Governors and Stability Theory

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Governors and Stability Theory

1. Introduction
2. Maxwell and Routh
3. Vyshnegradskii, Stodola and Hurwitz
4. The Routh-Hurwitz Theorem
5. Lyapunov
6. More recent results
7. Summary

Theme: Controlling the speed of mechanical machines and encountering instability.

Lectures

		1940	1960	2000
1	Introduction			
2	Governors	|	|	|
3	Process Control	|	|	|
4	Aerospace	|	|	|
5	Feedback Amplifiers	|	|	|
6	Harry Nyquist	|	|	|
		|		
7	Servomechanisms	←	|	|
8	The Second Phase	←	←	|
9	The Third Phase	←	←	←
10	The Swedish Scene			
11	The Lund Scene			

The Power of Feedback

Feedback has some amazing properties, it can

- make a system insensitive to disturbances and component variations,
- make good systems from bad components,
- stabilize an unstable system,
- create desired behavior, for example linear behavior from nonlinear components.

The major drawbacks are that

- feedback can cause instabilities
- sensor noise is fed into the system

Governors

- Industrialization created the need for controlling the speed: clocks, telescopes, windmills (Mead 1787), steam engines (Watt 1788), hydroelectric turbines.
- Creative creation of devices (governors) mostly mechanical and interactions between engineers and mathematicians
- Instability emerged as a central problem and basic theory emerged
  - Simplified models, P, I and D control action
  - Linearization, normalization, time constants
  - Stability theory (Rout-Hurwitz)
  - Construction practice (design rules)
  - Textbooks Tolle 1905, Zhukovsky 1909
- Industrialization (mechanical, hydraulic)
  - Strong hardware limitations, ≈ 75000 governors in 1868
  - many companies XXX

Introduction

- Typical problems
  - Keeping constant speed of machines and instruments
  - Telescope drives, clocks Huygens Hookes late 1600
  - Wind mills, Lee 1745, Mead 1787
  - Steam engines: Watt's flyball governor
  - Water turbines Stodola
- Beginning of PID Control
- Integration of sensing control and actuation
- Problems
  - Gravity
  - Friction
  - Hunting
- Industrialization ≈ 75000 governors in 1868: Siemens, Woodward, ABB, ...
- Interaction between theory and practice

The Centrifugal Governor < 1900

- Mead late 1600
- Watt 1798
- Integrating action Perier ≈ 1790
- Derivative action (Inertial action) Hicks 1840
- Siemens 1866
- Stodola 1894

Theory

- Maxwell 1868
- Vyshnegradskii 1876
- Hermite 1856
- Routh 1877
- Hurwitz 1895

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Maxwell and Routh

James Clarke Maxwell 1831-1879

- Born Glenlair South of Glasgow
- University of Edinburgh 1847
- Graduate Trinity College Cambridge 1854
- Professor Natural Philosophy Aberdeen 1856
- Professor Physics and Astronomy Kings College London 1860
- Professor Experimental Physics Cavendish Laboratory 1871
- A Treatise on Electricity and Magnetism 1873
- Broad interests
  - Color vision
  - Saturns rings
  - Control 1868 paper

Maxwells 1868 Paper

- Stability Concept
- Simple Mathematical Models
- Importance of integral action
- Linearization
- Stability is an algebraic problem
- Criteria for first, second and third order systems
- Posed stability problem as a competition

Taxonomy - Stability Concept

It will be seen that the motion of a machine with its governor consist in general of a uniform motion, combined with a disturbance which may be expressed as the sum of several component motions. These components may be of four different kind:

1. The disturbance may continually increase.
2. It may continually diminish.
3. It may be an oscillation of continually increasing amplitude.
4. It may be an oscillation of continually decreasing amplitude.

The first and third cases are evidently inconsistent with the stability of the motion: and the second and fourth alone are admissible in a good governor. *This condition is mathematically equivalent to the condition that all the possible roots, and all the possible parts of the impossible roots of a certain equation shall be negative.*

Proportional, Derivative and Integral Action

" Most governors depend on the centrifugal force of a piece connected with a shaft of the machine. When the velocity increases, this force increases, and either increases the pressure of the piece against a surface or moves the piece, and so acts on a break or a valve.

In one class of regulators of machinery, which we may call *moderators*, the resistance is increased by a quantity depending on the velocity.

But if the part acted on by centrifugal force, instead of acting directly on the machine, sets in motion a contrivance which continually increases the resistance as long as the velocity is above its normal value, and reverses its action when the velocity is below that value, the governor will bring the velocity to the same normal value whatever variation (within the working limits of the machine) be made in the driving-power or the resistance."

Proof

First order and second order

$$x + a_1 = 0, \qquad x^2 + a_1 x + a_2 = 0, \qquad a_1 > 0, a_2 > 0$$

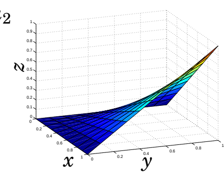
Third order

$$(x+a)(x^2+bx+c) = x^3 + (a+b)x^2 + (ab+c)x + ac$$
$$= x^3 + a_1 x^2 + a_2 x + a_3$$

Hence: $a_1 = a + b, \quad a_2 = ab + c, \quad a_3 = ac$

What does $a, b, c > 0$ imply for a_1, a_2 and a_3?

- ab-plane, $c = 0$, $a_3 = 0$
- bc-plane, $a = 0$, $a_3 = 0$
- ac-plane, $b = 0$, $a_1 = a$, $a_2 = c$, $a_3 = a_1 a_2$



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Vyshnegradskii Stodola and Hermite

Ivan Alexeyevich Vyshnegradskii 1831-1895

- Born Vyshny Volochyok 1831
- Tver Theological Seminary
- Taught maths and mechanics at St. Petersburg military ducational institutions
- Director St. Petersburg Technological Institute 1875
- Mémoire sur la théorie générale des régulateurs. Academie des Sciences 1876
- Member of the Council of Ministers of Public Instruction 1884
- Member of the State Council 1886
- Minister of Finance 1888-1892
  Reduce budget deficit, interference with private railways, nationalisation of least profitable railways, support of domestic industry and preparation of monetary reform
  Increasing taxes export and railway lines
- Honorary Member of St. Petersburg Academy of Sciences (1888)

Vyshnegradskii 1876

More detailed analysis than Maxwell, strong engineering flavor. Sur la théorie générale des régulateurs. Compe. Rend. Acad. Sci. Paris Vol 83, 1876, 318–321. Expanded versions in Russin, German and French 1977-1879.

$$J\frac{d\omega}{dt} = k(\varphi_0 - \varphi) - T_d, \qquad \text{steam engine}$$

$$m\ell\frac{d^2\varphi}{dt^2} + b\frac{d\varphi}{dt} + \frac{m(g\sin\varphi - \ell n^2\omega^2\sin 2\varphi)}{2} = k(\varphi_0 - \varphi) - T \qquad \text{governor}$$

Closed loop system

$$\frac{d^3\omega}{dt^3} + \frac{b}{m\ell}\frac{d^2\omega}{dt^2} + \frac{g\sin^2\varphi_0}{\ell\cos\varphi_0}\frac{d\omega}{dt} + \frac{2kg\sin\varphi_0}{\ell\omega_0 J}\omega = \frac{2kg\sin\varphi_0}{\ell\omega_0 J}\varphi_0 - \frac{2g\sin\varphi_0}{\ell\omega_0 J}T$$

Stablity condition

$$\frac{bJ}{m} > \frac{2k\cos\varphi_0}{\omega_0\sin\varphi_0}$$

Normalization and Stability Diagram

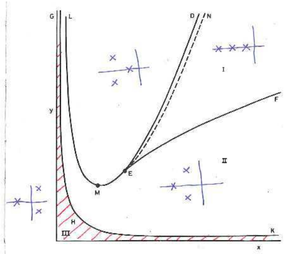
Linearized closed loop system

$$\frac{d^3\omega}{dt^3} + \frac{b}{m\ell}\frac{d^2\omega}{dt^2} + \frac{g\sin^2\varphi_0}{\ell\cos\varphi_0}\frac{d\omega}{dt} + \frac{2kg\sin\varphi_0}{\ell\omega_0 J}\omega = \frac{2kg\sin\varphi_0}{\ell\omega_0 J}\varphi_0 - \frac{2g\sin\varphi_0}{\ell\omega_0 J}T$$

Normalization

$$z^3 + az^2 + bz + 1 = 0$$

Distinguish different pole configurations (early pole placement)
Stability diagram



Design Rules

Linearized model

$$\frac{d^3\omega}{dt^3} + \frac{b}{m\ell}\frac{d^2\omega}{dt^2} + \frac{g\sin^2\varphi_0}{\ell\cos\varphi_0}\frac{d\omega}{dt} + \frac{2kg\sin\varphi_0}{\ell\omega_0 J}\omega = \frac{2kg\sin\varphi_0}{\ell\omega_0 J}\varphi_0 - \frac{2g\sin\varphi_0}{\ell\omega_0 J}T$$

Stablity condition

$$\frac{bJ}{m} > \frac{2k\cos\varphi_0}{\omega_0\sin\varphi_0} = \frac{2F}{\omega_0}$$

Design rules

1. Increase of the mass of balls harmful for stability
2. Decrease of friction in governor harmful for stability
3. Decrease of moment of inertia of flywheel is harmful for stability
4. Decrease of nonuniformity F harmful for stability

Integral action was not emphasized

Aurel Boleslaw Stodola 1859-1942

- Studies and industrial work in Prague, Berlin, Paris, Zurich
- Professor Polytechnikum (now ETH) Zurich 1892
- Regulation of high pressure water turbines
- Modeling
  Followed Vyshnegradskii used his results for 3rd order
  Normalize with imensionless quantities (percentage deviations)
  Introduce time constants and use them for model reduction
- 7th order system with help from Hurwitz
- Publication of 1894 Hurwitz criterion without proof
- Used derivative action (inertia governor). No integral action but proportional gain could be increased significantly.
- Engineering Philosophy: Gedanken zu einer Weltanschaung von Standpunkte des Ingenieurs, Springer, Berlin 1931

Max Tolle 1905, 1909, 1921

A. A. Andronov: Of compilatory and pedagogical nature.

Nikolai Egorovich Zhukovskii 1847-1921

- Father of modern aero- and hydrodynamics
- Graduated from Moscow University in 1868
- Professor at the Imperial Technical School 1972
- Established the world's first Aerodynamic Institute in 1904 in Kachino near Moscow
- Head TsAGI (Central Aero HydroDynamics Institute)
- Built the first wind tunnel in Russia
- Kutta-Joukokowski wing profile (rounded leading edge sharp trail)
- The water hammer effect (Joukowsky equation)
- Zhukovskii. Theory of the regulation of the motion of machines, 1909,...,1923 Similar to Tolle's 1905 book. Remained in circulation for a long time. Revised edition by Kotelnikov and Smirnov 1933.

Engineering impact

Bennet: Knowledge of the (stability) conditions did not, for many years spread beyond the scientific circle of Maxwell, Thomson, Airy, Siemens and others. Ordinary engineers remained unaware of this theoretical work. Maxwell had been concerned with special governors (instruments) and not with the typical engine governor. Little practical implication, no real models, but good insight for example of integral action.

Vyshnegradskii and Stodola were strongly engineering oriented. Hurwitz criterion was first published by Stodola 1894 without proof in an engineering journal, see Bissell..

Vyshnegradskii's idea of using time constants and dimension-free variables very useful.

Results inspired textbooks by M. Tolle 1905 and Die Regelung der Zhoukovsky 1909.

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1. J. C. Maxwell. On Governors. Proc. Roy. Soc. 16 (1868) 270-283.
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3. L. S. Pontryagin. Ordinary Differential Equations. Addison Wesley 1962. 213–220 (readable but minor errors).
4. A. A. Andronov, I. A. Vyshnegradskii and his role in the creation of automatic control theory (100 years from the day of publication of I. A. Vyshnegradskii's work on automatic control theory. Avtomatika i Telemekhanika 1978 (4) 5–17.
5. C. C. Bissell Control engineer and much more: aspects of the work of Aurel Stodola. Measurement and Control Vol. 22, 1989, 117–122.
6. O. Mayr Origines
7. Stuart Bennet A History of Control Engineering 1800-1930, Peter Peregrinius

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The Stability Problem

- Roots of an algebraic equation in LHP
- Routh was given the problem by Maxwell
- Hurwitz was given the problem by Stodola
- Hermite had already solved the problem
- What should be taught?
- Should you learn the proofs?
- Gantmacher Matrix Theory Vol 2 Chapter 15 The Problem of Routh-Hurwitz and Related Questions 172–248

Routh, Hurwitx and Hermite



Edward John Routh 1831-1907

- Born in Quebeck
- Senior Wrangler Mathematical Tripos Cambridge (Maxwell second!)
- Maxwells problem 3rd order included in Routh's book
- Adams Prize competition: The criterion of Dynamic Stability. 1875-77. Maxwell Judge
- Routh won with the 1876 paper: A treatise on the stability of the given state of motion.
- Good teacher

Adolf Hurwitz 1859-1919

- Born Hildesheim realgymnasium
- Universities
  - Munich (Klein)
  - Berlin (Kummer,Weierstrass, Kronecker)
  - Leipzig 1980 (Klein)
  - PhD for Klein on elliptic modular functions 1981
  - Göttigen
- Professor Albertus Universität Köningsberg 1984 (Hilbert, Minkowski)
- Professor after Frobenius at Eidgenössige Polytechnikum Zuric (ETH) 1892
- Riemann surfaces, algebraic curves, fix point theorems, quarternions

Charles Hermite 1822-1901

- College de Nancy
- École Polytechnique 1842 (leg problems left after a year)
- Independent studies (Jacobi, Bertrand, Liouville)
- École Polytechnique 1948
- Académie des Sciences 1956
- Professor professor of mathematics École Polytechnique and Faculty of Sciences of Paris 1869 –
- École Normale Supérieure 1862-1873
- Grand officer of the Légion d'honneur 1892
- Hermitian: matrices, operators, splines and wavelets

Routh's Array

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} \cdots + a_n = g(x^2) + xh(x^2)$$

Form the polynomials

$$f_1(x) = \begin{cases} g(x) & \text{if } n \text{ even} \\ h(x) & \text{if } n \text{ odd} \end{cases} \qquad f_2(x) = \begin{cases} h(x) & \text{if } n \text{ odd} \\ g(x) & \text{if } n \text{ even} \end{cases}$$

$$f_{k+1}(x) = f_{k-1}(x) \operatorname{mod} f_k(x)$$

Leading coefficient of $f_1(x)$ is a_0, degrees are decreasing.

Routh's Theorem: The number of real roos in RHP ($\operatorname{Re} x > 0$) equal to sign changes of leading coefficient of $f_k(s)$.

Routh array:

$$\begin{matrix} a_0 & a_2 & a_4 & a_6 & \dots \\ a_1 & a_3 & a_5 & a_7 & \dots \end{matrix}$$

The Hurwitz Determinant

$$f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_n$$

The polynomial has all its roots in the LHP if all subdeterminants of

$$A = \begin{pmatrix} a_1 & a_3 & a_5 & \ldots \\ a_0 & a_2 & a_4 & \ldots \\ 0 & a_1 & a_3 & \ldots \\ 0 & a_0 & a_1 & \ldots \\ \vdots & & & \end{pmatrix}$$

are positive.

Compare Routh's array.

Charles Hermite 1822-1901

Interested in roots with positive imaginary part: Paper 1854: On the number of roots of an algebraic equation between two limits.

Complex coefficients! Key idea: Use of quadratic forms

$$f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_n$$

$$\frac{f(x)f(y) - f(-x)f(-y)}{x+y} = \sum_{i,j=1}^{n} h_{ij}x^{n-i}y^{n-j}$$

$$\begin{pmatrix} x^{n-1} & \vdots & 1 \end{pmatrix} H \begin{pmatrix} y^{n-1} & \vdots & 1 \end{pmatrix}$$

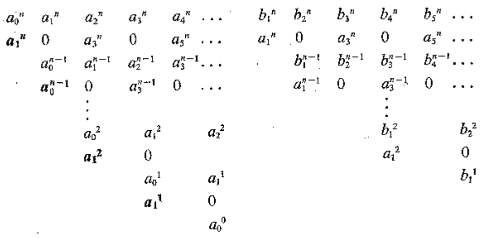
Example n=3

$$H = \begin{pmatrix} a_0a_1 & 0 & a_0a_3 \\ 0 & a_1a_2 - a_0a_3 & 0 \\ a_0a_3 & 0 & a_2a_3 \end{pmatrix} \Rightarrow \begin{pmatrix} a_0a_1 & 0 & a_0a_3 \\ 0 & a_1a_2 - a_0a_3 & 0 \\ 0 & 0 & \frac{a_3(a_1a_2 - a_0a_3)}{a_1} \end{pmatrix}$$

Relations to Quadratic Forms

The Routh array is related to quadratic forms.

$$I = \frac{1}{2\pi i}\int_{i\infty}^{i\infty} \frac{b(s)b(-s)}{a(s)s(-s)}ds = \sum_k \frac{(b_1^k)^3}{2a_0^k a_1^k}$$



K. J Åström Introduction to Stochastic Control Theory. Academic Press 1970, Dover 2006. Also proof of Routh-Hurwitz.

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Alexandr Mihailovich Lyapunov 1857-1918

- University of St. Petersburg: Euler, Chebyshev, Markov, Lyapunov, Yakubovich, Fradkov, Megretski, Andrey Ghulchak
- St. Petersburg University, Maters thesis: On the stability of ellipsoidal forms of equilibrium of rotating fluids 1884
- Privat-Dozent Mechanics Kharkov 1885
- Doctoral dissertation 1892 The General Problem of the Stability of Motion. Opponent: Joukowski. Back to Kharkov.
- Academician University of St. Petersburg 1902, Chebychev's chair.
- To Odessa on doctors order in 1917, wife tuberculosis. Lyapunov in poor shape, bad eyesight.
- Family estate burned during Russian revolution including a great library built by his father and grand father. Suicide 1918.

Some Results

- Stability concept; stability, asymptotic stability
- Lyapunov's first method - Linearize
- Lyapunov's Second Method - Find asymptotic stability without using an explicit solution

$$\frac{dx}{dt} = f(x), \quad \frac{dV}{dt} = \frac{\partial V}{\partial x}f(x) < 0$$

- Linear systems $A^TP + PA = -Q$
- Lyapunov exponents

Connections Routh-Hurwitz-Lyapunov

Routh-Hurwitz matrix manipulations

Use Hermite's matrix as a Lyapunov function with A on companion form

$$\begin{pmatrix} a_0a_1 & 0 & a_0a_3 & \ldots \\ 0 & -a_0a_3 + a_1a_2 & 0 & \ldots \\ a_0a_3 & 0 & a_0a_5 - a_1a_4 + a_2a_3 & \ldots \\ \vdots & & & \\ \ldots & \ldots & \ldots & a_na_{n-1} \end{pmatrix}$$

$$V = x^THx$$

Impact of Lyapunov 1

- Very strong interest of Lyapunov's work in Russia and USSR
- Nikolai Egorovich Zhukovskii (Joukowsky) Theory of regulation of the motion of machines, 1909. Equivalent of Tolle's book. Remained in circulation for a long time revised edition 1933 Kotelnikov and Smirnov.
- Zhukovskii First Aeronatical Institute 1902
- Automation and Remote Control Commission, 1934
- Institute of Automation and Remote Control 1939 - A premier research institute of the Academy of Sciences. Now Institute of Control Sciences (ICS)
  - A. A. Andronov Started influential seminar on nonlinear dynamics and control at the institute 1944, Lurje, Aizerman, Tsypkin, Alexander B. Kurzhanskiy
  - Andronov, Chaikin de Witt
- Kazan Aviation Institute
  - Chetaev 1946, Malkin 1952

Impact of Lyapunov 2

- Solomon Lefschetz at Princeton translated Krylov and Bogolyubovs book 1943
- Cover-to-cover translation of Automatica I Telemechanica began 1957, a consequence of Sputnik.
- Lefschetz quote to USAF 1959:“ A point of particular importance here is the development and application of Lyapunov's theory of stability and use of methods and techniques that have been almost completely ignored outside the USSR”
- Special efforts to propagate ideas by Lefschetz and LaSalle in US, Cartwright and Parks in England
- La Salle Princeton-RIAS-Brown University. La Salle Lefschets Stability by Liapunovs Direct Method 1961
- Impact of IFAC
- Kalman and Bertram ASME paper 1960

Kalman (RIAS) Bertram (IBM) Paper

Control System Analysis and Design via the “Second Method of Lyapunov”. ASME Journal of Basic Engineering 1960, pp 304-499.

The original work of Lyapunov in French is difficult to read because of the obsolescent mathematical terminology. An authoritative survey is given by Massera. The recent monograph by Hahn (in German) covers existing results and includes a few applications. There is now an English translation of the the well-known book by Malkin. This book, allegedly addressed to engineers, is primarily a detailed and rigorous mathematical treatment of classical problems. ...

Over the past 10 years, a great deal has been published about the “second method” in the journals Avtomatica i Telemechanika and Prikladnaya matematika i Mekhanika (English translation).

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More Recent Development

- Nyquist (Bell Labs)
- Discrete time systems Schur-Cohn-Jury (Columbia)-Tsypkin (ICS Moscow)
- The Lurje problem (
- Aizerman's conjecture 1949 (ICS)
- Popov 1961 Lesson from LaSalle
- Kalman-Yakubowitz (St. Petersburg) Lemma
- Small gain theorem and passivity Zames (Mc Gill) and Sandberg (Bell Labs)
- Passivity
- Kharitonov

Later Contributions

- Matrix polynomials
- LaSalle (Princeton-RIAS-Brown)
- Linear Matrix inequalities (LMI)
- Kalman-Yakubovich-Popov
- Polynomials and numerical calculations
  Polynomials great for low order systems!
  Be aware of numerics for high order systems!

$$x^n = 0, \qquad x^n = \epsilon, \qquad |x| = \epsilon^{1/n}$$
$$n = 20, \qquad \epsilon = 10^{-10}, \qquad |x| = \sqrt{0.1} \approx 0.3$$

- Kharitonov's theorem for interval polynomials

Kharitonov and Interval Polynomials

Let $a_k^- \le a_k \le a_k^+$ and

$$f(x) = a_0 x^n + a_1 x_{n-1} + a_2 x^{n-2} + a_3 x^{n-3} + \cdots + a_n$$

Introduce

$$f_1(x) = a_0^+ x^n + a_1^+ x^{n-1} + a_2^- x^{n-2} + a_3^- x^{n-3} + a_4^+ x^{n-4} + \cdots$$
$$f_2(x) = a_0^- x^n + a_1^- x_{n-1} + a_2^+ x^{n-2} + a_3^+ x^{n-3} + a_4^- x^{n-4} + \cdots$$
$$f_3(x) = a_0^+ x^n + a_1^- x_{n-1} + a_2^- x^{n-2} + a_3^+ x^{n-3} + a_4^+ x^{n-4} + \cdots$$
$$f_4(x) = a_0^- x^n + a_1^+ x_{n-1} + a_2^+ x^{n-2} + a_3^- x^{n-3} + a_4^- x^{n-4} + \cdots$$

Theorem: The polynomial $f(x)$ is stable for all coefficients in the intervals if $F_1(x)$, $f_2(x)$, $f_3(x)$ and $f_4(x)$ are stable.

- Useful for robustness analysis
- Discrete systems, other regions and Anders Rantzer

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Summary

- Development of governors driven by industrial needs
- An impressive development
- The early engineers formulated problems and asked mathematicians for help
- Insight and understanding
- Embryo of a design theory
  Physical modeling, normalization, linearization
  Stability theory
  Design methods
  Textbooks Tolle 1905 Joukovski 1909
- Research institutes
- Industrialization
  Patents and companies (Siemens)
  Implementation: Clever mechanical devices attached to the process
- Lyapunove theory is key element of control eduction
- Routh-Hurwitz useful for low order systems.

Lessons Learned

- It pays to know the literature and to interact with other researchers
  Neither Routh nor Hurwitz knew that Hermite had solved the problem. Imagine what could have happened if Routh-Hurwitz and Lyapunov had met!
- Do not forget the customers and how the results are packaged and presented. Compare the impact of Maxwell and Stodola!
- Do not forget to feed industrial problemss to reserach
- Stability test - an eigenvalue computation
- Useful to have algebraic tests for low order systems
- Polynomials and numerical calculations
  Polynomials great for low order systems!
  Numerically illconditioned for high order systems

$$x^n = 0, \qquad x^n = \epsilon, \qquad |x| = \epsilon^{1/n}$$
$$n = 20, \qquad \epsilon = 10^{-10}, \qquad |x| = \sqrt{0.1} \approx 0.3$$

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A. A. Andronov, I. A. Vyshnegradskii and his role in the creation of automatic control theory (100 years from the day of publication of I. A. Vyshnegradskii's work on automatic control theory. Avtomatika i Telemekhanika 1978 (4) 5–17.

R. Bellman and R. Kalaba Selected papers on mathematical trends in control theory. Dover 1964. e.g. Maxwell, Hurwitz, Nyquist etc.

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