# Exercise session 4

*Internal Stability of LFT. Structured Singular Value $\mu$. Structured Robust Stability and Performance. $\mu$ Synthesis via $D-K$ iterations.*

## Reading Assignment

Read [Zhou] Ch. 10. Optional reading:

- Stability Theory for LFT — [Francis] Ch. 3,4, [Zhou] Ch. 11.
- $\mu$ — [Skogestad,Postlethwaite] Ch. 8.7–8.14 (Many examples!!!).

## Exercises

**E4.1** [Zhou] 10.1

**E4.2** [Zhou] 10.4

**E4.3** [Zhou] 10.9

**E4.4** [Zhou] 10.12

**E4.5** Consider a system $P$ and a controller $K$

$$P(s) = \frac{1}{75s+1}\begin{pmatrix} -87.8 & 1.4 \\ -108.2 & -1.4 \end{pmatrix}, \quad K(s) = \frac{75s+1}{s}\begin{pmatrix} -0.0015 & 0 \\ 0 & -0.075 \end{pmatrix}$$

and a diagonal uncertainty $\Delta = \mathrm{diag}\{\delta_1, \delta_2\}$.

(a) With the help of Robust Toolbox calculate $\mu_\Delta(T)$ $(= \min_D \|DTD^{-1}\|$. Why?) and $\|T\|$ at the frequency $\omega_0 = 0.2$ for $T = KP(I+KP)^{-1}$. Estimate the conservatism.

(b) Analyze $T(j\omega_0)$ and $D_{\min}T(j\omega_0)D_{\min}^{-1}$ and indicate the property that you think most contributes to this difference.

(c) Assume the multiplicative uncertainty model

$$P_\Delta = P(I + W\Delta), \quad W(s) = \frac{s+0.2}{0.5s+1}, \; \|\Delta\|_\infty < 1$$

and the performance criterion to be

$$\|W_p(I + P_\Delta K)^{-1}\|_\infty \leq 1, \quad W_p(s) = \frac{s+0.1}{2s}.$$

1. Test stability robustness ignoring the structure of $\Delta$.
2. Test stability robustness taking into account the structure of $\Delta$.
3. Test nominal performance.
4. Test robust performance taking into account the structure of $\Delta$.

## Hand-In problems:

**H4.1** [Zhou] 10.3

**H4.2** Consider a stable nominal plant $P$ and an uncertainty model

$$P_\Delta = (I + W_1\Delta_1)P + W_2\Delta_2, \quad \|\Delta_i\|_\infty < 1.$$

The robust performance objective is to achieve

$$\|W_3(I + P_\Delta K)^{-1}\|_\infty \leq 1$$

for all $P_\Delta$.

(a) Make a block diagram for the closed loop system showing all weights and uncertain blocks.

(b) Pull out all uncertainties and redraw the block diagram as upper LFT for uncertainties and lower LFT for $K$ with respect to a generalized plant $G$. Determine the generalized plant $G$.

(c) Close $G$ by $K$ and find the resulting closed loop function $M$ (in terms of LFT).

(d) Giva a condition for stability robustness, ignoring the structure of $\Delta$.

(e) Giva a condition for stability robustness, taking into account the structure of $\Delta$.

(f) Repeat last item for robust performance.

(g) Under condition that the plant is SISO find the analytical expressions for robust stability and robust performance.