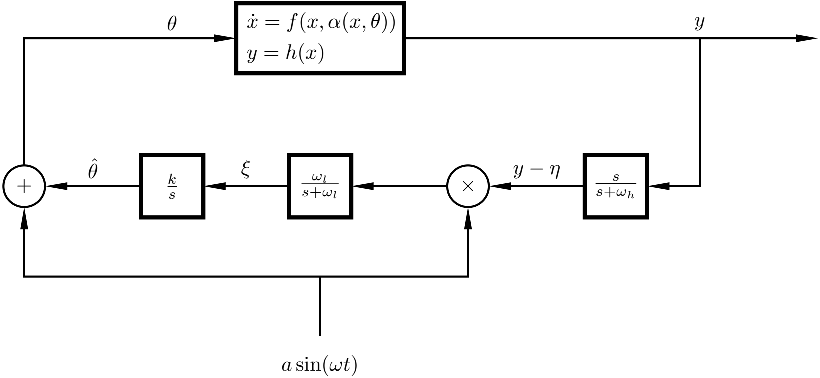

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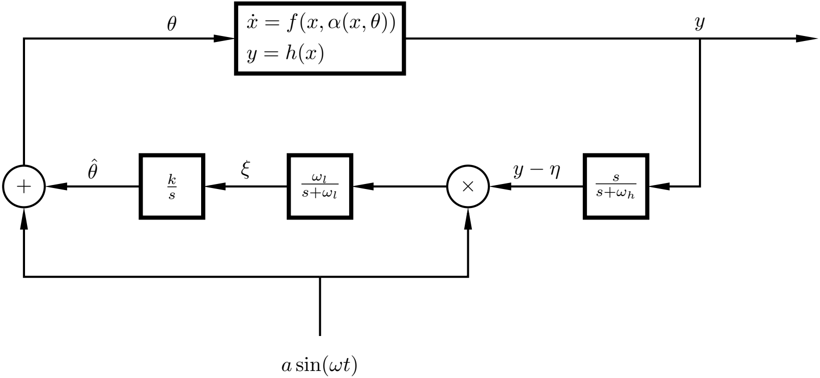

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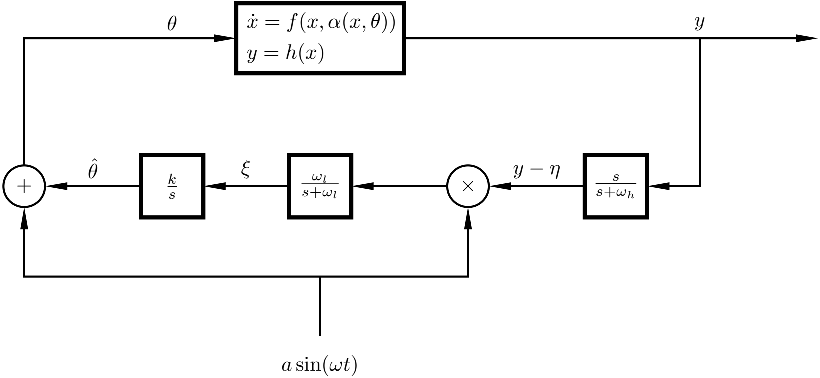

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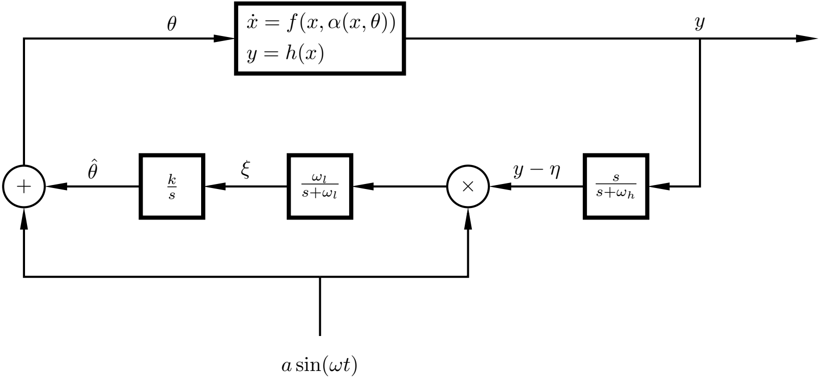

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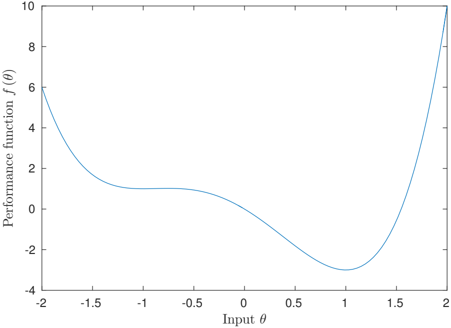
General advise: Keep all parameters small!

A simulation example - the performance function



$$f(\theta) = \theta^4 + \theta^3 - 2\theta^2 - 3\theta$$

- Local minimum $f(-1) = 1$, local maximum $f(-3/4) = 261/256$ and global minimum $f(1) = -3$
- Simulations performed with $\omega_l = \omega_h = 1$, $k = -0.8$ and $\omega = 3$, $a = 0.1$ or 0.3
- Simulations initialized both at $\theta = 0$ and $\theta = -1.5$

Speed of convergence vs resulting oscillations

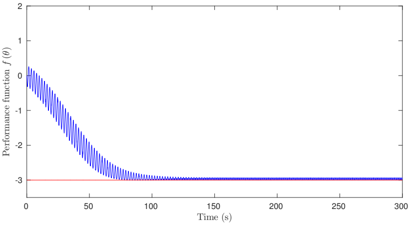


Figure: Simulations performed with perturbation amplitude $a = 0.1$.

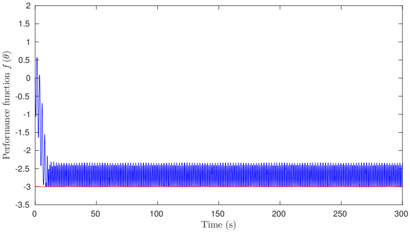


Figure: Simulations performed with perturbation amplitude $a = 0.3$.

Movie time!

PROD. NO
SCENE
TAKE
ROLL
DATE
SOUND
PROD. CO.
DIRECTOR
CAMERAMAN

Reaching the global minimum I

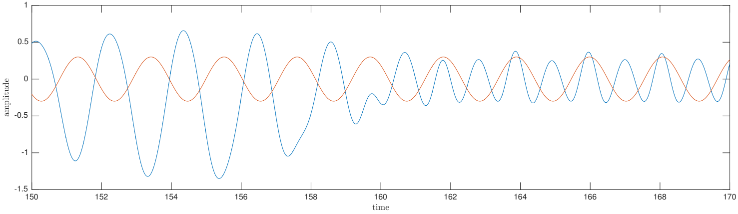


Figure: High-pass filtered output (blue) and perturbation signal (red).

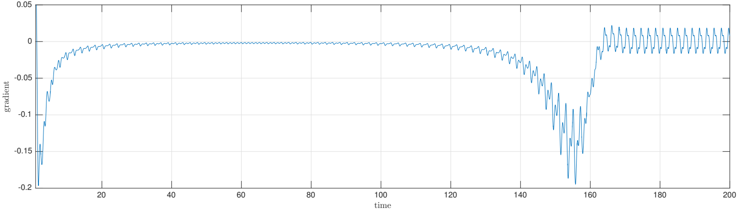
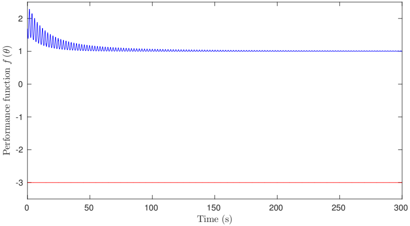


Figure: Estimated gradient over time.

Reaching the global minimum II



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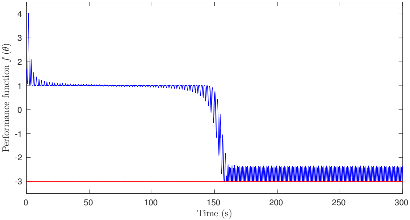

Figure: Simulations performed with perturbation amplitude $a = 0.3$.

Questions?

Class dismissed!