Week 3

Exercise 1

Consider the water tank model

$$\begin{cases} x_1(t+1) = 0.7x_1(t) + w_1(t) + u_1(t) \\ x_2(t+1) = 0.3x_1(t) + 0.7x_2(t) + w_2(t) + u_2(t) \\ x_3(t+1) = 0.3x_2(t) + 0.7x_3(t) + w_3(t) + u_3(t) \end{cases}$$

a) At stationarity and for constant disturbances $w_1 = 1$, $w_2 = 0$, $w_3 = 1$, determine u_1, u_2, u_3 that minimize the cost

$$J_d = \sum_{i=1,2,3} |x_i|^2 + |u_i|^2$$

b) At equilibrium, introduce prices to separate the three tanks by dual decomposition. What are the equilibrium prices? Can you give an interpretation?

c) Solve the differential equation for the saddle dynamics starting with $u_1 = u_2 = u_3 = 0$

d) Assume instead that w_1, w_2, w_3 are independent white noises with zero mean and unit variance. Compute the centralized state feedback control $u = -Lx$ that minimizes at stationarity the cost

$$J_a = \mathbb{E} \sum_{i=1,2,3} |x_i|^2 + |u_i|^2$$

e) Approximate the optimal feedback law with one where $u_i = \mu_i(x_i), i = 1, 2, 3$, and with another where $u_1 = \mu_1(x_1, x_2)$, $u_2 = \mu_2(x_1, x_2, x_3)$, $u_3 = \mu_3(x_2, x_3)$. Compare the performance with respect to the optimal controller.

f) Use the algorithm proposed in *Dynamic Dual Decomposition for Distribute Control* to estimate the optimal controller in the case $u_1 = \mu_1(x_1)$, $u_2 = \mu_2(x_2)$, $u_3 = \mu_3(x_3)$

Exercise 2

Consider a simple network with two sources and one destination. The two sources need to communicate through a link with capacity c, and can dynamically adapt their window size on the basis of loss probability. Denote by x_1 and x_2 the two flows and w_1 and w_2 the two windows sizes.

a) Following the paper *Internet Congestion Control*, implement the TCP Reno protocol with RED queue management model for fixed round-trip times $\tau_1 = 1$ and $\tau_2 = 2$, and link capacity $c = 1$. Assume for sake of simplicity $r = b$, and use the following parameters: $\underline{b} = \overline{b} = 0.6$

b) Simulate the (average) system

Under the distributed optimization viewpoint, assume that the two agents have utility functions

$$U_i(x_i) = \frac{\sqrt{2}}{\tau_i} \tan^{-1}\left(\frac{\tau_i x_i}{\sqrt{2}}\right), i = 1, 2$$

and the social benefit is

$$U_1(x_1) + U_2(x_2)$$

c) Formulate a market mechanism to achieve the social optimum at the equilibrium

d) Run the saddle point algorithm

Compare the results.