Control System Design - Interaction

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Interaction

1. Introduction
2. Interaction of Simple Loops
3. Multivariable zeros
4. Decoupling
5. Parallel Systems
6. Summary

Theme: When the wires get crossed

Introduction

- **Centralized or distributed**
- The process control experience
  - A few important variables were controlled using the single loop paradigm: one sensor, one actuator and one controller
  - Loops were added, feedforward, cascade, midrange, selectors
  - Not obvious how to associate sensors and actuators - The pairing problem
- Early process control systems were distributed and decentralized, centralization came with computer control
- The state-feedback paradigm - centralized control
- Decentralized control popular for large systems
- What happens when loops are interacting?
- Interaction measures - The relative gain array RGA, Singular values
- Decoupling: static, dynamic (different time scales), different physical mechanisms, mass balance, energy balance

A Large Process

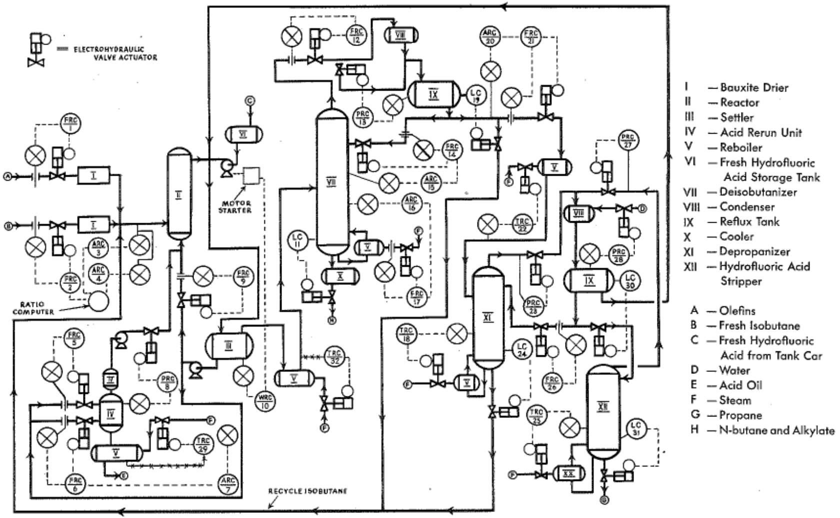


Figure 13–6. Automatic control system for Perco motor fuel alkylation process.

The Shell Standard Control Problem

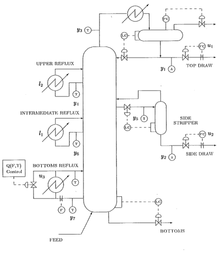


Figure A.1: Diagram of a Heavy Oil Fractionator (Shell Control Problem).

$$\frac{K e^{-\theta s}}{\tau s + 1}$$

Units for θ, τ are in minutes.

	TOP DRAW	SIDE DRAW	BOTTOMS REFLUX DUTY	INTER. REFLUX DUTY	UPPER REFLUX DUTY
TOP END POINT	$K = 4.05$ $\tau = 50$ $\theta = 27$	$K = 1.77$ $\tau = 60$ $\theta = 28$	$K = 5.88$ $\tau = 50$ $\theta = 27$	$K = 1.20$ $\tau = 45$ $\theta = 27$	$K = 1.44$ $\tau = 40$ $\theta = 27$
SIDE END POINT	$K = 5.39$ $\tau = 50$ $\theta = 18$	$K = 5.72$ $\tau = 60$ $\theta = 14$	$K = 6.90$ $\tau = 40$ $\theta = 15$	$K = 1.52$ $\tau = 25$ $\theta = 15$	$K = 1.83$ $\tau = 20$ $\theta = 15$
TOP TEMP	$K = 3.66$ $\tau = 9$ $\theta = 2$	$K = 1.65$ $\tau = 30$ $\theta = 20$	$K = 5.53$ $\tau = 40$ $\theta = 2$	$K = 1.16$ $\tau = 11$ $\theta = 0$	$K = 1.27$ $\tau = 6$ $\theta = 0$
UPPER REFLUX TEMP	$K = 5.92$ $\tau = 12$ $\theta = 11$	$K = 2.54$ $\tau = 27$ $\theta = 12$	$K = 8.10$ $\tau = 20$ $\theta = 2$	$K = 1.73$ $\tau = 5$ $\theta = 0$	$K = 1.79$ $\tau = 19$ $\theta = 0$
SIDE DRAW TEMP	$K = 4.13$ $\tau = 8$ $\theta = 5$	$K = 2.38$ $\tau = 19$ $\theta = 7$	$K = 6.23$ $\tau = 10$ $\theta = 2$	$K = 1.31$ $\tau = 2$ $\theta = 0$	$K = 1.26$ $\tau = 22$ $\theta = 0$
INTER. REFLUX TEMP	$K = 4.06$ $\tau = 13$ $\theta = 8$	$K = 4.18$ $\tau = 33$ $\theta = 4$	$K = 6.53$ $\tau = 9$ $\theta = 1$	$K = 1.19$ $\tau = 19$ $\theta = 0$	$K = 1.17$ $\tau = 24$ $\theta = 0$
BOTTOMS REFLUX TEMP	$K = 4.38$ $\tau = 33$ $\theta = 20$	$K = 4.42$ $\tau = 44$ $\theta = 22$	$K = 7.20$ $\tau = 19$ $\theta = 0$	$K = 1.14$ $\tau = 27$ $\theta = 0$	$K = 1.26$ $\tau = 32$ $\theta = 0$

Paper Machine Control

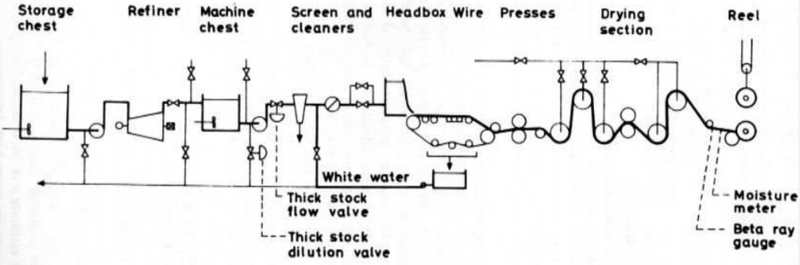
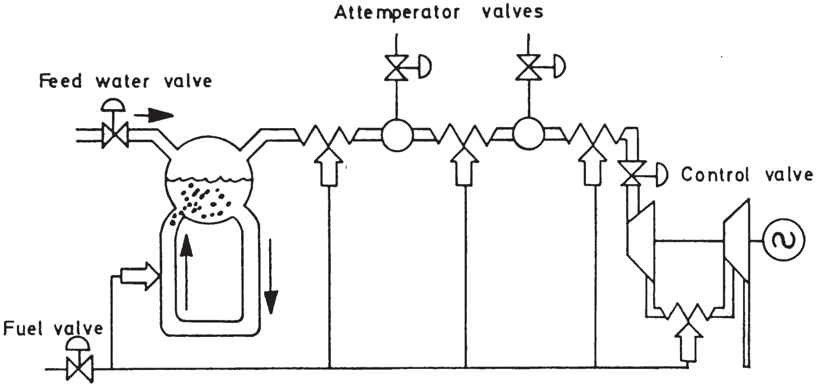


Fig. 6.9. Simplified diagram of a kraft paper machine.

- Primary quality: Basis weight and moisture
- Head box control: Pressure and flow rate

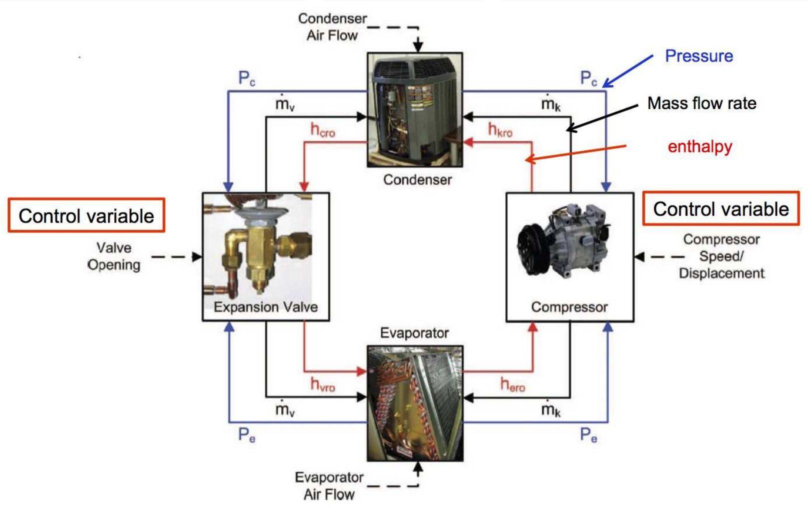
Boiler Control



Schematic diagram of the boiler-turbine unit.

Who is the boss? Boiler leading turbine or turbine leading boiler.

A Small Process – Chiller



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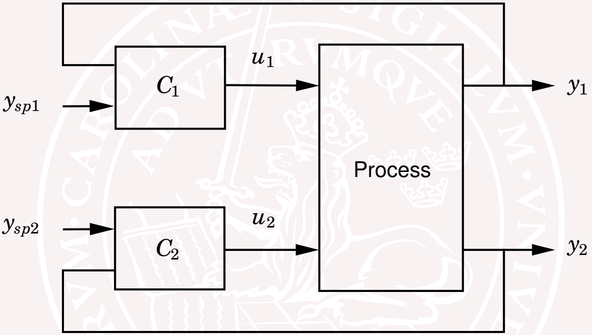
Interaction of Simple Loops

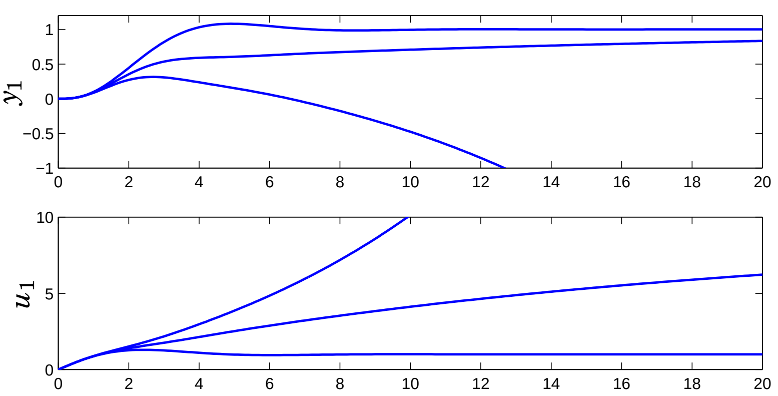


$$Y_1(s) = p_{11}(s)U_1(s) + p_{12}U_2(s)$$
$$Y_2(s) = p_{21}(s)U_1(s) + p_{22}U_2(s),$$

What happens when the controllers are tuned individually?

An Example

Controller C_1 is a PI controller with gains $k_1 = 1$, $k_i = 1$, and the C_2 is a proportional controller with gains $k_2 = 0$, 0.8, and 1.6.



The second controller has a major impact on the first loop!

Analysis

$$Y_1(s) = \frac{1}{(s+1)^2} U_1(s) + \frac{2}{(s+1)^2} U_2(s)$$
$$Y_2(s) = \frac{1}{(s+1)^2} U_1(s) + \frac{1}{(s+1)^2} U_2(s).$$

P-control of second loop $U_2(s) = -k_2 Y_2(s)$ gives

$$Y_1(s) = g_{11}^{cl}(s) U_1(s) = \frac{s^2 + 2s + 1 - k_2}{(s+1)^2(s^2 + 2s + 1 + k_2)} U_1(s).$$

The gain k_2 in the second loop has a significant effect on the dynamics in the first loop. The static gain is

$$g_{11}^{cl}(0) = \frac{1 - k_2}{1 + k_2}.$$

Notice that the gain decreases with increasing k_2 and becomes negative for $k_2 > 1$.

Bristol's Relative Gain

- A simple way of measuring interaction based on static properties
- Edgar H. Bristol On a new measure of interaction for multivariable process control IEEE TAC 11(1967) 133–135
- Idea: What is effect of control of one loop on the steady state gain of another loop?
- Consider one loop when the other loop is under perfect control

$$
\begin{aligned}
Y_1(s) &= p_{11}(s)U_1(s) + p_{12}U_2(s) \\
0 &= p_{21}(s)U_1(s) + p_{22}U_2(s).
\end{aligned}
$$

Bristol's Relative Gain

Consider the first loop $u_1 \to y_1$ when the second loop is in perfect control ($y_2 = 0$)

$$\begin{aligned} Y_1(s) &= p_{11}(s)U_1(s) + p_{12}U_2(s) \\ 0 &= p_{21}(s)U_1(s) + p_{22}U_2(s). \end{aligned}$$

Eliminating $U_2(s)$ from the second equation gives

$$Y_1(s) = \frac{p_{11}(s)p_{22}(s) - p_{12}(s)p_{21}(s)}{p_{22}(s)}U_1(s).$$

The ratio of the static gains of loop 1 when the second loop is open and closed is

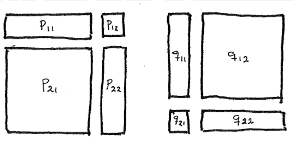
$$\lambda = \frac{p_{11}(0)p_{22}(0)}{p_{11}(0)p_{22}(0) - p_{12}(0)p_{21}(0)}.$$

Parameter λ is called *Bristol's interaction index*

Many Loops

Assume n inputs and n outputs. Pick an input output pair and relabel so that the output is y_1, let the remaining outputs be $y_2 = 0$. Let the input be u_2 and the remaining inputs be u_1.

$$
\begin{aligned}
y_1 &= p_{11}u_1 + p_{12}u_2 \\
0 &= p_{21}u_1 + p_{22}u_2
\end{aligned}
$$



Solving for y_1 gives (notice the Schur complement)

$$
y_1 = (p_{12} - p_{11}p_{21}^{-1}p_{22})u_2, \qquad r_{12} = \frac{p_{12}}{p_{12} - p_{11}p_{21}^{-1}p_{22}}
$$

Compare

$$
P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}, \qquad P^{-1} = Q = \begin{pmatrix} \cdots & \cdots \\ p_{12} - p_{11}p_{21}^{-1}p_{22} & \cdots \end{pmatrix}
$$

The relative gain array is $R = P \bullet P^{-T}$ (Hadamard or Schur product)

Bristol's Relative Gain Array

Let $P(s)$ be an $n \times n$ matrix of transfer functions. The relative gain array is

$$\Lambda = P(0) \bullet P^{-T}(0)$$

The product is element by element product (Schur or Hadamard product). Properties

- $(A \bullet B)^T = A^T \bullet B^T$
- P diagonal gives $\Lambda = I$

Insight and use

- A measure of static interactions for square systems which tells how the gain in one loop is influenced by perfect feedback on all other loops
- Dimension free. Row and column sums are 1.
- Negative elements correspond to sign reversals due to feedback of other loops

Pairing

When designing complex systems loop by loop we must decide what measurements should be used as inputs for each controller. This is called the *pairing* problem. The choice can be governed by physics but the relative gain can also be used

Consider the previous example

$$P(0) = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, \qquad P^{-1}(0) = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$$

$$\Lambda = P(0) \bullet P^{-T}(0) = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix},$$

- Negative sign indicates the sign reversal found previously
- Better to use reverse pairing, i.e. let u_2 control y_1

Pairing ...

Consider

$$P(s) = \begin{pmatrix} \frac{1}{(s+1)^2} & \frac{2}{(s+1)^2} \\ \frac{1}{(s+1)^2} & \frac{1}{(s+1)^2} \end{pmatrix}$$

Introducing the feedback $u_1 = -k_2 y_2$ gives

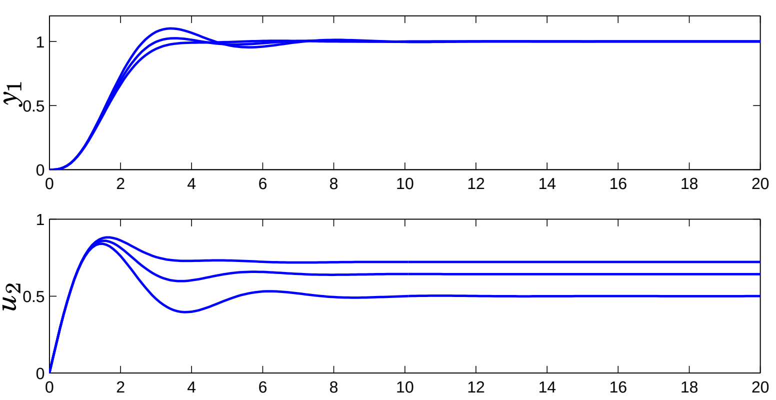
$$Y_1(s) = g_{12}^{cl}(s)U_2(s) = \frac{2s^2 + 4s + 2 + k_2}{(s+1)^2(s^2 + 2s + 1 + k_2)} U_2(s),$$

Zero frequency gain decreases from 2 to 1 when k_2 ranges from 0 to ∞.

Discuss how dynamics changes with k_2!

Use rootlocus!

Step Responses with Reverse Pairing



- $U_2 = \left(1 + \frac{1}{s}\right)(Y_{sp1} - Y_1)$
- $u_1 = -k_2 y_2$ with $k_2 = 0$, 0.8, and 1.6.

Summary for 2×2 Systems

$\lambda = 1$ No interaction

$\lambda = 0$ Closed loop gain $u_1 \rightarrow y_1$ is zero. Pair u_1 and y_2 instead

$0 < \lambda < 1$ Closed loop gain $u_1 \rightarrow y_1$ is larger than open loop gain. Interaction strongest for $\lambda = 1$

$\lambda > 1$ Closed loop gain $u_1 \rightarrow y_1$ is smaller than open loop gain. Interaction increases with increasing λ. Very difficult to control both loops independently if λ is very large.

$\lambda < 0$ The closed loop gain $u_1 \rightarrow y_1$ has different sign than the open loop gain. Opening or closing the second loop has dramatic effects. The loops are counteracting each other. Such pairings should be avoided for decentralized control and the loops should be controlled jointly as a multivariable system.

Singular Values

Let A be an $k \times n$ matrix whose elements are complex variables. The singular value decompostion of the matrix is

$$A = U \Sigma V^*$$

where $*$ denotes transpose and complex conjugation, U and V are unitary matrices ($UU^* = I$ and $VV^* = I$ is. The matrix Σ is a $k \times n$ matrix such that $\Sigma_{ii} = \sigma_i$ and all other elements are zero. The elements σ_i are called singular values. The largest $\overline{\sigma} = \max_i \sigma_i$ and smallest $\underline{\sigma} = min_i \sigma$ singular values are of particular interest. The number $\bar{\sigma}/\underline{\sigma}$ is called the condition number. The singular values are the square roots of the eigenvalues of A^*A.

Example: A real 2×2 matrix can be written as

$$A = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -sin\theta_2 & \cos\theta_2 \end{pmatrix}$$

Singular Decomposition $A = U\Sigma V^*$

- The columns u_i of U represent the output directions
- The columns v_i of V represent the input directions
- We have $AV = U\Sigma$, or $Av_i = \sigma_i u_i$. An input in the direction v_i thus gives the output $\sigma_i u_i$, i.e. in the direction u_i
- Since the vectors u_i and v_i are of unit length the gain of A for the input u_i is σ_i
- The largest gain is $\bar{\sigma} = \max_i \sigma_i$
- There are efficient numerical algorithms `svd` in Matlab
- Singular values can be applied to nonsquare matrices
- A natural way to define gain for matrices A and transfer function matrices $G(s)$

$$\text{gain} = \max_v \frac{||Av||}{||v||} = \overline{\sigma}(A), \qquad \text{gain} = \max_\omega \overline{\sigma}(G(i\omega))$$

Interaction Analysis

Consider a system with the scaled zero frequency gain

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0.48 & 0.90 & -0.006 \\ 0.52 & 0.95 & 0.008 \\ 0.90 & -0.95 & 0.020 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

Relative gain array

$$\Lambda = \begin{pmatrix} 0.7100 & -0.1602 & 0.4501 \\ -0.3557 & 0.7925 & 0.5632 \\ 0.6456 & 0.3677 & -0.0133 \end{pmatrix}$$

Singular values: $\sigma_1 = 1.6183$, $\sigma_2 = 1.1434$ and $\sigma_3 = 0.0097$. Condition number $\kappa = 166$. Only two outputs can be controlled. What variables should be chosen?

Interaction Analysis

We have $y = U\,S\,V^T$. How to pick two input output pairs

$$SV^T = \begin{pmatrix} -0.088 & -1.616 & 0.010 \\ 1.142 & -0.062 & 0.018 \\ -0.000 & 0.000 & 0.010 \end{pmatrix} \quad U = \begin{pmatrix} -0.571 & 0.377 & -0.729 \\ -0.604 & 0.409 & 0.684 \\ 0.556 & 0.831 & -0.007 \end{pmatrix}$$

The matrix SV^T shows that u_1 and u_2 are obvious choices of inputs. There are two choices of outputs: y_1, y_3 or y_2, y_3, based on the angles between rows, y_1, y_2 is not a good choice because the corresponding rows of US are almost parallel. Singular values:

Selection $y_1, y_3 \;\leftarrow\; u_1, u_2$ Condition number $\kappa = 1.51$

$$\Lambda = \begin{pmatrix} 0.3602 & 0.6398 \\ 0.6398 & 0.3602 \end{pmatrix}$$

Selection $y_2, y_3 \;\leftarrow\; u_1, u_2$ Condition number $\kappa = 1.45$

$$\Lambda = \begin{pmatrix} 0.3662 & 0.6338 \\ 0.6338 & 0.3662 \end{pmatrix}$$

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Zeros of Multivariable Systems

Roughly speaking transmission zeros are values of s where the transmission of the signal e^{st} is blocked

$$Y(s) = P(s)\, v\, e^{st}, \qquad 0 = P(s)\, v$$

There is a nontrivial v only if the matrix $P(s)$ is singular. For a square system the zeros s_i are given by

$$\det P(s) = 0$$

and the *zero directions* are the corresponding right eigenvectors of $P(s_i)$.

Rosenbrock's Example

There is a nice collection of linear multivariable systems with interesting properties. Here is one of them

$$P(s) = \begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+3} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{pmatrix}$$

Very benign subsystems. Relative gain array

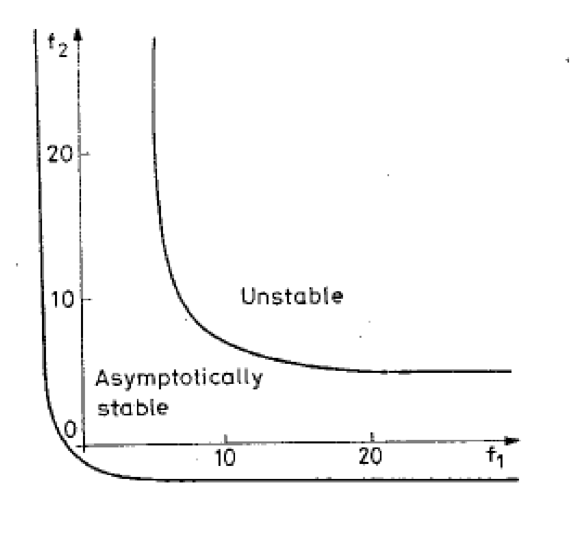
$$R = \begin{pmatrix} 1 & 2/3 \\ 1 & 1 \end{pmatrix} \bullet \begin{pmatrix} 3 & -3 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix},$$

The transmission zeros are given by

$$\det P(s) = \frac{1}{s+1}\Big(\frac{1}{s+1} - \frac{2}{s+3}\Big) = \frac{1-s}{(s+1)^2(s+3)} = 0.$$

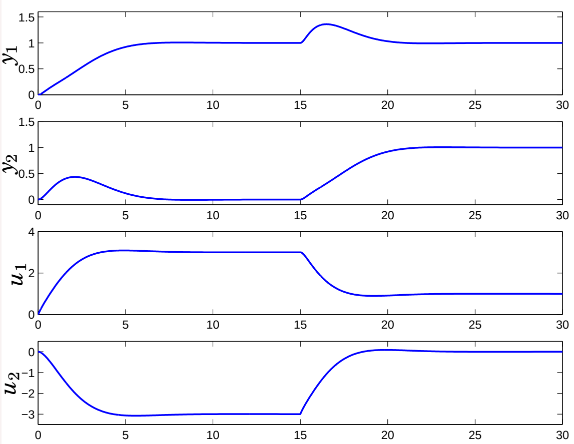
Difficult to control the system with gain crossover frequencies larger than $\omega_{gc} = 0.5$.

Stability Region - P in Both Loops



Discuss commissioning and windup!

Rosenbrock’s Example



PI controllers with $k_p = 2$ and $k_i = 2$ in both loops. Systems becomes unstable if gains are increased by a factor of 3.

Interactions Can be Beneficial

$$P(s) = \begin{pmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{pmatrix} = \begin{pmatrix} \dfrac{s-1}{(s+1)(s+2)} & \dfrac{s}{(s+1)(s+2)} \\ \dfrac{-6}{(s+1)(s+2)} & \dfrac{s-2}{(s+1)(s+2)} \end{pmatrix}.$$

The relative gain array

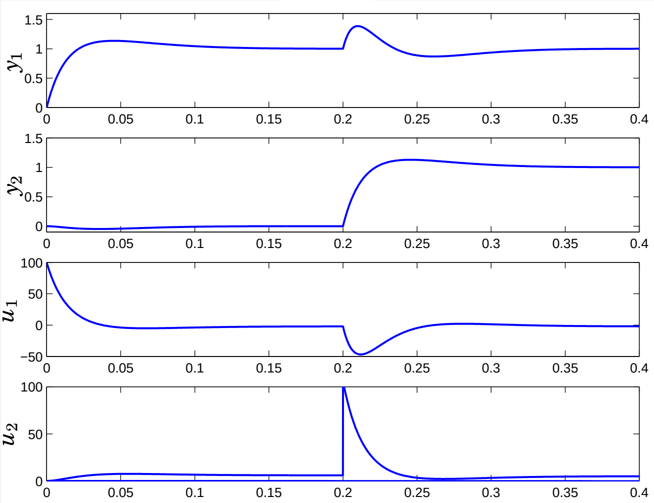
$$R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

Transmission zeros

$$\det P(s) = \frac{(s-1)(s-2)+6s}{(s+1)^2(s+2)^2} = \frac{s^2+4s+2}{(s+1)^2(s+2)^2}$$

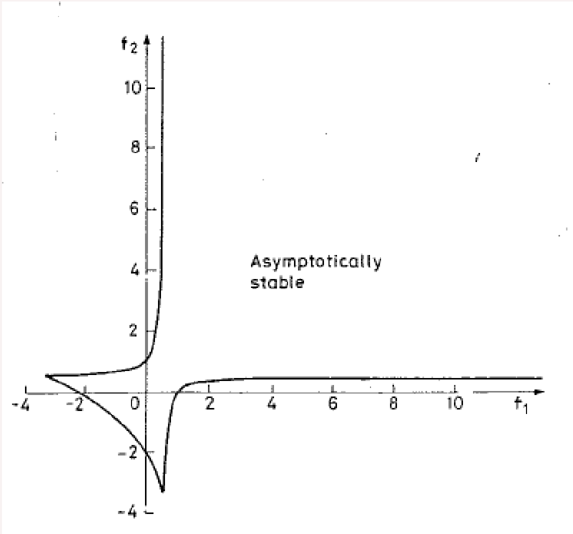
Difficult to control individual loops fast because of the zero at $s = 1$. Since there are no multivariable zeros in the RHP the multivariable system can easily be controlled fast but the system is not robust to loop breaks.

Interactions Can be Beneficial ...



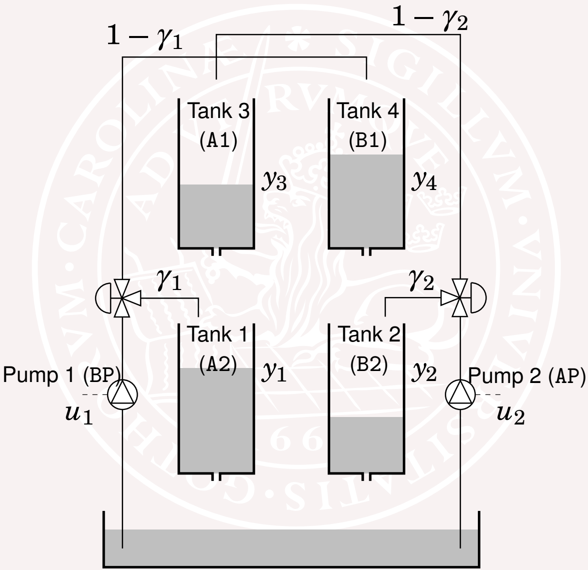
PI controllers with gains $k = 100$ and $k_i = 2000$ in both loops

Stability Region - P in Both Loops



Discuss commissioning and windup!

The Quadruple Tank



Transfer Function of Linearized Model

Transfer function from u_1, u_2 to y_1, y_2

$$P(s) = \begin{pmatrix} \dfrac{\gamma_1 c_1}{1+sT_1} & \dfrac{(1-\gamma_2)c_1}{(1+sT_1)(1+sT_3)} \\ \dfrac{(1-\gamma_1)c_2}{(1+sT_2)(1+sT_4)} & \dfrac{\gamma_2 c_2}{1+sT_2} \end{pmatrix}$$

Transmission zeros

$$\det P(s) = \frac{(1+sT_3)(1+sT_4) - \dfrac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}}{(1+sT_1)(1+sT_2)(1+sT_3)(1+sT_4)}$$

- No interaction if $\gamma_1 = \gamma_2 = 1$
- Minimum phase if $1 \leq \gamma_1 + \gamma_2 \leq 2$
- Nonminimum phase if $0 < \gamma_1 + \gamma_2 \leq 1$.
- Intuition?

The Quadruple Tank

Zero frequency gain

$$P(0) = \begin{pmatrix} \gamma_1 c_1 & (1-\gamma_2)c_1 \\ (1-\gamma_1 c_2 & \gamma_2 c_2 \end{pmatrix}$$

Relative gain array

$$R = P(0) \bullet P(0)^{-1} = \begin{pmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{pmatrix}$$

where

$$\lambda = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - 1}$$

Recall RHP zero if $\gamma_1 + \gamma_2 < 1$. Physical interpretation!

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Decoupling

Simple idea: Find a compensator so that the system appears to be without coupling.

Many versions

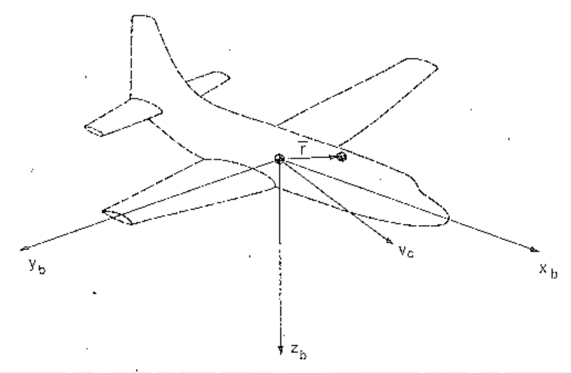
- Input decoupling $Q = PD$ or output decoupling $Q = DP$
- Conventional (Feedforward)
- Inverse (Feedback)
- Static

Important to consider windup, manual control and mode switches.

- Keep the decentralized philosophy

Decoupling may not be useful – Flight Control

Think about what happens when the plane turns!



An autopilot is typically split into two subsystems:

- lateral (pitch)
- longitudinal (yaw and roll)

Feedforward (Conventional) Decoupling

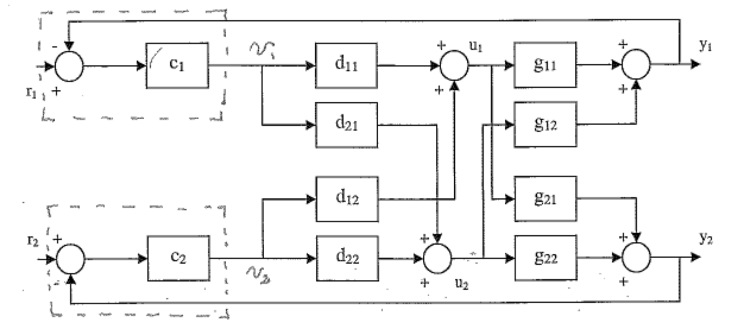
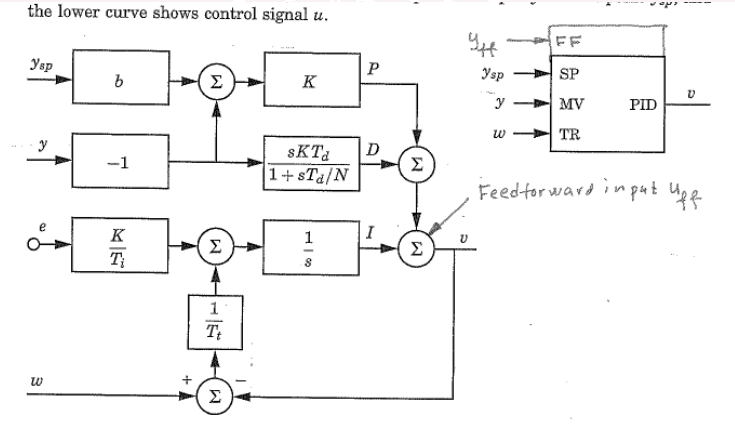


Figure 1 – Conventional decupling control system of a TITO process

$$D = P^{-1}(s)Q(s) = \frac{1}{\det P(s)} \begin{pmatrix} p_{22}(s) & -p_{12}(s) \\ -p_{21}(s) & p_{11}(s) \end{pmatrix} Q(s)$$

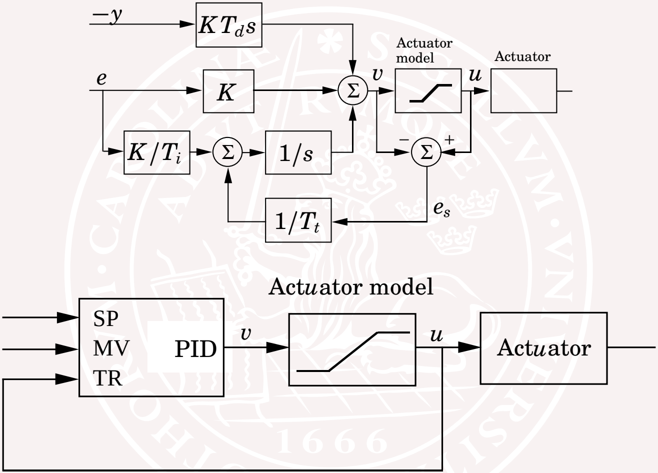
$Q(s)$ is the desired transfer function of decoumpled process. This decoupler has difficulties to deal with anti-windup, manual control and mode changes (auto-tuning). Controller C_1 has no information about v_2.

PID Controller with Tracking and Feedforward



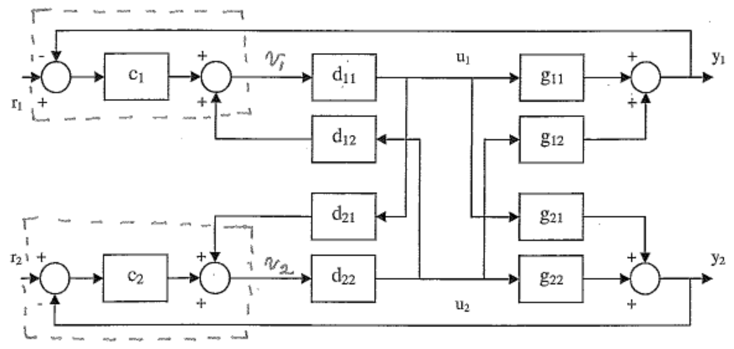
No tracking if $w = v$!

Anti-windup for Controller with Tracking Mode



- Notice that there is no tracking effect if $u = v$!
- The tracking input can be used in many other ways

Feedback (Inverted) Decoupling

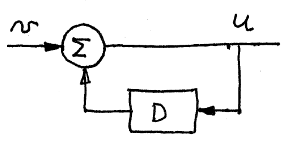


Simple decoupler, easy to deal with anti-windup, manual control and mode changes (auto-tuning) if $d_{11} = d_{22} = 1$. Why?

Inverted (Feedback) Decoupling

$$u = v + Du$$
$$(I - D)u = v$$
$$u = (I - D)^{-1}v$$
$$Q = P(I - D)^{-1}$$
$$P = Q - QD$$

Blockdiagram vector case



Easy to solve for D_{fb} also for systems with many inputs and outputs.
Example 2×2, pick $Q = \mathrm{diag}(p_{11}, p_{22})$, why?

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 0 \\ 0 & p_{22} \end{pmatrix} - \begin{pmatrix} p_{11} & 0 \\ 0 & p_{22} \end{pmatrix} \begin{pmatrix} 0 & d_{12} \\ d_{21} & 0 \end{pmatrix}$$

Hence

$$d_{12} = -\frac{p_{12}}{p_{11}} \qquad d_{21} = -\frac{p_{21}}{p_{22}}$$

The Wood-Berry Distillation Column

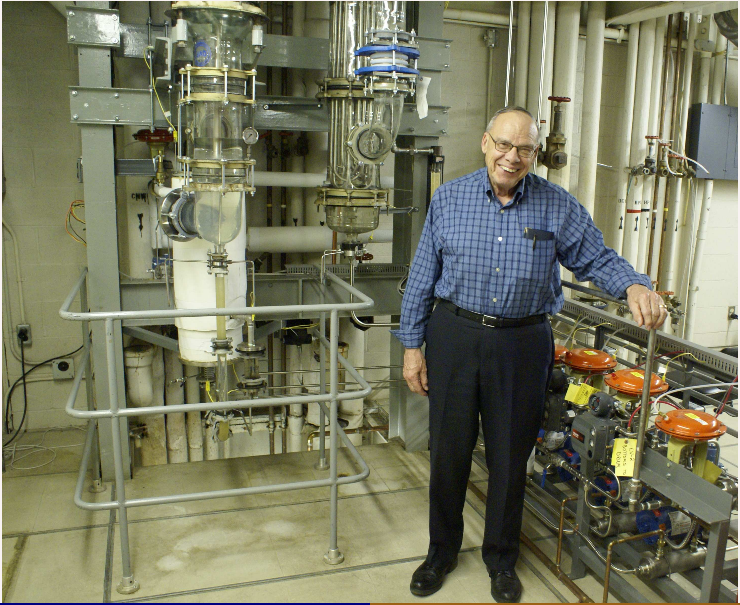
$$P(s) = \begin{pmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.60e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{pmatrix}.$$

Choose $d_{11} = d_{22} = 1$

$$\begin{aligned} d_{11} &= 1 \\ d_{12} &= -\frac{p_{12}}{p_{11}} = 0.68\,\frac{16.7s+1}{21.0s+1}\,e^{-2s} \\ d_{21} &= -\frac{p_{21}}{p_{22}} = 0.34\,\frac{14.4s+1}{10.9s+1}\,e^{-4s} \\ d_{22} &= 1 \end{aligned}$$

Easy to implement. What can go wrong?

Wood and His Column – U Alberta



Properties of Inverted (Feedback) Decoupling

- Simple decoupler even for systems with many inputs and many outputs
- Easy to deal with anti-windup, manual control and other mode changes (auto-tuning)
- Decoupler may be noncausal (pure predictor). Try different pairing or add extra time delay in d_{ii}.
- Since it is a feedback coupling there may be instabilities

Interaction

1. Introduction
2. Interaction of Simple Loops
3. Multivariable zeros
4. Decoupling
5. Parallel Systems
6. Summary

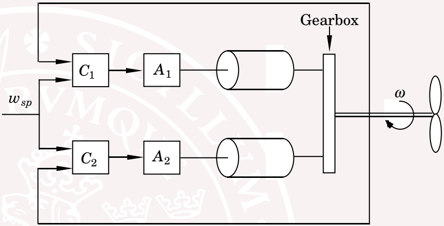
Theme: When the wires get crossed

Systems with Parallel Actuation

- Motor drives for papermachines and rolling mills
- Trains with several motors or several coupled trains
- Power systems
- Electric cars

A Prototype Example

$$J\frac{d\omega}{dt}+D\omega = M_1+M_2-M_L,$$



Proportional control

$$M_1 = M_{10} + K_1(\omega_{sp} - \omega)$$
$$M_2 = M_{20} + K_2(\omega_{sp} - \omega)$$

The proportional gains tell how the load is distributed

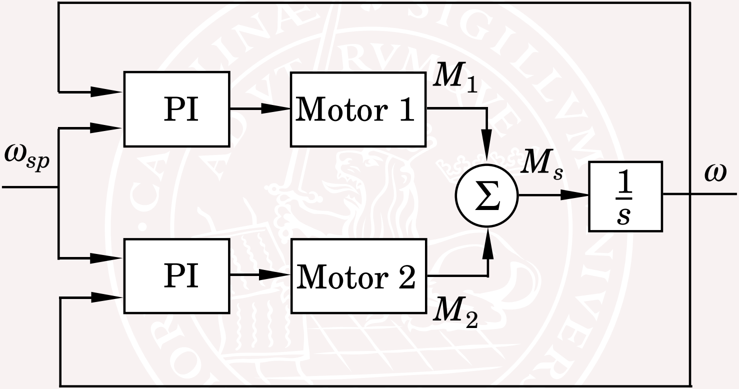
$$J\frac{d\omega}{dt} + (D + K_1 + K_2)\omega = M_{10} + M_{20} - M_L + (K_1 + K_2)\omega_{sp}.$$

A first order system with time constant $T = J/(D + K_1 + K_2)$

Discuss response speed, damping and steady state

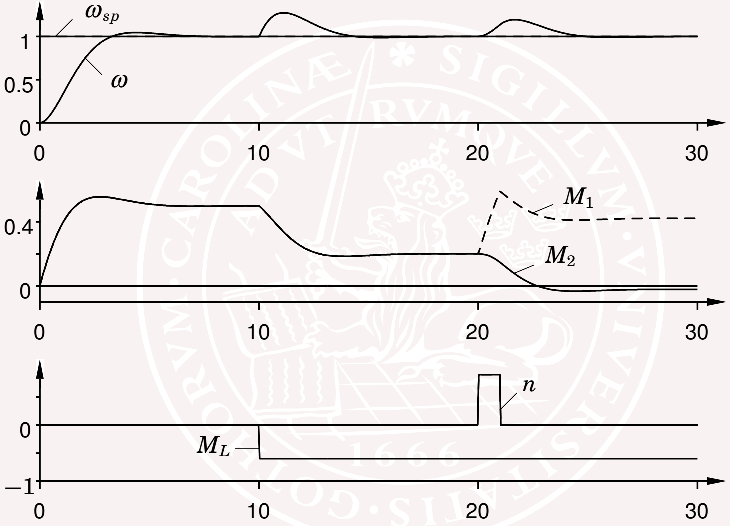
$$\omega = \omega_0 = \frac{K_1 + K_2}{D + K_1 + K_2}\,\omega_{sp} + \frac{M_{10} + M_{20} - M_L}{D + K_1 + K_2}.$$

Integral Action?



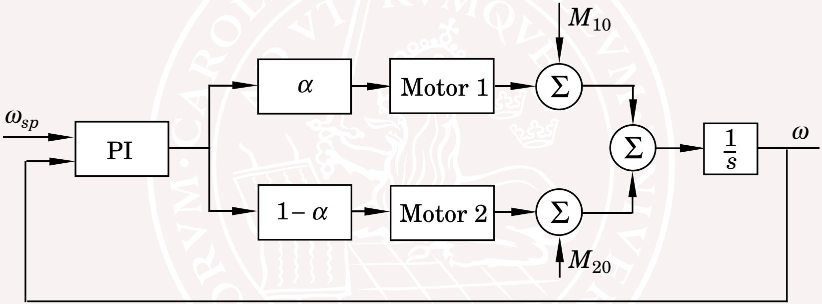
Prototypes for lack of reachability and observability!

Integral Action

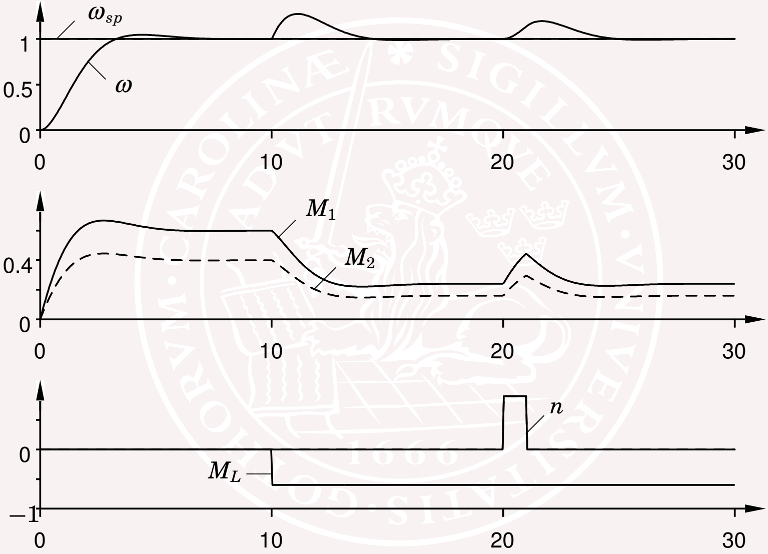


Notice that M_2 is breaking for $t > 22$

Better Integral Action



Better Integral Action?



Power Systems - Massive Parallellism

- Edison's experience

  Two generators with governors having integral action
- Many generators supply power to the net.

  Frequency control

  Voltage control
- Isochronous governors (integral action) and governors with speed-drop (no integral action)

Turbine Governors

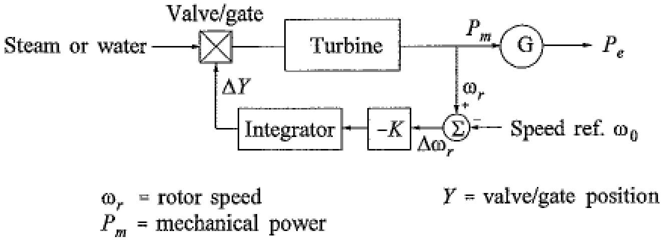


Figure 11.6 Schematic of an isochronous governor

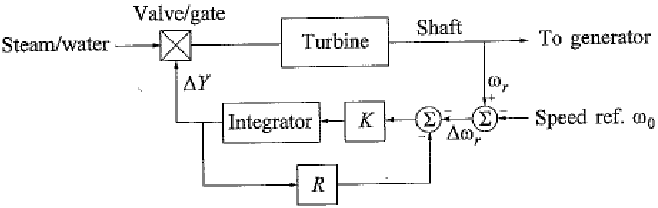


Figure 11.8 Governor with steady-state feedback

Load Sharing - Through Proportional Action

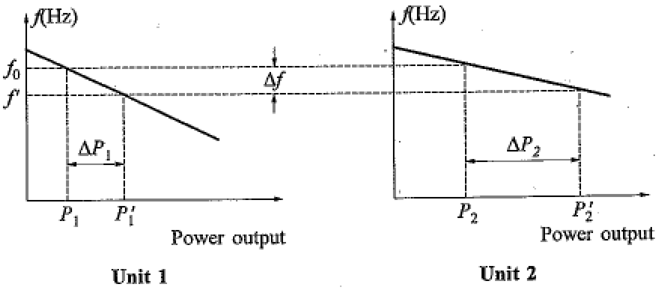


Figure 11.11 Load sharing by parallel units with drooping governor characteristics

In Sweden the final adjustment is made manually

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Summary

- All real systems are coupled
- Relative gain array and singular values give insight
- Never forget process redesign
- Multivariable zeros and zero directions
- Why decouple

  Simple system.
  SISO design, tuning and operation can be used
  What is lost?
- Multivariable design

  Dont forget windup and operational aspects (tuning, manual control, ...)
- Parallel systems

  One integrator only!

To Learn More

- F. G. Shinskey Controlling Multivariable Processes. ISA. Research Triangle Park 1981.
- P. Kundur Power System Stability and Control, 1994
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- M. Morari and E. Zafiriou Robust Process Control. Prentice Hall 1989.