Exercise 2

Well-posedness and internal stability. Coprime factorization over H_∞. Performance specifications in terms of H_2 and H_∞ norms.

Reading Assignment

Read [Zhou] Ch. 5, 6.1–4. Optional reading:

- Internal stability and performance — [Doyle,Francis,Tannenbaum] Ch. 3, 5.2–4.
- Performance specifications — H. Kwakernaak, "Symmetries in Control Systems Design" in *Trends in control: A European perspective*, A.Isidori (Ed.), 1995.
- Coprime factorization and internal stability — M. Vidyasagar, "Control System Synthesis: A Factorization Approach", MIT Press, 1985.

1. Let $G \in RL_\infty$ be a scalar function. Show that $G \in RH_\infty$ if and only if $y(s) = G(s)u(s)$ defines a linear bounded operator on H_2.
2. Suppose that a plant and a controller are scalar and consider the coprime *polynomial* (!) factorizations of them: $P = \frac{N}{M}$ and $K = \frac{U}{V}$. Prove that the closed-loop system is internally stable if and only if the characteristic polynomial
$$g(s) = M(s)V(s) - N(s)U(s)$$
is stable, i.e. it has no unstable roots.
3. [Zhou] 5.4
4. [Zhou] 5.5
5. [Zhou] 5.6
6. To understand how the sensitivity function $S = \frac{1}{1-PK}$ changes with respect to an uncertainty in the plant P estimate the relative derivative of S as
$$\lim_{\Delta P \to 0} \frac{\Delta S/S}{\Delta P/P}.$$

Hand-In problems:

1. [Zhou] 5.8
2. [Zhou] 6.8