Exercise Session 3

1. Describe your results on Handin 2.

2. a) Show that state feedback control $u = -L\hat{x} + l_r y_r$, where $\hat{x}$ is given by a Kalman filter, can be written as

$$U(s) = -C_{fb}(s)Y(s) + C_{ff}(s)Y_r(s)$$

with

$$\begin{aligned} C_{fb}(s) &= L(sI - A + BL - KC)^{-1}K \\ C_{ff}(s) &= (I - L(sI - A + BL - KC)^{-1}B)l_r \\ &= (I + L(sI - A + KC)^{-1}B)^{-1}l_r \end{aligned}$$

b) Show that the controller above can be written as

$$R(s)U = -S(s)Y + T(s)Y_r$$

with

$$\begin{aligned} R(s) &= \det(sI - A + BL + KC), \\ T(s) &= \text{const} \cdot \det(sI - A + KC). \end{aligned}$$

Hence the observer polynomial equals the zero polynomial of the feedforward controller $C_{ff}(s)$.

3. Skiss the level curves in the complex plane of $\text{Re}(\frac{1}{z}) = c$.

4. Consider control of the resonant system

$$P(s) = \frac{\omega_0^2}{s^2 + 2\zeta_0\omega_0 s + \omega_0^2}$$

with $\omega_0 = 1, \zeta_0 = 0.01$. Try to find a controller achieving as low M_{cs} as possible while giving $M_s < 2$ and $M_{ps} < 0.1$. Plot the GOF and compare with PID control as presented at the pole placement lecture.

5. Use QFT design on the uncertain system

$$P(s) = \frac{k}{(s+a)(s+b)}, \quad k \in [1, 10], \quad a \in [1, 5], \quad b \in [20, 30].$$

The specifications are

- $\max_\omega |T(i\omega)| < 1.2, \qquad \forall\omega$
- $|S(i\omega)| \leq |\frac{0.02s^3+1.28s^2+14.96s+48}{s^2+14.4+169}|_{s=i\omega}$ for $\omega \in [0, 10]$ rad/s
- $|PS(i\omega)| \leq 0.01, \qquad \omega \in [0, 50]$ rad/s

Hint: This is Example 1 in the manual for the QFTIT tool.

6. Leftover from exercise 2: Derive the limitations for the sensitivity for systems with an unstable complex pole pair $x_0 \pm iy_0$in the RHP

$$\omega_{sc} \geq \frac{M_t}{M_t^2 - 1}\left(x_0 + \sqrt{M_t^2 x_0^2 + (M_t^2 - 1)y_0^2}\right)$$

$$y_0 = 0, \omega_{sc} \geq \frac{M_t x_0}{M_t - 1}$$

$$x_0 = 0, \omega_{sc} \geq \frac{M_t y_0}{\sqrt{M_t^2 - 1}}$$