Exercise 4 Poleplacement and PID

1. Use Euclid's algorithm to find all solutions to the equation

$$7x + 5y = 6$$

where x and y are integers.

2. Use Euclid's algorithm to find all solutions to the equation

$$s^2x(s) + (0.5s + 1)y(s) = 1$$

where $x(s)$ and $y(s)$ are polynomials. Use the results to find a solution to the equation

$$s^2 f(s) + (0.5s + 1)g(s) = (s^2 + 2\zeta_c\omega_c s + \omega_c^2)(s^2 + 2\zeta_o\omega_o s + \omega_o^2)$$

such that the polynomials $f(s)$ and $g(s)$ have degrees 2 and 1 respectively. Compare with the calculations in the Poleplacement lecture.

3. Discuss what interface a (Matlab/Julia/Python,...) routine for doing 2DOF SISO polynomial design should have.

Ideas: One should be able to request a given factor of the numerator or denominators, e.g. enforcing integral action. One should be able to cancel process zeros or poles. Some inspiration can be found by studying the PPBOX described in the internal report TFRT-7454 by Gustafsson et.al.

4. Develop such a routine. Cooperation is encouraged.

5. Show that the following Matlab program can be used to generate the stability boundary for PI control of a process with the transfer function $P(s)$.

```
% stabregion PI.m
% Segments of the curve generated by this program
% is the boundary of the stability region for a
% process with transfer function P(s)
% The transfer function is defined symbolically
% for example as P='exp(-sqrt(s))';
% kja 091001

dw = (w2-w1)/1000; w=w1:dw:w2; s=i*w;
Pv = eval(P); r = abs(Pv); phi = angle(Pv);
kp = -cos(phi)./r; ki = -w.*sin(phi)./r;
plot(kp,ki,'Linewidth',1.5)
xlabel('kp'); ylabel('ki');
```

Plot the stability region for PI control for the system

$$P = 100(s + 6)^2/(s(s + 1)^2(s + 50)^2).$$

Also modify the program to give the stability region for a PID controller $k_p + k_i/s + k_d s$ with given derivative gain k_d.

6. Consider the interpretation of the PI controller as a loopshaping technique where one point on the open transfer function nyquist curve is mapped to a given value of the loop transfer function L. Let the process be $P(s)$ and the selected frequency be ω_p, introduce $P(i\omega_p) = r_P e^{\phi_P}$ and let the desired point value be $L(i\omega_p) = r_L e^{i\omega_p}$. Show that the parameters of the PI controller are given by

$$k_p = \frac{r_L}{r_P} \cos\left(\phi_P - \phi_L\right), \qquad k_i = \frac{r_L \omega_P}{r_P} \sin\left(\phi_P - \phi_L\right).$$

Discuss how the chosen point should depend on the process.

Illustrate your results on the process

$$P(s) = \frac{1}{(s+1)^4}.$$

Let the desired point be defined by the phase margin $\varphi_m = 60°$. Use the ILM tool PIDLoopshaping to explore the effects of different choices of ω_p guided by the insights you developed by the analysis of the first part.