Advanced PID Control - Interaction

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Interaction

K. J. Åström

Theme: When the wires get crossed

Introduction

- **To centralized or to decentralize?**
- The process control experience
  - A few important variables were controlled
  - The single loop paradigm: one sensor one actuator
  - Add loops to control more variables
  - Sometimes it was not obvious how to associate sensors and actuators - The pairing problem
- The state-feedback paradigm - centralized contol
- Complex systems - decentralize
- What happens when loops are interacting?
- Interaction measures
  - Bristol's relative gain array RGA
  - Singular values
- The pairing problem
- Decoupling: static, dynamic (different time scales), different physical mechanisms, mass balance, energy balance

Typical Process Control System

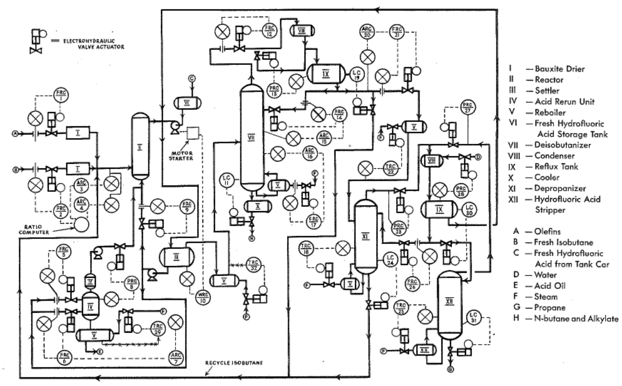

Figure 13–6. Automatic control system for Perco motor fuel alkylation process.

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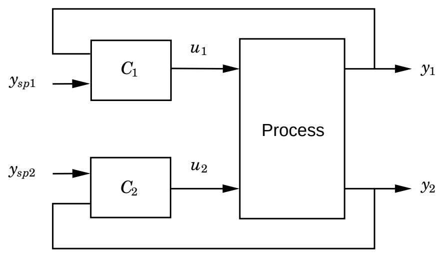
Interaction of Simple Loops

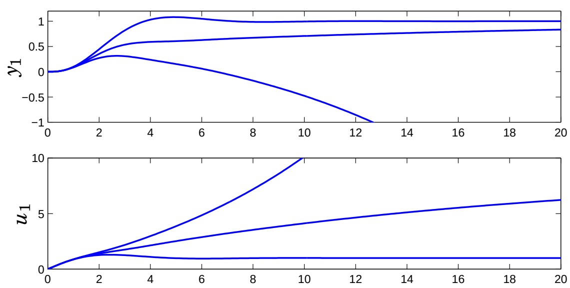


$$Y_1(s) = p_{11}(s)U_1(s) + p_{12}U_2(s)$$
$$Y_2(s) = p_{21}(s)U_1(s) + p_{22}U_2(s),$$

What happens when the controllers are tuned individually?

An Example

Controller C_1 is a PI controller with gains $k_1 = 1$, $k_i = 1$, and the C_2 is a proportional controller with gains $k_2 = 0$, 0.8, and 1.6.



The second controller has a major impact on the first loop!

Analysis

$$Y_1(s) = \frac{1}{(s+1)^2}U_1(s) + \frac{2}{(s+1)^2}U_2(s)$$
$$Y_2(s) = \frac{1}{(s+1)^2}U_1(s) + \frac{1}{(s+1)^2}U_2(s).$$

P-control of second loop $U_2(s) = -k_2 Y_2(s)$ gives

$$Y_1(s) = g_{11}^{cl}(s)U_1(s) = \frac{s^2+2s+1-k_2}{(s+1)^2(s^2+2s+1+k_2)}U_1(s).$$

The gain k_2 in the second loop has a significant effect on the dynamics in the first loop. The static gain

$$g_{11}^{cl}(0) = \frac{1-k_2}{1+k_2}.$$

changes from 1 to -1 as k_2 goes from 0 to ∞, it is zero for $k_2 = 1$.

A Bit of Intuition - Integral Control

- Diagonal dominance (Check the previous example)
- Integral control: Any stable SISO system can be controlled by an integrating controller provided that the zero frequency gain is not zero $P(0) > 0$. The characteristic polynomial for low integral gain is $s + k_i P(0)$
- The MIMO version for square systems. The characeristic equation and its low frequency approximation

$$\det(sI + P(s)K_i) \approx \det(sI + P(0)K_i)$$

where K_i is a diagonal matrix of integral gains.

- All eigenvalues of $P(0)K_i$ positive
- For positive integral gains $\det K_i > 0$ we must require $\det P(0) > 0$. (Niederlinski 1971)

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Bristol's Relative Gain

- Edgar H. Bristol On a new measure of interaction for multivariable process control IEEE TAC 11(1967) 133–135
- A simple way of measuring interaction based on static properties
- Idea: What is effect of control of one loop on the steady state gain of another loop?
- Idea: consider one loop when the other loop is under perfect control

$$\begin{aligned} Y_1(s) &= p_{11}(s)U_1(s) + p_{12}U_2(s) \\ 0 &= p_{21}(s)U_1(s) + p_{22}U_2(s). \end{aligned}$$

- Frequency dependent extensions

Bristol's Relative Gain

Consider the first loop $u_1 \to y_1$ when the second loop is in perfect control ($y_2 = 0$)

$$\begin{aligned} Y_1(s) &= p_{11}(s)U_1(s) + p_{12}U_2(s) \\ 0 &= p_{21}(s)U_1(s) + p_{22}U_2(s). \end{aligned}$$

Eliminating $U_2(s)$ from the first equation gives

$$Y_1(s) = \frac{p_{11}(s)p_{22}(s) - p_{12}(s)p_{21}(s)}{p_{22}(s)} U_1(s).$$

The ratio of the static gains of first loop when the second loop is open ($u_2 = 0$) and under perfect closed loop control ($y_2 = 0$).

$$r_{11} = \lambda = \frac{p_{11}(0)p_{22}(0)}{p_{11}(0)p_{22}(0) - p_{12}(0)p_{21}(0)}.$$

Bristol's Relative Gain ...

$$\begin{aligned} r_{11} &= \frac{p_{11}(0)p_{22}(0)}{p_{11}(0)p_{22}(0) - p_{12}(0)p_{21}(0)} = \lambda \\ r_{12} &= \frac{p_{12}(0)p_{21}(0)}{p_{11}(0)p_{22}(0) - p_{12}(0)p_{21}(0)} = 1 - \lambda \\ r_{21} &= \frac{p_{21}(0)p_{12}(0)}{p_{11}(0)p_{22}(0) - p_{12}(0)p_{21}(0)} = 1 - \lambda \\ r_{22} &= \frac{p_{22}(0)p_{11}(0)}{p_{11}(0)p_{22}(0) - p_{12}(0)p_{21}(0)} = \lambda \end{aligned}$$

The relative gain array

$$R = \begin{pmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{pmatrix}$$

The parameter λ is called *Bristol's interaction index*

Summary for 2×2 Systems

$\lambda = 1$ No interaction, decoupled design OK

$\lambda = 0$ Closed loop gain $u_1 \to y_1$ is zero Control y_1 by u_2 instead, decoupled design then OK

$0 < \lambda < 1$ Closed loop gain $u_1 \to y_1$ is larger than open loop gain. Interaction strongest for $\lambda = 1$

$\lambda > 1$ Closed loop gain $u_1 \to y_1$ is smaller than open loop gain. Interaction increases with increasing λ. Very difficult to control both loops independently if λ is very large.

$\lambda < 0$ The closed loop gain $u_1 \to y_1$ has different sign than the open loop gain. Opening or closing the second loop has dramatic effects. Use multivariable control.

Many Loops

Let $P(s)$ be an $n \times n$ matrix of transfer functions. The relative gain array is

$$R = P(0) \bullet P^{-T}(0) = P \bullet P^{-T} = P. * P^{-T}$$

The product is element by element product (Schur or Hadamard product). Properties

- $(A \bullet B)^T = A^T \bullet B^T$
- P diagonal gives $R = I$

Insight and use

- A measure of static interactions for square systems which tells how the gain in one loop is influenced by perfect feedback on all other loops
- Dimension free. Row and column sums are 1.
- Negative elements correspond to sign reversals due to feedback of other loops

Pairing

When designing complex systems loop by loop we must decide what measurements should be used as inputs for each controller. This is called the *pairing* problem. The choice can be governed by physics but the relative gain can also be used

Consider the previous example

$$P(0) = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, \qquad P^{-1}(0) = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$$

$$R = P(0) \bullet P^{-T}(0) = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix},$$

- Negative sign indicates the sign reversal found previously
- Better to use reverse pairing, i.e. let u_2 control y_1

$$R = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \bullet \begin{pmatrix} -1 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

Pairing ...

Consider

$$P(s) = \begin{pmatrix} \frac{1}{(s+1)^2} & \frac{2}{(s+1)^2} \\ \frac{1}{(s+1)^2} & \frac{1}{(s+1)^2} \end{pmatrix}$$

Introducing the feedback $u_1 = -k_2 y_2$ gives

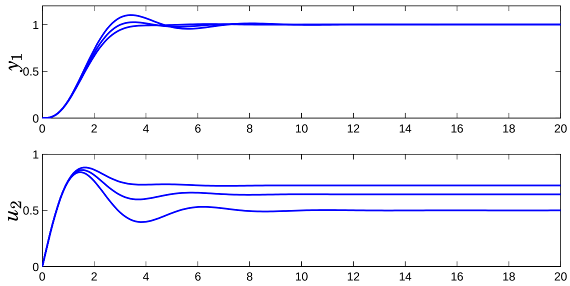
$$Y_1(s) = g_{12}^{cl}(s) U_2(s) = \frac{2s^2 + 4s + 2 + k_2}{(s+1)^2(s^2 + 2s + 1 + k_2)} U_2(s),$$

Zero frequency gain decreases from 2 to 1 when k_2 ranges from 0 to ∞, a significant improvement!

Discuss how dynamics changes with k_2!

Use rootlocus!

Step Responses with Reverse Pairing



- $U_2 = \left(1 + \frac{1}{s}\right)(Y_{sp1} - Y_1)$
- $u_1 = -k_2 y_2$ with $k_2 = 0$, 0.8, and 1.6.

Singular Values

Let A be an $k \times n$ matrix whose elements are complex variables. The singular value decompostion of the matrix is

$$A = U \Sigma V^*$$

where $*$ denotes transpose and complex conjugation, U and V are unitary matrices ($UU^* = I$ and $VV^* = I$ is. The matrix Σ is a $k \times n$ matrix such that $\Sigma_{ii} = \sigma_i$ and all other elements are zero. The elements σ_i are called singular values. The largest $\overline{\sigma} = \max_i \sigma_i$ and smallest $\underline{\sigma} = min_i \sigma$ singular values are of particular interest. The number $\bar{\sigma}/\underline{\sigma}$ is called the condition number. The singular values are the square roots of the eigenvalues of A^*A.

Example: A real 2×2 matrix can be written as

$$A = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -sin\theta_2 & \cos\theta_2 \end{pmatrix}$$

Singular Decomposition $A = U\Sigma V^*$

- The columns u_i of U represent the output directions
- The columns v_i of V represent the input directions
- We have $AV = U\Sigma$, or $Av_i = \sigma_i u_i$. An input in the direction v_i thus gives the output $\sigma_i u_i$, i.e. in the direction u_i
- Since the vectors u_i and v_i are of unit length the gain of A for the input u_i is σ_i
- The largest gain is $\bar{\sigma} = \max_i \sigma_i$
- There are efficient numerical algorithms `svd` in Matlab
- Singular values can be applied to nonsquare matrices
- A natural way to define gain for matrices A and transfer function matrices $G(s)$

$$\text{gain} = \max_v \frac{||Av||}{||v||} = \bar{\sigma}(A), \qquad \text{gain} = \max_\omega \bar{\sigma}(G(i\omega))$$

Interaction Analysis

Consider a system with the scaled zero frequency gain

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0.48 & 0.90 & -0.006 \\ 0.52 & 0.95 & 0.008 \\ 0.90 & -0.95 & 0.020 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

Relative gain array

$$R = \begin{pmatrix} 0.7100 & -0.1602 & 0.4501 \\ -0.3557 & 0.7925 & 0.5632 \\ 0.6456 & 0.3677 & -0.0133 \end{pmatrix}$$

Singular values: $\sigma_1 = 1.6183$, $\sigma_2 = 1.1434$ and $\sigma_3 = 0.0097$. Condition number $\kappa = 166$. Only two outputs can be controlled. What variables should be chosen?

Interaction Analysis

We have $y = U\,S\,V^T$. How to pick two input output pairs

$$SV^T = \begin{pmatrix} -0.088 & -1.616 & 0.010 \\ 1.142 & -0.062 & 0.018 \\ -0.000 & 0.000 & 0.010 \end{pmatrix} \qquad U = \begin{pmatrix} -0.571 & 0.377 & -0.729 \\ -0.604 & 0.409 & 0.684 \\ 0.556 & 0.831 & -0.007 \end{pmatrix}$$

The matrix SV^T shows that u_1 and u_2 are obvious choices of inputs. As far as the outputs are concerned. We have two choices y_1, y_3 or y_2, y_3 (angles between rows). Notice that y_1, y_2 is not a good choice because the corresponding rows of US are almost parallel. The singular values are

Selection $y_1, y_3 \leftarrow u_1, u_2$ Condition number $\kappa = 1.51$

$$R = \begin{pmatrix} 0.3602 & 0.6398 \\ 0.6398 & 0.3602 \end{pmatrix}$$

Selection $y_2, y_3 \leftarrow u_1, u_2$ Condition number $\kappa = 1.45$

$$R = \begin{pmatrix} 0.3662 & 0.6338 \\ 0.6338 & 0.3662 \end{pmatrix}$$

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Zeros of Multivariable Systems

Transmission zeros are values of s where the transmission of the signal e^{st} is blocked

$$Y(s) = P(s)\,v\,e^{st}, \qquad 0 = P(s)\,v$$

There is a nontrivial v only if the matrix $P(s)$ is singular. For a square system the zeros are the solutions to

$$\det P(s) = 0$$

and the *zero directions* are the corresponding right eigenvectors of $P(s)$.

Rosenbrock's Example

Process transfer function

$$P(s) = \begin{pmatrix} \dfrac{1}{s+1} & \dfrac{2}{s+3} \\ \dfrac{1}{s+1} & \dfrac{1}{s+1} \end{pmatrix}$$

Very benign subsystems. Relative gain array

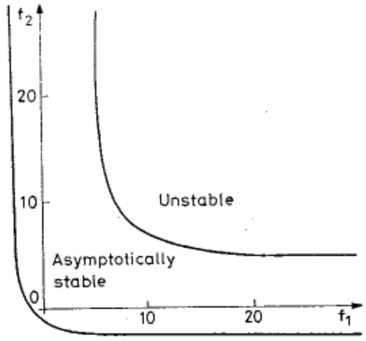
$$R = \begin{pmatrix} 1 & 2/3 \\ 1 & 1 \end{pmatrix} \bullet \begin{pmatrix} 3 & -3 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix},$$

The transmission zeros are given by

$$\det P(s) = \frac{1}{s+1}\left(\frac{1}{s+1} - \frac{2}{s+3}\right) = \frac{1-s}{(s+1)^2(s+3)} = 0.$$

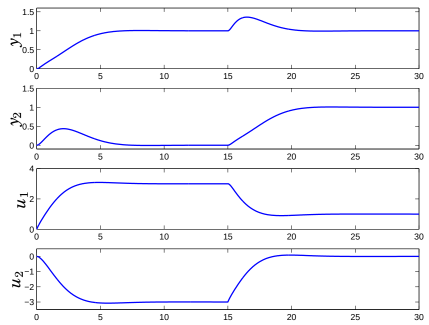
RHP zero at z=1, difficult to control the system with gain crossover frequencies larger than $\omega_{gc} = 0.5$.

Stability Region - P in Both Loops



Discuss commissioning and windup!

Rosenbrock's Example



PI controllers with $k_p = 2$ and $k_i = 2$ in both loops. Systems becomes unstable if gains are increased by a factor of 3.

Interactions Can be Beneficial

$$P(s) = \begin{pmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{pmatrix} = \begin{pmatrix} \dfrac{s-1}{(s+1)(s+2)} & \dfrac{s}{(s+1)(s+2)} \\ \dfrac{-6}{(s+1)(s+2)} & \dfrac{s-2}{(s+1)(s+2)} \end{pmatrix}.$$

The relative gain array

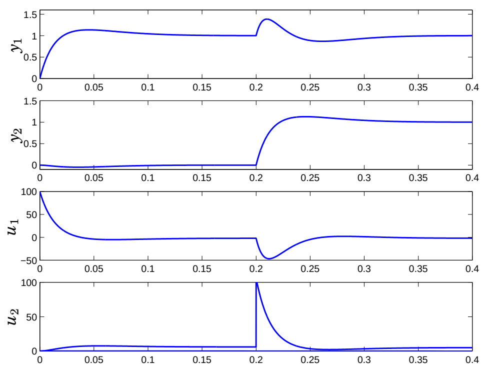
$$R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

Transmission zeros

$$\det P(s) = \frac{(s-1)(s-2)+6s}{(s+1)^2(s+2)^2} = \frac{s^2+4s+2}{(s+1)^2(s+2)^2}$$

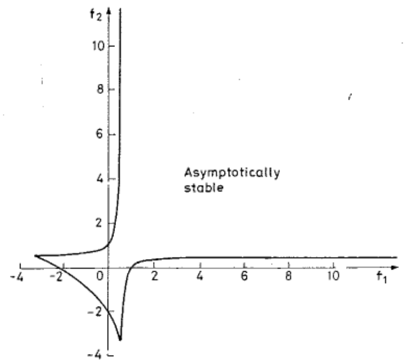
Difficult to control individual loops fast because of the zero at $s = 1$. Since there are no multivariable zeros in the RHP the multivariable system can easily be controlled fast but ths system is not robust to loop breaks.

Interactions Can be Beneficial ...



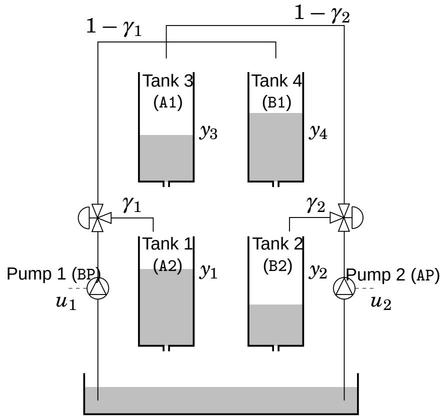
PI controllers with gains $k = 100$ and $k_i = 2000$ in both loops

Stability Region - P in Both Loops



Discuss commissioning and windup!

The Quadruple Tank



Transfer Function of Linearized Model

Transfer function from u_1, u_2 to y_1, y_2

$$P(s) = \begin{pmatrix} \dfrac{\gamma_1 c_1}{1+sT_1} & \dfrac{(1-\gamma_2)c_1}{(1+sT_1)(1+sT_3)} \\ \dfrac{(1-\gamma_1)c_2}{(1+sT_2)(1+sT_4)} & \dfrac{\gamma_2 c_2}{1+sT_2} \end{pmatrix}$$

Transmission zeros

$$\det P(s) = \frac{(1+sT_3)(1+sT_4) - \dfrac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}}{(1+sT_1)(1+sT_2)(1+sT_3)(1+sT_4)}$$

- No interaction of $\gamma_1 = \gamma_2 = 1$
- Minimum phase if $1 \le \gamma_1 + \gamma_2 \le 2$
- Nonminimum phase if $0 < \gamma_1 + \gamma_2 \le 1$.
- Intuition?

Relative Gain Array

Zero frequency gain matrix

$$P(0) = \begin{pmatrix} \gamma_1 c_1 & (1-\gamma_2)c_1 \\ (1-\gamma_1 c_2 & \gamma_2 c_2 \end{pmatrix}$$

The relative gain array

$$P(0) = \begin{pmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{pmatrix}$$

where

$$\lambda = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1}$$

- No interaction for $\gamma_1 = \gamma_2 = 1$
- Severe interaction if $\gamma_1 + \gamma_2 < 1$

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Theme: When the wires get crossed

Decoupling

Simple idea: Find a compensator so that the system appears to be without coupling.

Many versions

- Input decoupling $Q = PD$ or output decoupling $Q = DP$
- Conventional (Feedforward)
- Inverse (Feedback)
- Static

Important to consider windup, manual control and mode switches.

- Keep the decentralized philosophy

Feedforward (Conventional) Decoupling

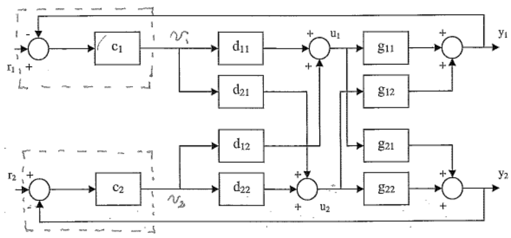
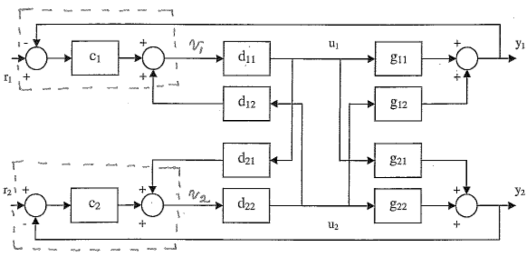


Figure 1 – Conventional decoupling control system of a TITO process

$$D = P^{-1}(s)Q(s) = \frac{1}{\det P(s)} \begin{pmatrix} p_{22}(s) & -p_{12}(s) \\ -p_{21}(s) & p_{11}(s) \end{pmatrix} Q(s)$$

Complicated decoupler, difficult to deal with anti-windup, manual control and mode changes (auto-tuning). Controller C_1 does not know what happens to u_1.

Feedback (Inverted) Decoupling

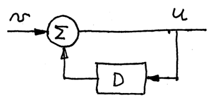


Simple decoupler, easy to deal with anti-windup, manual control and mode changes (auto-tuning) if $d_{11} = d_{22} = 1$. Why?

Inverted (Feedback) Decoupling

$$u = v + D_{fb}u$$
$$(I - D_{fb})u = v$$
$$u = (I - D_{fb})^{-1}v$$
$$Q = P(I - D_{fb})^{-1}$$
$$P = Q - QD_{fb}$$

Blockdiagram vector case



Easy to solve for D_{fb} also for systems with many inputs and outputs. Example 2×2, pick $Q = \mathrm{diag}(p_{11}, p_{22})$, why?

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 0 \\ 0 & p_{22} \end{pmatrix} - \begin{pmatrix} p_{11} & 0 \\ 0 & p_{22} \end{pmatrix} \begin{pmatrix} 0 & d_{12} \\ d_{21} & 0 \end{pmatrix}$$

Hence

$$d_{12} = -\frac{p_{12}}{p_{11}} \qquad d_{21} = -\frac{p_{21}}{p_{22}}$$

The Wood-Berry Distillation Column

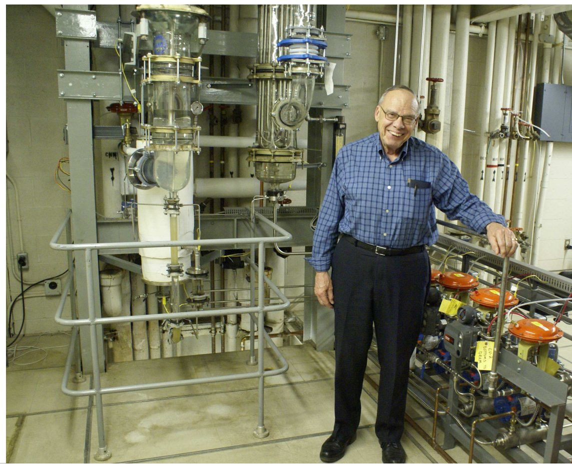
$$P(s) = \begin{pmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21.0s+1} \\ \dfrac{6.60e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{pmatrix}.$$

The decoupler is $d_{11} = d_{22} = 1$

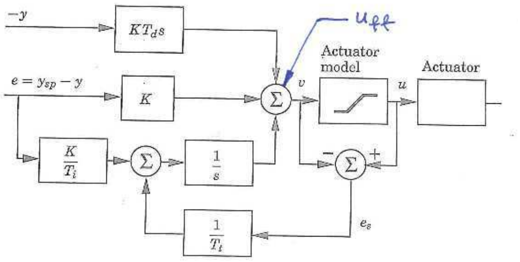
$$d_{11} = 1$$
$$d_{12} = -\frac{p_{12}}{p_{11}} = 0.68\,\frac{16.7s+1}{21.0s+1}e^{-2s}$$
$$d_{21} = -\frac{p_{21}}{p_{22}} = 0.34\,\frac{14.4s+1}{10.9s+1}e^{-4s}$$
$$d_{22} = 1$$

Easy to implement. What can go wrong?

Wood and His Column



Decoupling with Anti-windup



Properties of Inverted (Feedback) Decoupling

- Simple decoupler even for systems with many inputs and many outputs
- Easy to deal with anti-windup, manual control and other mode changes (auto-tuning)
- Decoupler may be noncausal (pure predictor). Try different pairing or add extra time delay in d_{ii}.
- Since it is a feedback coupling there may be instabilities

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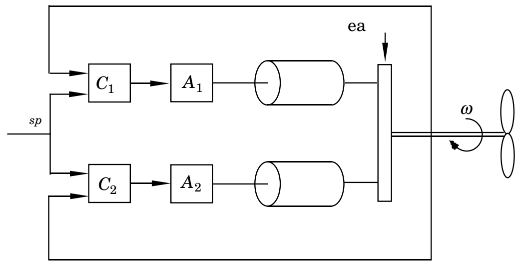
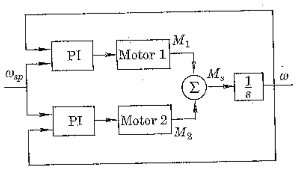
Systems with Parallel Actuation



- Motor drives for papermachines and rolling mills
- Trains with several motors or several coupled trains
- Electric cars with motors for each wheel
- Power systems, HVAC systems

A Prototype Example

$$J\frac{d\omega}{dt} + D\omega = M_1 + M_2 - M_L,$$



Proportional control

$$M_1 = M_{10} + K_1(\omega_{sp} - \omega)$$
$$M_2 = M_{20} + K_2(\omega_{sp} - \omega)$$

The proportional gains tell how the load is distributed

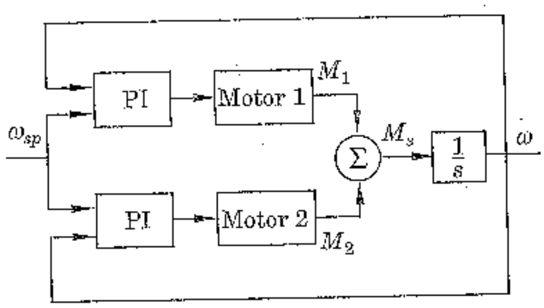
$$J\frac{d\omega}{dt} + (D + K_1 + K_2)\omega = M_{10} + M_{20} - M_L + (K_1 + K_2)\omega_{sp}.$$

A first order system with time constant $T = J/(D + K_1 + K_2)$

Discuss response speed, damping and steady state

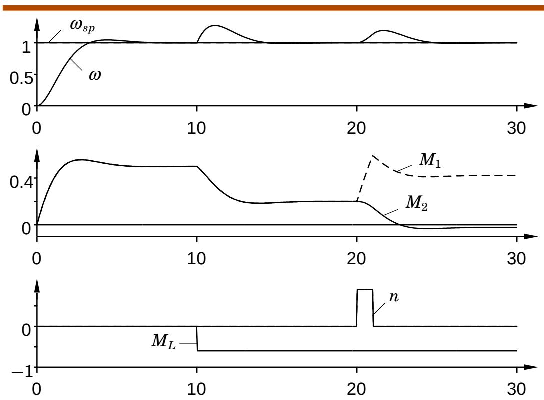
$$\omega = \omega_0 = \frac{K_1 + K_2}{D + K_1 + K_2}\omega_{sp} + \frac{M_{10} + M_{20} - M_L}{D + K_1 + K_2}.$$

Integral Action?



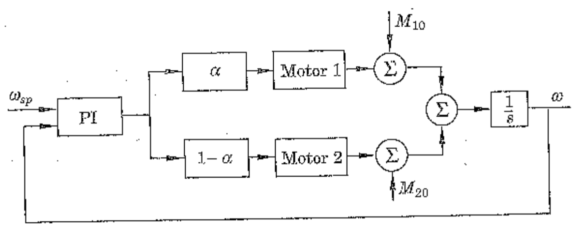
Prototype for lack of reachability and observability!

Integral Action

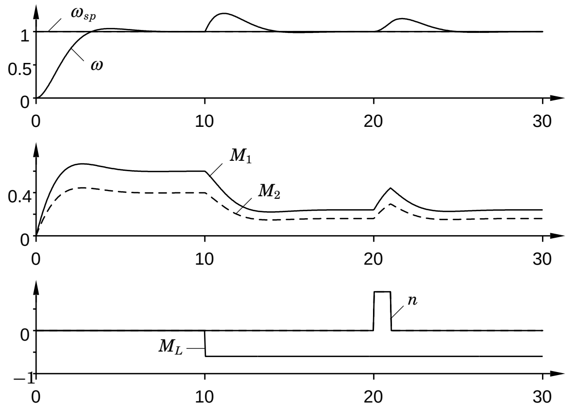


Notice that M_1 is driving and M_2 is breaking for $t > 22$

Better Integral Action



Better Integral Action?

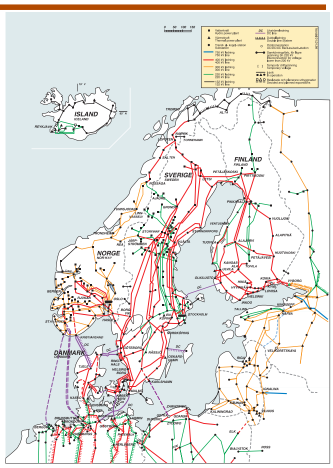


Electric Cars

One motor for each wheel

Power Systems - Massive Parallellism

- Edison’s experience
  - Two generators with governors having integral action
- Many generators supply power to the net.
  - Frequency control
  - Voltage control
- Isochronous governors (integral action) and governors with speed-drop (no integral action)



Turbine Governors

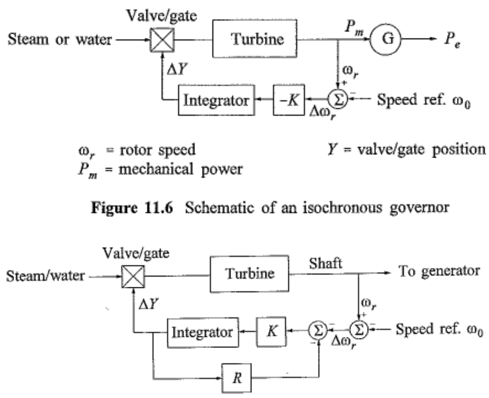


Figure 11.6 Schematic of an isochronous governor

Figure 11.8 Governor with steady-state feedback

Load Sharing - Through Proportional Action

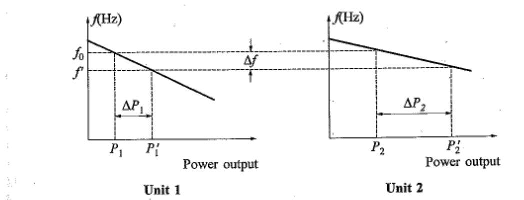


Figure 11.11 Load sharing by parallel units with drooping governor characteristics

Integral action through central action

Kundur: The isochronous governors cannot be used when there are two or more units connected to the same system since each governor would have to have precisely the same speed setting. Otherwise, they would fight each other, each trying to control system frequency to its own setting.

Interaction

K. J. Åström

1. Introduction
2. An Example
3. RGA and Singular Values
4. Multivariable zeros
5. Decoupling
6. Parallel Systems
7. More Examples
8. Summary

Theme: When the wires get crossed

A Gallery of Systems

- Rosenbrock 1 och 2
- Distillation columns
  - Wood-Berry
- Tyreus system
- Quadruple tank
- Basis weight and moisture control of paper machine
- Concentration and level control
- Boiler control
- Shell standard control problem

Paper Machine Control

Decoupling through physics

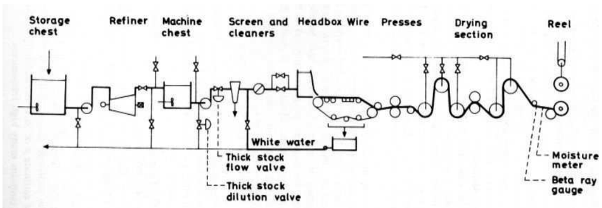


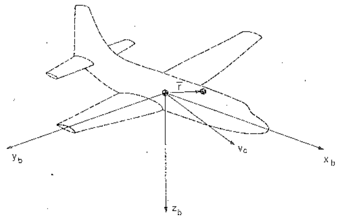
Fig. 6.9. Simplified diagram of a kraft paper machine.

- Primary quality: Basis weight and moisture
- Head box control: Pressure and flow rate

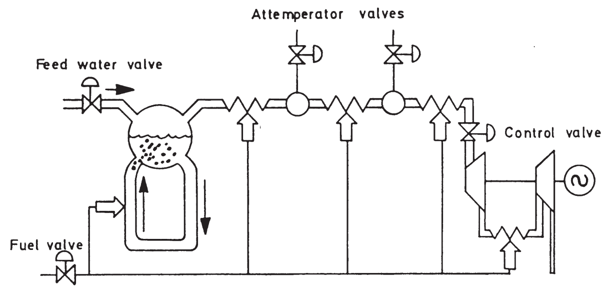
Flight Control

Decoupling through physics

- Longitudinal
- Lateral



Boiler Control



Schematic diagram of the boiler-turbine unit.

The Shell Standard Control Problem

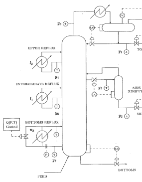


Figure A.1: Diagram of a Heavy Oil Fractionator (Shell Control Problem).

$$\frac{Ke^{-\theta s}}{\tau s+1}$$

Units for θ, τ are in minutes.

	TOP DRAW	SIDE DRAW	BOTTOMS REFLUX DUTY	INTER. REFLUX DUTY	UPPER REFLUX DUTY
TOP END POINT	$K=4.05$ $\tau=50$ $\theta=27$	$K=1.77$ $\tau=60$ $\theta=28$	$K=5.88$ $\tau=50$ $\theta=27$	$K=1.20$ $\tau=45$ $\theta=27$	$K=1.44$ $\tau=40$ $\theta=27$
SIDE END POINT	$K=5.39$ $\tau=50$ $\theta=18$	$K=5.72$ $\tau=60$ $\theta=14$	$K=6.90$ $\tau=40$ $\theta=15$	$K=1.52$ $\tau=25$ $\theta=15$	$K=1.83$ $\tau=20$ $\theta=15$
TOP TEMP	$K=3.66$ $\tau=9$ $\theta=2$	$K=1.65$ $\tau=30$ $\theta=20$	$K=5.53$ $\tau=40$ $\theta=2$	$K=1.16$ $\tau=11$ $\theta=0$	$K=1.27$ $\tau=6$ $\theta=0$
UPPER REFLUX TEMP	$K=5.92$ $\tau=12$ $\theta=11$	$K=2.54$ $\tau=27$ $\theta=12$	$K=8.10$ $\tau=20$ $\theta=2$	$K=1.73$ $\tau=5$ $\theta=0$	$K=1.79$ $\tau=19$ $\theta=0$
SIDE DRAW TEMP	$K=4.13$ $\tau=8$ $\theta=5$	$K=2.38$ $\tau=19$ $\theta=7$	$K=6.23$ $\tau=10$ $\theta=2$	$K=1.31$ $\tau=2$ $\theta=0$	$K=1.26$ $\tau=22$ $\theta=0$
INTER. REFLUX TEMP	$K=4.06$ $\tau=13$ $\theta=8$	$K=4.18$ $\tau=33$ $\theta=4$	$K=6.53$ $\tau=9$ $\theta=1$	$K=1.19$ $\tau=19$ $\theta=0$	$K=1.17$ $\tau=24$ $\theta=0$
BOTTOMS REFLUX TEMP	$K=4.38$ $\tau=33$ $\theta=20$	$K=4.42$ $\tau=44$ $\theta=22$	$K=7.20$ $\tau=19$ $\theta=0$	$K=1.14$ $\tau=27$ $\theta=0$	$K=1.26$ $\tau=32$ $\theta=0$

Interaction Analysis

$$P(s)=\begin{pmatrix}
\frac{4.05e^{-27s}}{1+50s} & \frac{1.77e^{-28s}}{1+60s} & \frac{5.88e^{-27s}}{1+50s} & \frac{1.20e^{-27s}}{1+45s} & \frac{1.44e^{-27s}}{1+40s}\\
\frac{5.39e^{-18s}}{1+50s} & \frac{5.72e^{-14s}}{1+60s} & \frac{6.90e^{-15s}}{1+40s} & \frac{1.52e^{-15s}}{1+25s} & \frac{1.83e^{-15s}}{1+20s}\\
\frac{3.66e^{-2s}}{1+9s} & \frac{1.65e^{-20s}}{1+30s} & \frac{5.53e^{-2s}}{1+40s} & \frac{1.16}{1+11s} & \frac{1.27}{1+6s}\\
\frac{5.92e^{-11s}}{1+12s} & \frac{2.54e^{-12s}}{1+27s} & \frac{8.10e^{-2s}}{1+20s} & \frac{1.73}{1+5s} & \frac{1.79}{1+19s}\\
\frac{4.13e^{-5s}}{1+8s} & \frac{2.38e^{-7s}}{1+19s} & \frac{6.23e^{-2s}}{1+10s} & \frac{1.31}{1+2s} & \frac{1.26}{1+22s}\\
\frac{4.06e^{-8s}}{1+13s} & \frac{4.18e^{-4s}}{1+33s} & \frac{6.53e^{-s}}{1+9s} & \frac{1.19}{1+19s} & \frac{1.17}{1+24s}\\
\frac{4.38e^{-20s}}{1+33s} & \frac{4.42e^{-22s}}{1+44s} & \frac{7.20}{1+19s} & \frac{1.14}{1+27s} & \frac{1.26}{1+32s}
\end{pmatrix}$$

Controls: top draw, side draw, bottoms reflux duty. inter reflux duty, upper reflux duty. Outputs are: top end draw, side end point top temperature, upper reflux temperature, side draw temperature, inter reflux temperature, bottoms reflux temperature.

Interaction Analysis ...

Singular values

$$\sigma_1=23.7,\quad \sigma_2=3.23,\quad \sigma_3=0.97,\quad \sigma_4=0.23,\quad \sigma_4=0.15$$

Condition number is $\kappa=162$

$$SV^T=\begin{pmatrix}
-12.0740 & -8.8747 & -17.6193 & -3.5152 & -3.8122\\
0.6437 & -2.9896 & 0.9498 & 0.3149 & 0.2407\\
0.6436 & 0.0103 & -0.5729 & 0.2125 & 0.3896\\
-0.1065 & 0.0030 & 0.0241 & 0.0198 & 0.2005\\
0.0308 & -0.0048 & 0.0028 & -0.1402 & 0.0300\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

All variables cannot be controlled, choose y_1 and y_2 and u_2 and u_3, condition number $\kappa=5.34$

$$R=\begin{pmatrix}-0.57 & 1.57\\ 1.57 & -0.57\end{pmatrix}$$

Significant interaction, use multivariable control!

Interaction

K. J. Åström

Theme: When the wires get crossed

Summary

- All real systems are coupled
- Relative gain array and singular values give insight
- Never forget process redesign
- Multivariable zeros and zero directions
- Why decouple

  Simple system.
  SISO design, tuning and operation can be used
  What is lost?
- Multivariable design

  Dont forget windup and operational aspects (tuning, manual control, ...)
- Parallel systems

  One integrator only!

To Learn More

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