Readings and exercises

limit cycles, existence and uniqueness, Lyapunov functions, regions of attraction

Reading assignment

Khalil Chapter 1–3.1, (not 2.7), 4–4.6

Comments to text [Khalil]

Chapter 2.6

The main topic is about existance of periodic orbits for planar systems and the most important subjects are the Poincaré-Bendixson Criterion and the Bendixson Criterion. Lemma 2.3 and Corollary 2.1 can also be used to rule out the existence of limit cycles.

Chapter 3.1

The topics in Chapter 3.1 concerns (local) existence and uniqueness of solutions to differential equations, where the Lipschitz condition plays a major role.

Chapter 4.1-...

This chapter is devoted to the study of equilibrium points of nonlinear autonomous systems. The main issues are the following.

- The use of Lyapunov functions and invariant sets for proofs of asymptotic stability (LaSalle's theorem). Consider in particular its application to the pendulum, Example 4.4.
- The use of Lyapunov functions for proofs of instability (Chetaev's theorem).
- Stability analysis by linearization.

Chapter 4.5–...

Here the main new ingredient is time-variations. The terms *uniform asymptotic stability* and *exponential stability* are introduced to specify time dependence in the stability behaviour. Main results:

- Uniform asymptotic stability can be proved from time-invariant bounds on the Lyapunov function. This is Theorem 4.9.
- For linear systems, uniform asymptotic stability is equivalent to exponential stability (Theorem 4.10).

- Exponential stability implies input-output stability
- The center manifold theorem, which we will cover later, on is a powerful complement to stability analysis by linearization. Chapter 8 should therefore be read in connection to Section 4.3.

Exercises on Chapters 2, 3.1, & 4

Exercise 1.1 = Kha. 2.20 (3,5)

Exercise 1.2 Kha 3.1 (1)

Exercise 1.3 = Kha. 3.2 (4)

Exercise 1.4 (a) = Kha. 4.9 (Radial boundedness)
(b) What is the region of attraction for the origin in **(a)**?
You may use simulation tools like e.g., *pplane* (see
`http://math.rice.edu/~dfield/`)

Exercise 1.5 = Kha 4.10 (Krasovskii's method, can also see pp.84–84 [Slotine&Li])

Exercise 1.6 = Kha. 4.38 (time-varying Lyap fcn)

Exercise 1.7 = Kha 4.39 (time-varying Lyap fcn)

Exercise 1.8 = Kha. 4.36 (uniform asymptotic stability, (or not))

Exercise 1.9 = Kha. 4.19 (Robot manipulator)

Exercise 1.10 = Kha. 4.37 (1),(2) (Quadratic Lyap functions)

Exercise 1.11 Find a quadratic simultaneous Lyapunov function for the linear time-varying system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 1 & -8 \\ 6 & -13 \end{bmatrix} x + \begin{bmatrix} 8 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} \delta_1(t) & 0 \\ 0 & \delta_2(t) \end{bmatrix} y \\ y &= \begin{bmatrix} 1 & -1 \\ -11 & 16 \end{bmatrix} x \end{aligned}$$

where $|\delta_k(t)| \leq 1$, $k = 1, 2$,
using e.g., Matlab's LMI-lab and the IQCbeta toolbox (see lecture slides).