Week 4

Exercise

Consider again the water tank model

$$\begin{cases} x_1(t+1) = 0.7x_1(t) + w_1(t) + u_1(t) \\ x_2(t+1) = 0.3x_1(t) + 0.7x_2(t) + w_2(t) + u_2(t) \\ x_3(t+1) = 0.3x_2(t) + 0.7x_3(t) + w_3(t) + u_3(t) \end{cases}$$

Assume that $\{w_i(t)\}_{i=1,2,3,t\geq 0}$ is a white stochastic process with zero mean and unitary variance.

We want to control the water tanks making use of Distributed Model Predictive Control. We want to minimize the usual quadratic cost

$$W = \sum_{\tau=0}^{N} \sum_{i=1}^{J} (x_i^2 + u_i^2)$$

Use the algorithm proposed during this week's lecture. At time t

1. Agent i measures its local state $x_i(t)$
2. Agents use the gradient algorithm (with K steps and step size γ) to compute
   - the price prediction sequences $\{p_i(t,\tau)\}_{\tau=0}^{N}$
   - the state prediction sequences $\{x_i(t,\tau)\}_{\tau=1}^{N}$
   - the control prediction sequences $\{u_i(t,\tau)\}_{\tau=0}^{N}$
3. Agent i applies the input $u_i(t,0)$

Experiment for different values of the parameters N, K and γ. Compare the results with classical LQ solutions.

OBS! Use $\boldsymbol{x}_i = Q_i^{-1}(\sum_j A_{ij}^* \boldsymbol{p}_j - \boldsymbol{p}_i)$