Exercises for Chapter 2

1. Why is the FOTD model

$$P(s) = \frac{K_p}{1+sT}\, e^{-sL}$$

so widely used in process control?

2. Consider the FOTD processes

$$P(s) = \frac{e^{-sL}}{1+sT},$$

The rise time T_r is defined as $T_r = P(0)/\max_t \dot{y}$, where y is the step response. Show that $T_r = T$. The settling time is defined as the time it takes the step response to come witin 2% of the steady state value, show that $T_{2\%} \approx L + 4T$.

3. It is useful to have simple estimates of the response times for the closed-loop system. The average residence time is a good measure that is easy to calculate. It is of course only reasonable if the closed-loop system is well damped.

Calculate T_{ar} for the transfer function between setpoint and process output when process $P(s)$ is controlled by the PID controller $C(s)$. It is assumed that $P(0) = K_P \neq 0$ and $k_i = K/T_i \neq 0$.

4. Consider a process with the transfer function

$$P(s) = \frac{(s+1)(s+10)}{(s+0.1)(s+2)(s+5)^2}$$

Develop an approximate low order model of the system and explore the frequency range where it is valid.

5. Heat propagation in a semi-infinite, thin metal rod can be modeled by the partial differential equation

$$\rho c \frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial^2 x}, \qquad \frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial^2 x}, \tag{1}$$

where ρ [kg/m^3] is the density, c [J/kgK] the specific heat capacity, k [W/mK] the thermal conductivity, and α [m^2/s] the thermal diffusivity. Assume that the control signal u is the temperature at the left end and that the output y is the temperature at a point $x = a$.

a) Show that the transfer function of the system is $P(s) = e^{-\sqrt{sT}}$, where the time constant is $T = a^2/\alpha$ [s]. Also show that all the derivatives of the impulse response vanishes at $t = 0$. Determine the average residence time of the system.

b) Show that the transfer function changes to $P(s) = \frac{1}{\cosh\left(\sqrt{sT}\right)}$, where the time constant is $T = a^2/\alpha$ [s] if the heat rod is insulated prefectly at $x = a$. Hint: The boundary condition for an isolated rod is that the gradient of the temperature is zero. Determine the average residence time of the system.

c) Compare the properties of the transfer functions in a) and b).

6. The dynamics of heat propagation in a liquid flowing with velocity v [m/s] in a heated pipe is governed by the partial differential equation

$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial x} + \alpha(T_w - T), \qquad \alpha = \frac{Dh}{\rho AC}$$

where D [m] is the circumference of the tube, h [W/m^2/°C] is the heat transfer coefficient, ρ [kg/m^3] is the density of the liquid, A [m^2] is the cross section of the pipe and C [Ws/kg/°C] is the specific heat capacity of the liquid.

Consider a pipe of length a [m] show that the transfer function from wall temperature to the outlet temperature is

$$P(s) = \frac{\alpha}{s+\alpha}\left(1 - e^{-\frac{(s+\alpha)a}{v}}\right)$$