Session 2

Transition matrix properties. Change of coordinates. Periodic Systems.

Reading Assignment

Rugh Ch 5 (and Ch 21).

Exercise 2.1 = Rugh 4.3
Exercise 2.2 = Rugh 4.4
Exercise 2.3 = Rugh 4.6
Exercise 2.4 = Rugh 4.9 (you don't have to use the hint)
Exercise 2.5 = Rugh 5.13
Exercise 2.6 = Rugh 5.14
Exercise 2.7 = Rugh 20.1
Exercise 2.8 = Rugh 20.12

Exercise 2.9 Is it possible for a time-varying system $\dot{x}(t) = A(t)x(t)$ to have **all** its eigenvalues in the right half plane and also be stable in the sense that $\|\Phi(t, t_0)\| \to 0, \;\; t \to \infty$.

Exercise 2.10 The Wronski-determinant of the functions $u(t)$ and $v(t)$ is definied by

$$W(u, v) = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$$

The functions are called *independent* in an interval if $W \neq 0$ in this interval. Let u and v be solutions to a second order homogenuous linear equation

$$y'' + a(t)y' + b(t)y = 0$$

a. Find a first order differential equation for W.

b. Show that if $a(t) \equiv 0$ then W is constant.

c. Show that u and v are independent if W is non-zero for a single t-value.

Hand in problem - to be handed in at the exercise session

Exercise 2.11 Consider two well-mixed tanks in series with given time-varying flow $q(t)$ and constant volumes V_1 and V_2 (see example in Lecture 1). Regard the inlet concentration as input and the tank concentrations as state variables. Determine the transition matrix as a function of $q(t)$, V_1, and V_2. Express the relationship between inlet and outlet concentrations by an integral equation.

Exercise 2.12 Compute $\Phi(t, \tau)$ for

$$A(t) = \begin{bmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{bmatrix}$$

Hint: Note that $A(t) = a_1(t)A_1 + a_2(t)A_2$ where A_1 and A_2 commute.