ADDITIONAL EXERCISES 1

Exercise 1

A group of fishermen concentrate their activity on a certain restricted area. The fishing returns in that area depend on the total hours worked by the whole group. Thus, letting h denote the number of hours worked by each fisherman $i = 1, 2, ..., n$ and $H = \sum_{i=1}^{n} h_i$ the total number of hours worked by the group, it is postulated that the hourly returns of each of them is given by a concave function of H, $\rho : \mathbb{R} \to \mathbb{R}$, with $\lim_{H\to\infty} \rho'(H) = 0$. On the other hand, each worker incurs an individual cost (or disutility) per hour worked, c, that is the same for all fishermen. Overall, the payoffs of each fisherman i are given by a function $u_i(\cdot)$ defined as follows:

$$u_i(h_1, ..., h_n) = \rho(H)h_i - c \cdot h_i.$$

Show that this game is a potential game.

Exercise 2

Consider the following congestion game, taken from Monderer and Shapley (1996a). There are four cities located around a lake with a single road joining them in the following clockwise order: A-B-C-D-A. There are two agents, 1 and 2, the first living in city A and the second in city B. Individual 1 wants to go to city C, whereas individual 2 wants to reach city D. The cost of travel depends on congestion, i.e., how many individuals (one or two) use the same segment of the road joining any two adjacent cities. Costs are additive across travel segments, with $c_\xi(k)$ denoting the cost of travelsegment $\xi \in \Xi = \{AB, BC, CD, DA\}$ when there are k individuals using it.

Model the situation as a game where each player i has two possible strategies: travel clockwise or travel counterclockwise. For any strategy profile $s = (s_1, s_2)$, define $h_\xi(s) \in \{0, 1, 2\}$ as the number of individuals using segment ξ. Moreover, define the function $\Phi : \mathcal{S} \to \mathbb{R}$ as follows:

$$\Phi(s) = \sum_{\xi\in\Xi} \sum_{k=0}^{h_\xi(s)} c_\xi(k)$$

where we make $c_\xi(0) = 0$. Prove that $\Phi(\cdot)$ is a potential for the game described.