Game Theory - Discussion Session 2

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Mixed strategies

- Σ_i set of probability measures over S_i
- $\sigma_i \in \Sigma_i$ mixed strategy of player i, $\sigma \in \Sigma = \Pi_{i \in \mathcal{I}} \Sigma_i$
- $\sigma_{-i} \in \Sigma_i = \Pi_{j \neq i} \Sigma_j$
- **Players randomize independently**

Mixed strategies - Battle of sexes

Player 1 \Player 2	ballet	football
ballet	(2,1)	(0,0)
football	(0,0)	(1,2)

For player 1 to be indifferent between ballet and football, y prob that player 2 plays ballet

$$2y + 0(1-y) = 0y + 1(1-y)$$
$$y = 1/3$$

Two pure and one mixed NE.

Mixed strategies

Definition (Mixed Nash Equilibrium)

A mixed strategy profile σ^* is a mixed strategy Nash Equilibrium if for each player i,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*) \quad \forall \sigma_i \in \Sigma_i$$

Proposition

A mixed strategy profile σ^* is a mixed strategy Nash Equilibrium if and only if for each player i,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(s_i, \sigma_{-i}^*) \quad \forall s_i \in S_i$$

Mixed strategies

Proposition

For a finite strategic form game, $\sigma^* \in \Sigma$ is a NE if and only if for each player $i \in \mathcal{I}$, every pure strategy in the support of σ_i^* is a best response to σ_{-i}^*.

Mixed strategies

Definition (Strict Domination by Mixed Strategies)

An action s_i is strictly dominated if there exits a mixed strategy $\sigma_i' \in \Sigma_i$ such that $u_i(\sigma_i', s_{-i}) > u_i(s_i, s_{-i})$, for all $s_{-i} \in S_{-i}$.

Iterative Elimination of Strictly Dominated Strategies

- Let $S_i^0 = S_i$ and $\Sigma_i^0 = \Sigma_i$
- For each player $i \in \mathcal{I}$ and for each $n \geq 1$, we define S_i^n as

$$S_i^n = \{s_i \in S_i^{n-1} | \nexists \sigma_i \in \sigma_i^{n-1} \text{ such that } u_i(\sigma_i, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i} \in S_{-i}^{n-1}\}$$

- Independently mix over S_i^n to get Σ_i^n.
- Let $D_i^\infty = \cap_{n=1}^\infty S_i^n$
- D_i^∞ strategies of player i that survive iterated strict dominance

Belifs

A rational player would only play those strategies that are best responses to some beliefs he might have about his opponent. Leads to an infinite regress.

Definition (Belief)

A belief of player i about the other players' action is a probability measure $\sigma_{-i} \in \Pi_{j \neq i} \Sigma_i$.

Beliefs

Definition (Never-best response)

A pure strategy s_i is a never-best response if for all beliefs σ_{-i} there exists $\sigma_i \in \Sigma_i$ such that

$$u_i(\sigma_i, \sigma_{-i}) > u_i(s_i, \sigma_{-i})$$

Strictly dominated strategy $\Rightarrow$ Never best response (N.B $\Leftarrow$ not true (if the number of players are more than 2) see Lecture Slides))

Rationalizable Strategies

- Let $\tilde{S}_i^0 = S_i$ and $\tilde{\Sigma}_i^0 = \Sigma_i$
- For each player $i \in \mathcal{I}$ and for each $n \geq 1$, let

$$\tilde{S}_i^n = \{s_i \in \tilde{S}_i^{n-1} | \exists \sigma_{-i} \in \Pi_{j \neq i} \tilde{\Sigma}_j^{n-1} \text{ such that } u_i(s_i, \sigma_{-i} \geq u_i(s_i', \sigma_{-i}) \quad \forall s_i' \in \tilde{S}_i^{n-1}\}$$

- Independently mix over $\tilde{S}_i^n$ to get $\tilde{\Sigma}_i^n$.
- Let $R_i^\infty = \cap_{i=1}^\infty \tilde{S}_i^n$ be the set of rationalizable strategies for player i.

Let NE_i denode the set of **pure** strategies of player i used with positive probability in any mixed NE.

$$NE_i \subseteq R_i^\infty \subseteq D_i^\infty$$

Correlated Rationalizability

The player might believe that the other players' are in coalition, and the theirs actions are correlated. Then

$$R_i^{\infty} = D_i^{\infty}$$

See additional Lecture Notes.

Correlated Equilibrium

Definition (Correlated Equilibrium)

A correlated equilibrium of a finite game is a joint probability distribution $\pi \in \Delta(s)$ such that if R is a random variable distributed according to π then

$$\sum_{s_{-i} \in S_{-i}} \mathrm{Prob}(R = s | R_i = s_i)[u_i(s_i, s_{i-1}) - u_i(s_i, s_{-i})] \geq 0$$

for all players i, all $s_i \in \S_i$ such that $\mathrm{Prob}(R_i = s_i) > 0$, and all $t_i \in S_i$.