Discussion session 1

Strategic Form Games

September 4, 2014

Introduction

Game setup:

- strategic (normal form)

Ways to find equilibria:

- Iterated strict dominance
- Nash equilibrium

Strategic form games

Setup: three elements
Finite set of players $i \in \{1, \ldots, I\}$
Pure strategy space S_i
Payoff/utility functions $u_i(s)$ for each strategy profile $s = (s_1, .., s_I)$

Further notation
Player $i's$ opponents: s_{-i} (not necessarily enemy).
Finite game: $S = \Pi_i S_i$ is a finite space.
Strategy profile: $(s_i, s_{-i}) \in S$.

Strategic form game, example

Prisoner's dilemma:
C - confess and D - defect.

$$\begin{pmatrix} (C, C) & (C, D) \\ (D, C) & (D, D) \end{pmatrix} = \begin{pmatrix} (1,1) & (-1,2) \\ (2,-1) & (0,0) \end{pmatrix}$$

Strategic form game, zero-sum game

Two player zero-sum game means that

$$\sum_{i=1}^{2} u_i(s) = 0 \text{ for all } s$$

- sum of utilities is a constant, normalization.
- true opponents. Whatever one wins, the other one looses.

Strategies

- No communictaion
- Common knowledge: All have full information on structure of the game

Dominant strategies

Dominant strategy:
A strategy $s_i \in S_i$ is dominant for player i if

$$u_i(s_i, s_{-i}) \geq u_i(s_i^w, s_{-i}) \text{ for all } s_i^w \in S_i \text{ and for all } s_{-i} \in S_{-i}$$

Dominated strategies

Dominated strategies:
s_i is strictly dominated for player i if there exists $s_i^d \in S_i$ such that

$$u_i(s_i^d, s_{-i}) > u_i(s_i, s_{-i}) \text{ for all } s_{-i} \in S_{-i}.$$

weakly dominated if weak inequality holds for at least one $s_{-i} \in S_{-i}$.

Iterated strict dominance

Def. Dominant Strategy Equilibrium A strategy profile s^* is the dominant strategy equilibrium if for each player i, s_i^* is a dominant strategy.

Iterated strict dominance: Dependent on where you start, no.

Example, Iterated strict dominance

Find equilibria by iterated strict dominance:
Prisoner's dilemma:
C - confess and D - defect.

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Example, Iterated strict dominance

Find equilibria by iterated strict dominance:
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Strategies

Dominated strategy: when a strategy is strictly worse than another, it is dominated.

Mixed strategy: probability distribution over mixed pure strategies (next time).

Nash equilibrium

Many games not solvable by iterated strict dominance
$\rightarrow$ Nash eq. exists in a large class of games.

A **Nash equilibrium** *is a profile of strategies such that each player's strategy is an optimal response to the other player's strategies.*

Nash equilibrium

A pure strategy profile s^* is a Nash equilibrium if for all players i

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \text{ for all } s_i \in S_i.$$

strict if $>$.

Examples: Cournot equilibrium (quantities) and Bertrand equilibrium (price).